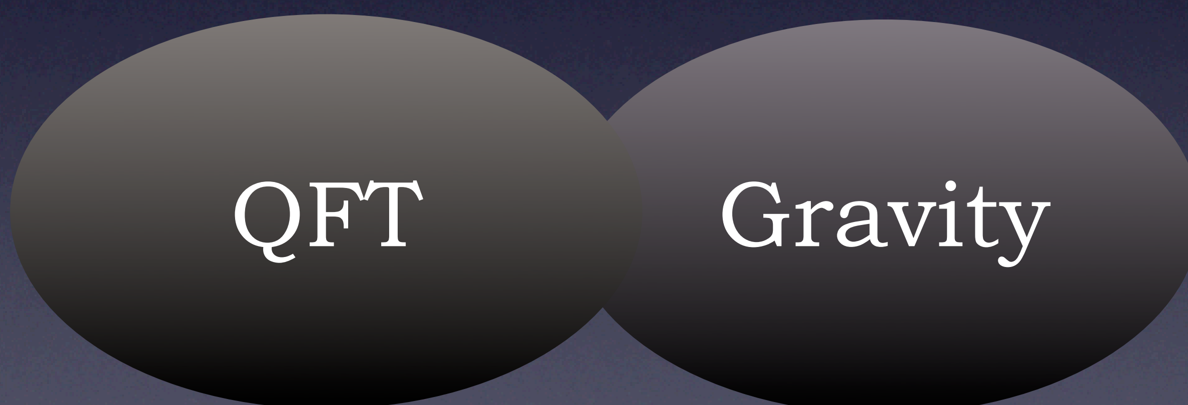
Introduction to String Theory

Universität Frankfurt, HGS-HIRe Powerweek, July 16-21st 2012



by Matthias Kaminski
(University of Washington)

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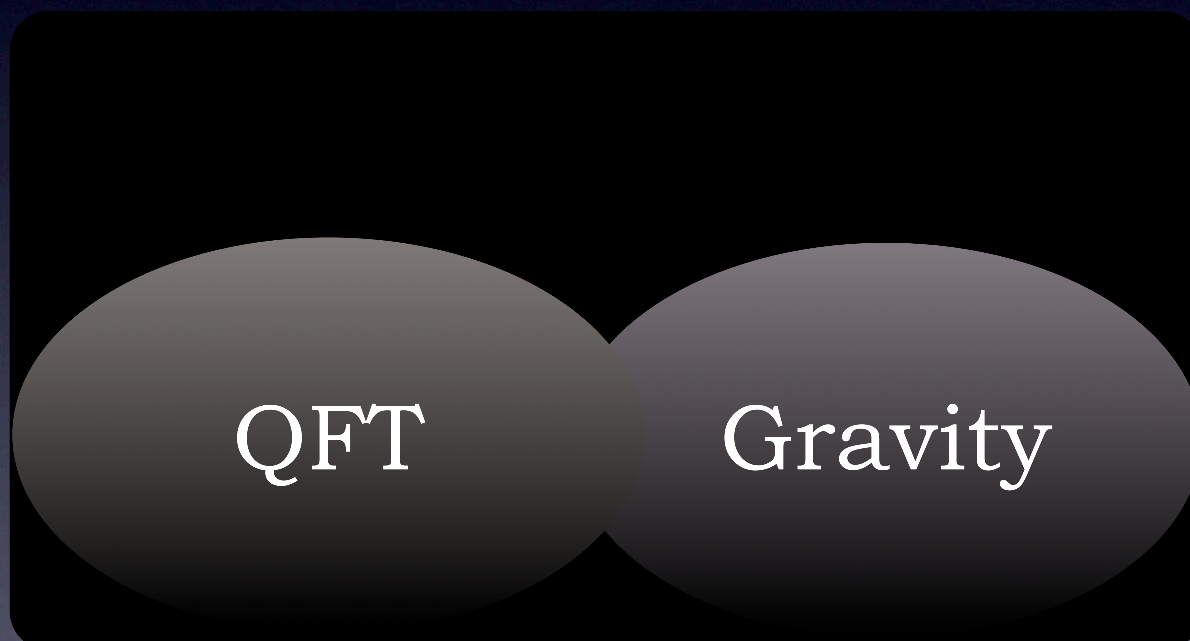

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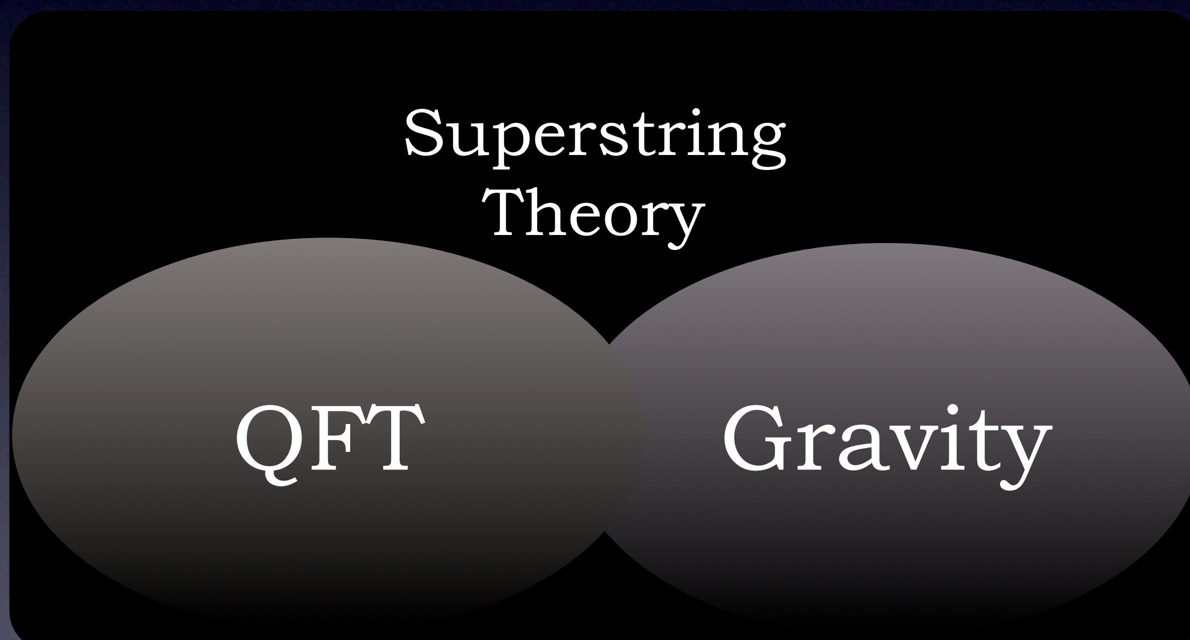

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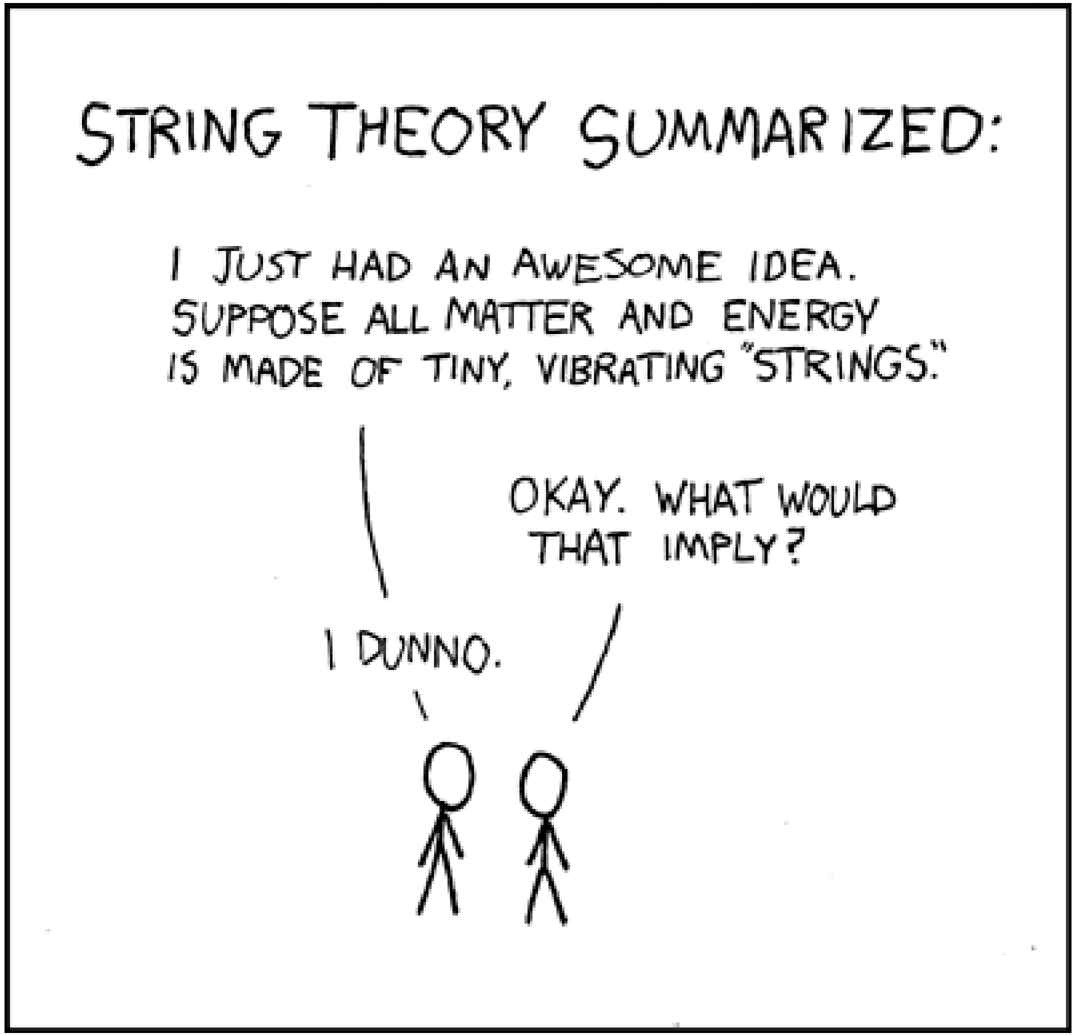
Lecture I: Overview of String Theory & Applications

Overview


STRING THEORY SUMMARIZED:
I JUST HAD AN AWESOME IDEA. SUPPOSE ALL MATTER AND ENERGY IS MADE OF TINY, VIBRATING "STRINGS."
OKAY. WHAT WOULD THAT IMPLY?
I DUNNO.

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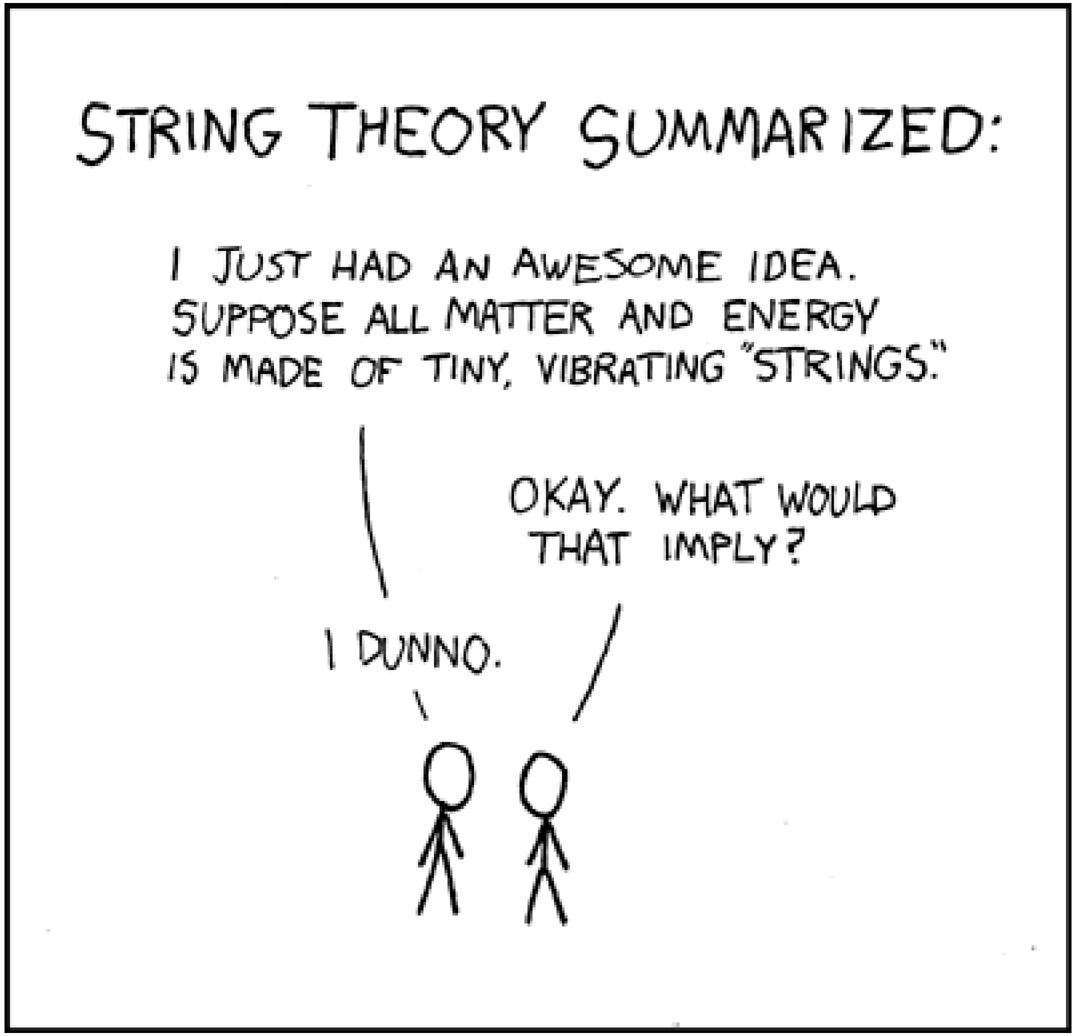
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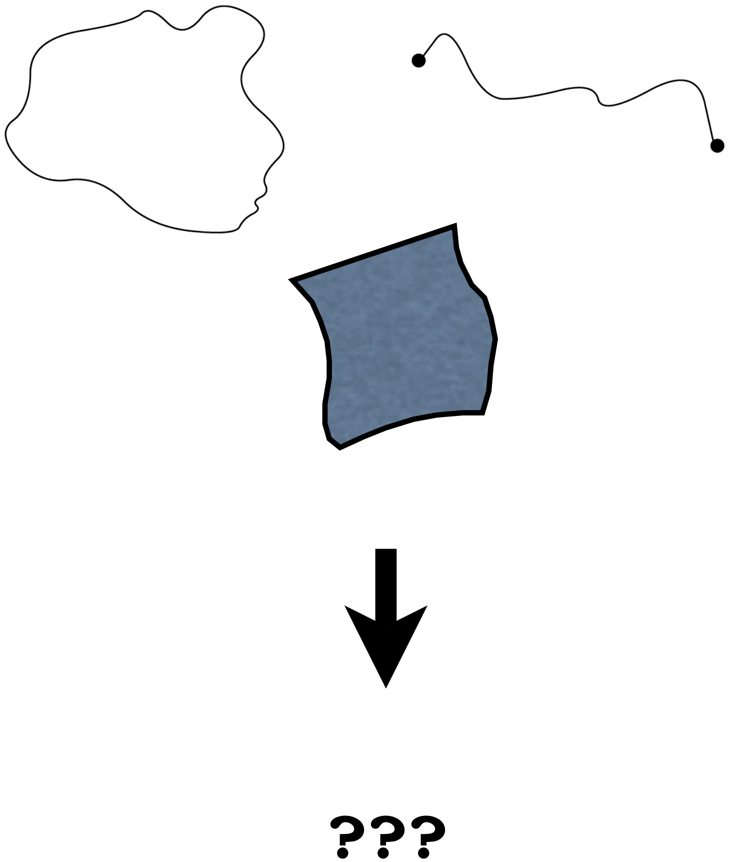

???

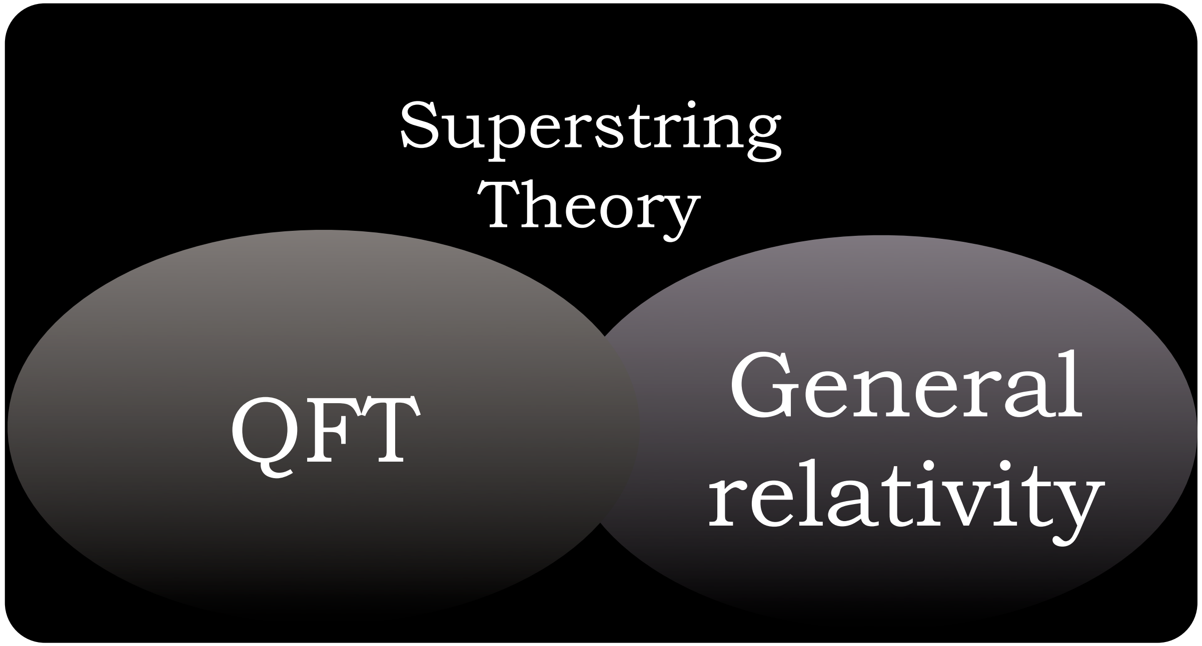
Overview

What is string theory?

Overview

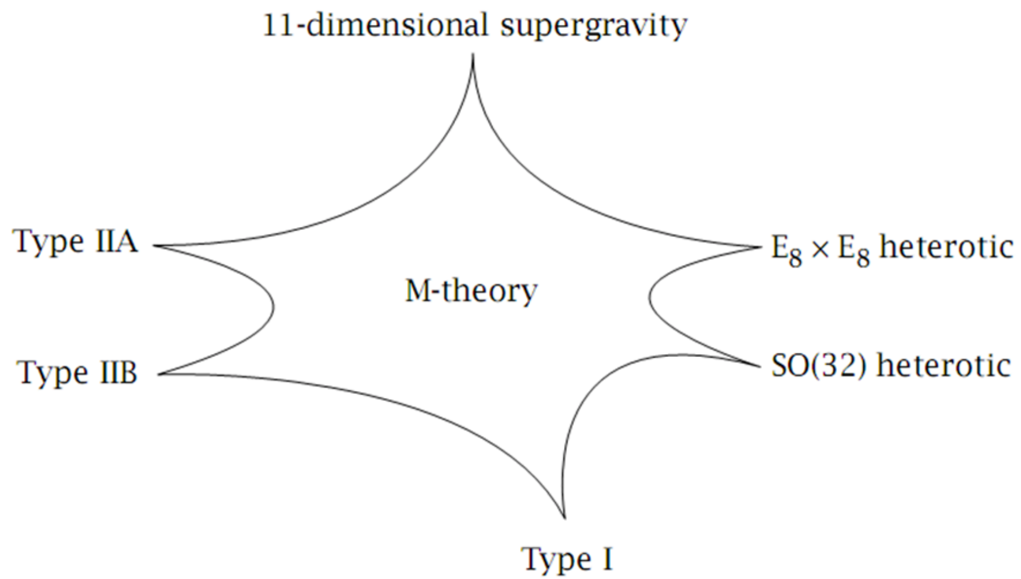
What is string theory?

String theory is an active research framework in particle physics that attempts to reconcile quantum mechanics and general relativity (invariance principles). It is a contender for a theory of everything, a self-contained mathematical model that describes all fundamental forces and forms of matter, a theory of strings, branes, and their dynamics.



Overview

Structure of string theory - limits & dualities



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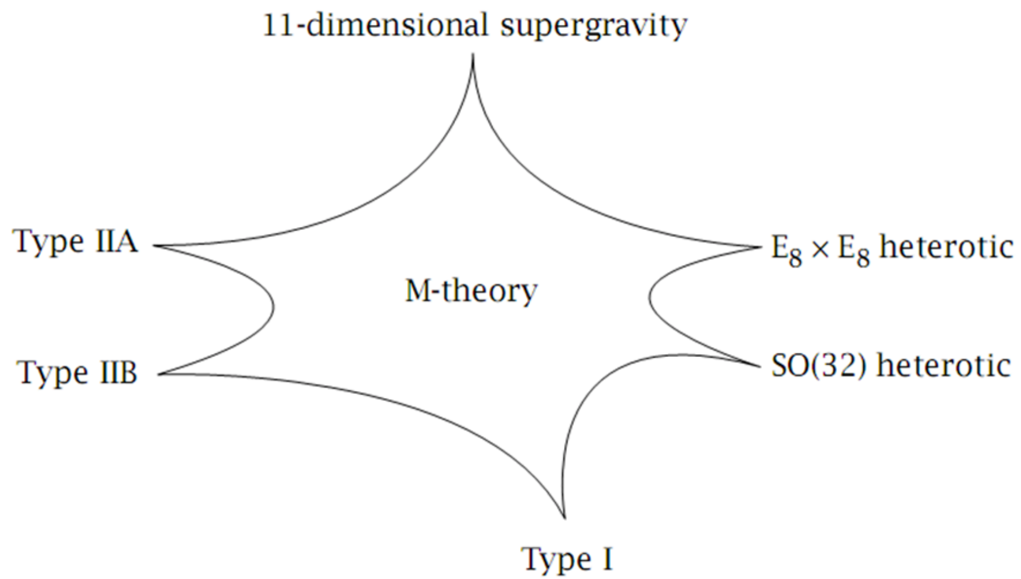

- supergravity theories are low-energy limits of these string theories (e.g. type IIA supergravity is derived from type IIA string theory by taking the massless tree-level approximation)

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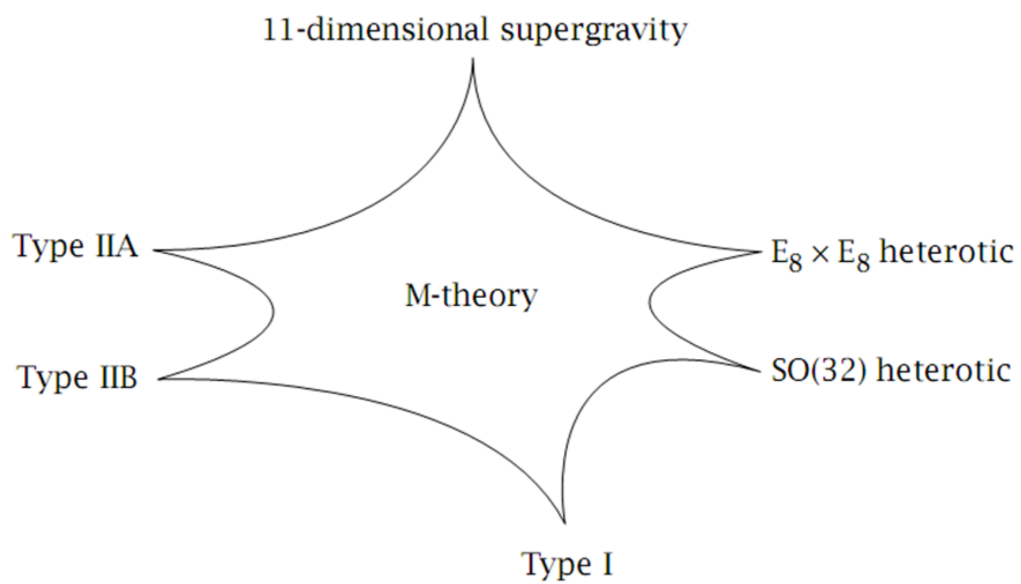

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- supergravity (SUGRA) emerges when making SUSY local

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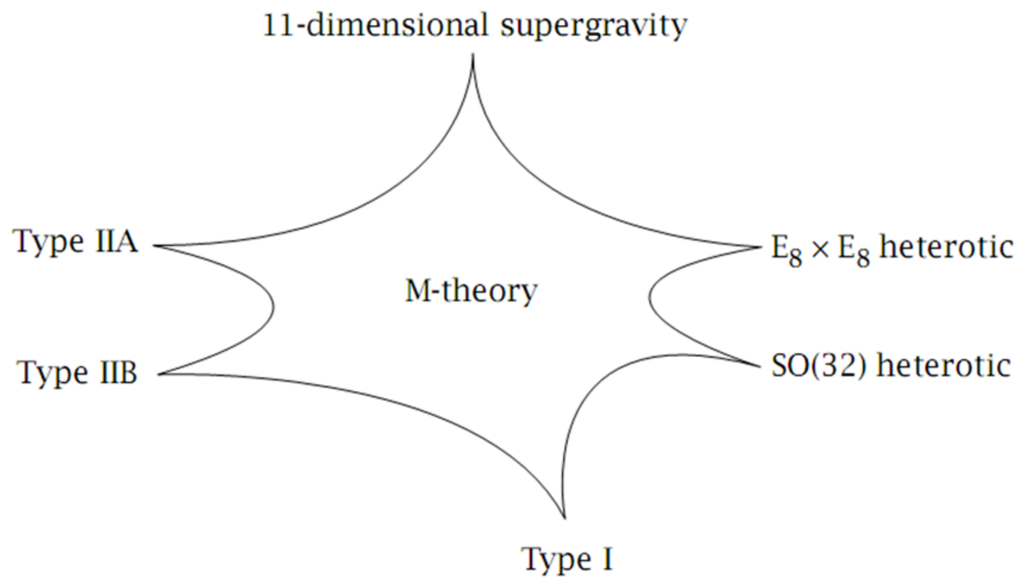

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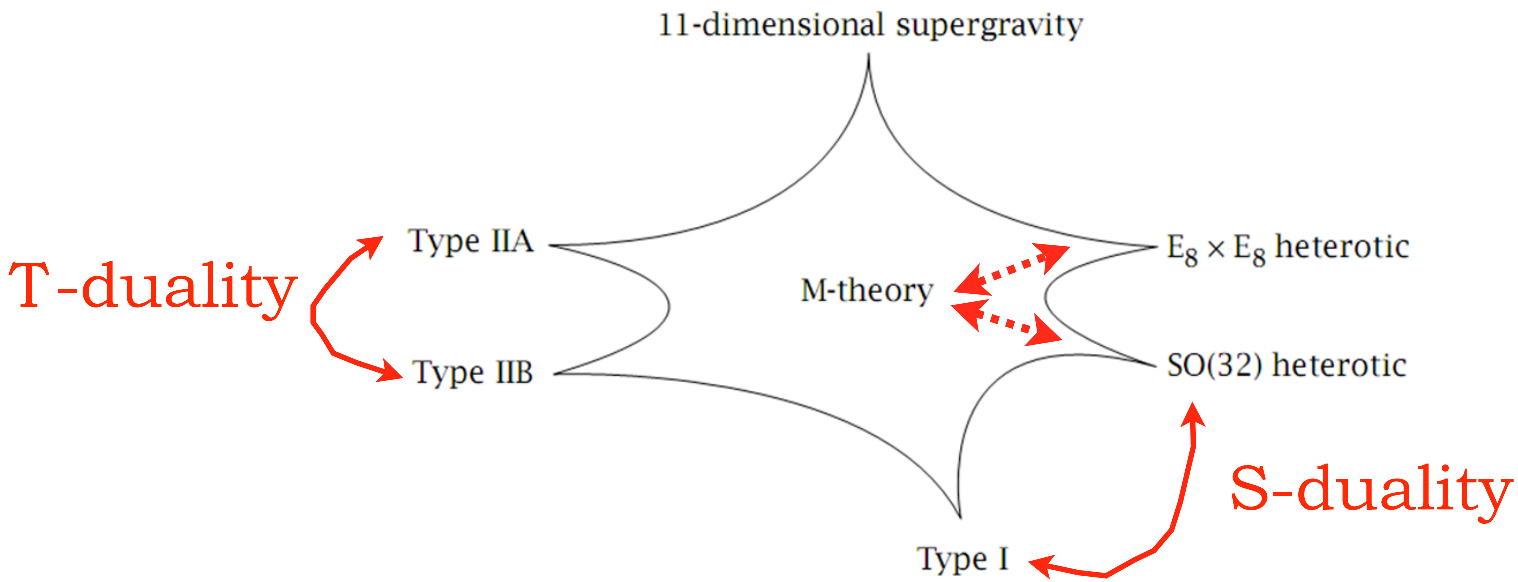

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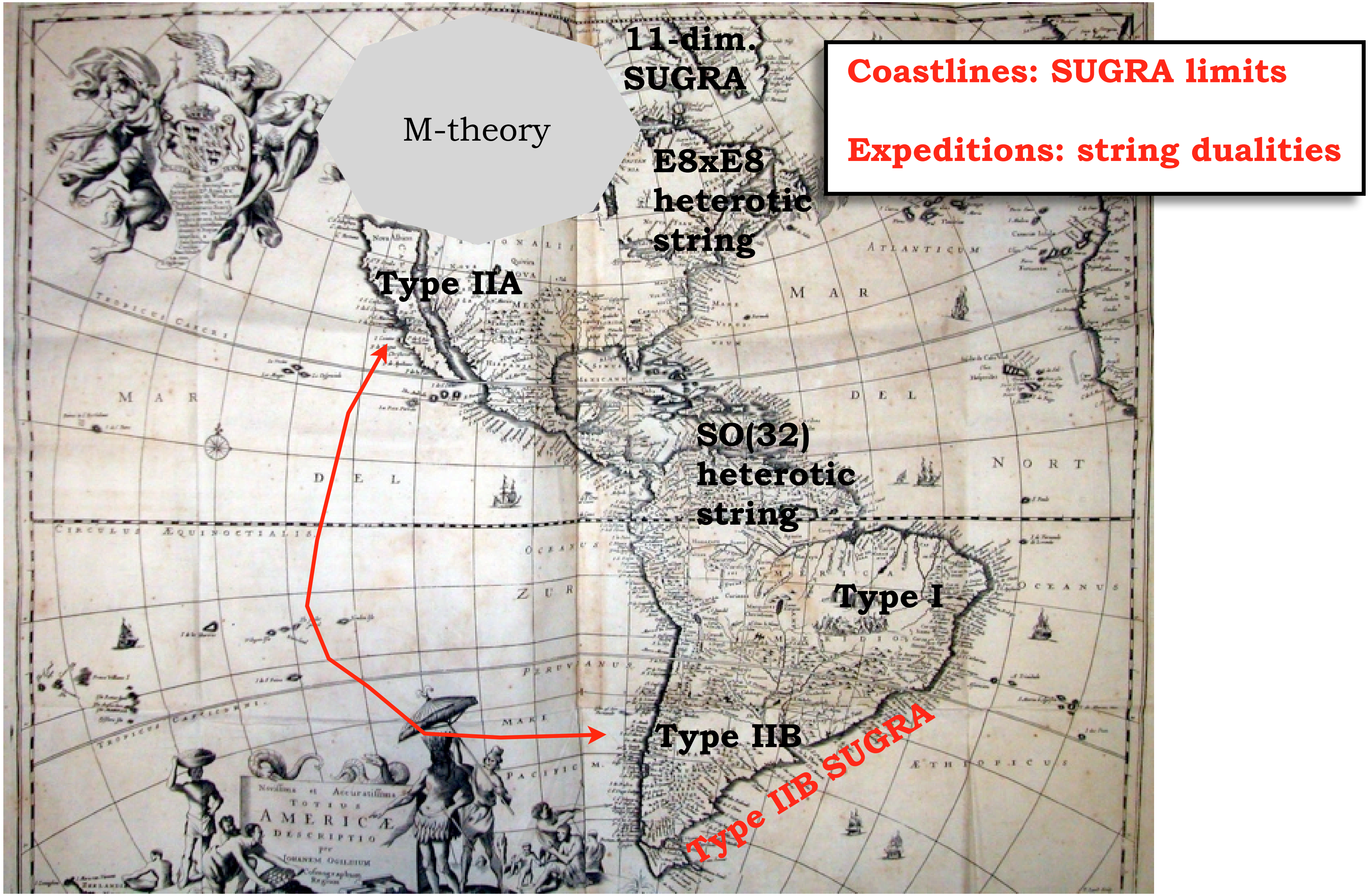
- supergravity (SUGRA) emerges when making SUSY local
- 1+1 dim CFT on world-sheets of strings
- dualities relate these “corners of superstring theory”

Terra incognita

Terra incognita



Overview

Modern research areas

- conformal field theory (CFT)
- integrability
- applications to heavy-ion-collisions
- dS/CFT
- black holes, black branes
- string phenomenology *
- gauge/gravity correspondence (duality) *
- AdS/CMT *
- higher spin holography *
- scattering amplitudes *

** group projects*

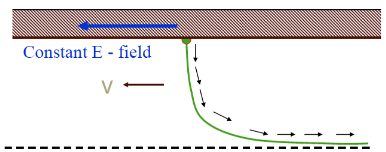
Overview

An eye on applications

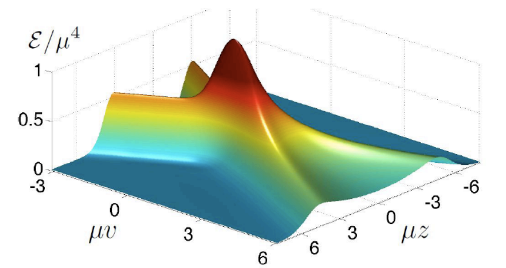
- MHV amplitudes
- applications to heavy-ion-collisions
  - shear viscosity over entropy density (hydro, dragging string)

$$\frac{\eta}{s} = \frac{1}{4\pi} \qquad (\hbar = k_B = c = 1)$$

$$\eta = \lim_{\omega \to 0} \langle [T^{xy}, T^{xy}] \rangle (\omega, \mathbf{k} = 0)$$



  - thermalization



  - chiral magnetic effect   chiral vortical effect

Overview

An eye on applications

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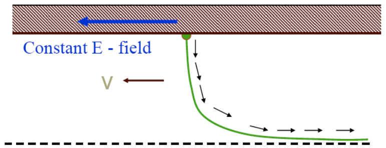

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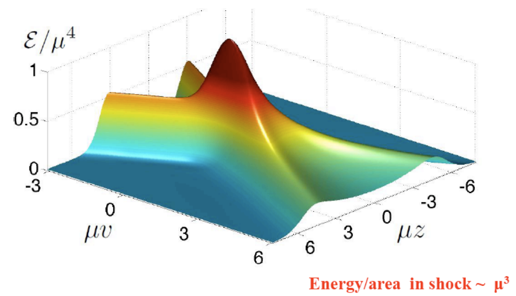

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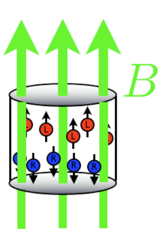
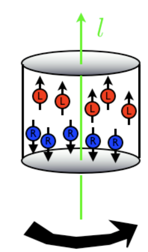


chiral vortical effect



Schedule

Monday, Tuesday, Thursday
09:00-10:00 Lecture
Break & Free discussion
10:30-11:30 Lecture
11:30-12:30 Hands-on tasks
Lunch
14:00-17:00 Hands-on tasks
17:00-19:00 Discussion of solutions

Wednesday, Friday: Group projects from 16:00-18:00

Saturday: Group projects discussion from 09:00-13:00

Monday, Tuesday: Visiting Strings2012 in München

Lectures

Monday-Friday 9:00 - 10:00, 10:30 - 11:30

Lecture I: Overview of String Theory & Modern Research

(general relativity, action principles)

Lecture II: Bosonic String Theory

(open/closed strings, quantization, string spectrum)

Lecture III: Bosonic String Theory

(CFT, operator product expansion, Virasoro algebra, mode expansions, vertex operators)

Lecture IV: Bosonic String Theory

(Polyakov path integral, string scattering amplitudes, string S-matrix)

Lecture V: Bosonic String Theory

(compactification, moduli, spectrum)

Lectures

Monday-Friday 9:00 - 10:00, 10:30 - 11:30

Lecture VI: Bosonic String Theory
(one-loop amplitudes, T-duality, D-branes, outlook on superstrings)

Lecture VII: Superstring Theory
(SUSY, type I, type II, Calabi-Yau compactification, T-duality, flux compactification, string geometry)

Lecture VIII: Superstring Theory
(string dualities, M-theory, string phenomenology)

Lecture IX: Gauge/Gravity Duality & Black Holes
(AdS/CFT conjecture, extensions)

Lecture X: Gauge/Gravity Duality & Black Holes
(applications to heavy-ion collisions and cond-mat)

Exercises

Monday-Friday 11:30 - 17:00

Exercise I: General relativity, action principles, CFT

Exercise II: CFT, string scattering, string spectrum

Exercise III: Compactification, T-duality, spinors & SUSY

Exercise IV: Anomaly cancellation

Exercise V: AdS spacetime, correlation functions

Goals

Participants of the powerweek will

- be able to carry out simple computations in string theory

- have an overview of string theory basics & current string theory research

- know the origins of gauge/gravity duality

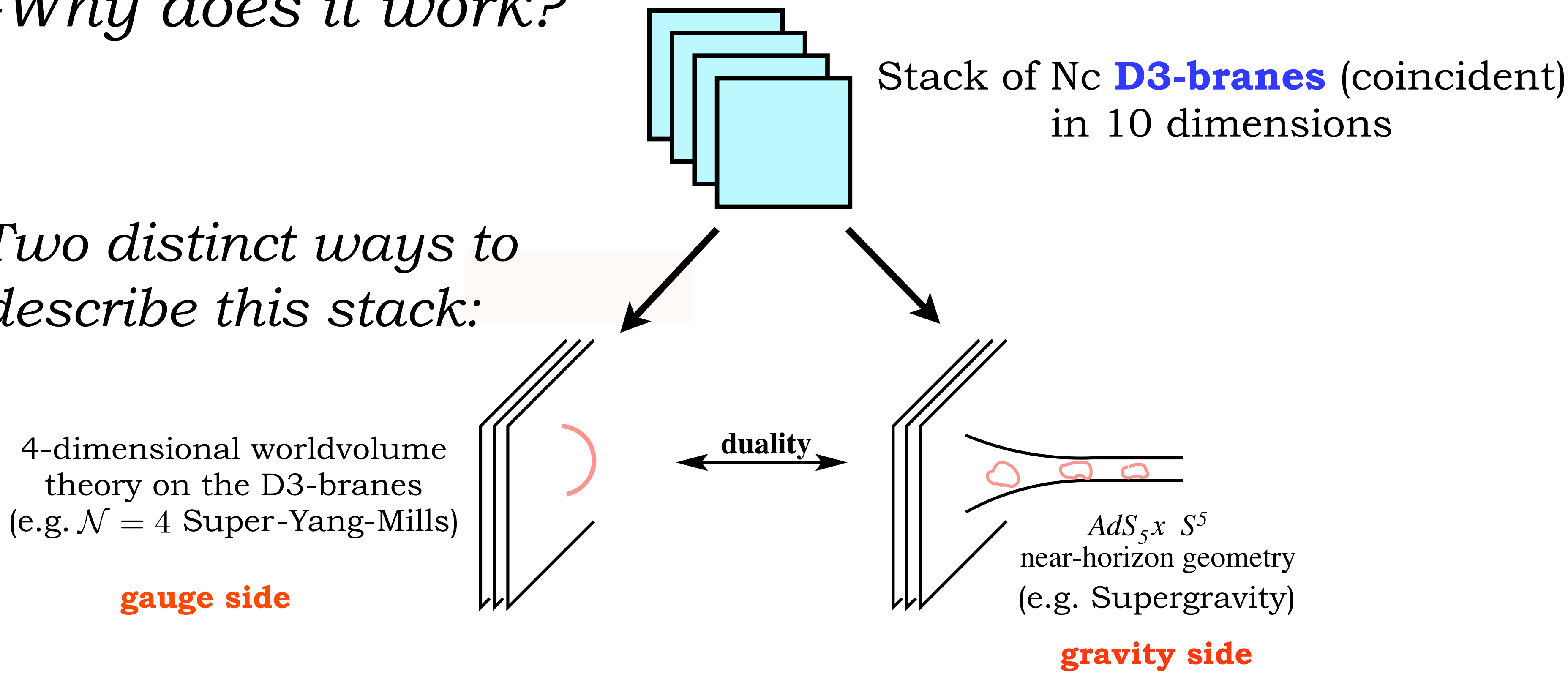
- have the knowledge needed to create simple string setups in order to investigate problems in current research

→ **Blackboard**

Invitation: Gauge/gravity correspondence

-Why does it work?

Two distinct ways to describe this stack:



Invitation: Gauge/gravity correspondence

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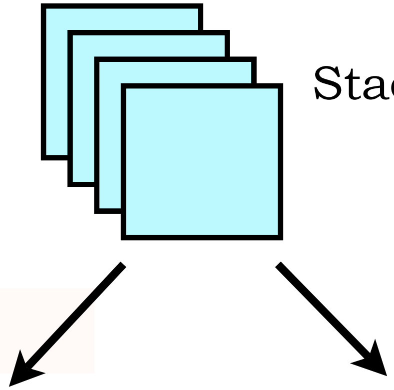
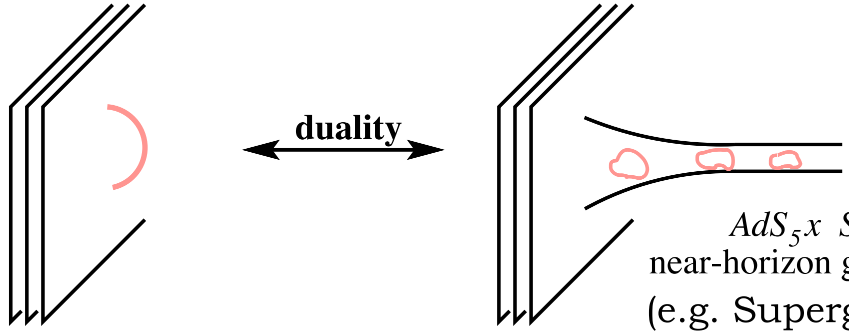


Stack of Nc **D3-branes** (coincident)
in 10 dimensions

Two distinct ways to describe this stack:

4-dimensional worldvolume theory on the D3-branes (e.g. $\mathcal{N} = 4$ Super-Yang-Mills)

gauge side



$AdS_5 x\ S^5$
near-horizon geometry
(e.g. Supergravity)

gravity side

-How does it work?

Add/change geometric objects on 'gravity side':

Geometric setup: Strings/Branes → Find solution configuration → Field Theory result

Example: Schwarzschild radius corresponds to temperature

Solutions
(to some of the exercises)

Exercise 1.2

In the following exercises one has the usual open or closed string boundary conditions (Neumann or periodic) on X^μ for $\mu = 0, \ldots, 24$ but a different boundary condition on X^{25}. Each of these has an important physical interpretation, and will be developed in detail in chapter 8. Find the mode expansion, the mass spectrum, and (for the closed string) the constraint from σ-translation invariance in terms of the occupation numbers. In some cases you need the result of exercise 1.5.

1.6 Open strings with

$$X^{25}(\tau, 0) = 0 \,, \quad X^{25}(\tau, \ell) = y$$

with y a constant. This is an open string with both ends on D-branes.

1.7 Open strings with

$$X^{25}(\tau, 0) = 0 \,, \quad \partial^\sigma X^{25}(\tau, \ell) = 0 \,.$$

This is an open string with one end on a D-brane and one end free.

Solution to exercise 1.2 (1)

The mode expansion satisfying the boundary conditions is

$$X^{25}(\tau,\sigma) = \sqrt{2\alpha'}\sum_n \frac{1}{n}\alpha_n^{25}\exp\left[-\frac{i\pi nc\tau}{l}\right]\sin\frac{\pi n\sigma}{l}, \tag{20}$$

where the sum runs over the half-odd-integers, $n = 1/2, -1/2, 3/2, -3/2, \ldots$. Note that there is no p^{25}. Again, Hermiticity of X^{25} implies $\alpha_{-n}^{25} = (\alpha_n^{25})^\dagger$. Using (1.3.18),

$$\boxed{\Pi^{25}(\tau,\sigma) = -\frac{i}{\sqrt{2\alpha'}l}\sum_n \alpha_n^{25}\exp\left[-\frac{i\pi nc\tau}{l}\right]\sin\frac{\pi n\sigma}{l}.} \tag{21}$$

We will now determine the commutation relations among the α_n^{25} from the equal time commutation relations (1.3.24b). Not surprisingly, they will come out the same as for the free string (1.3.25b). We have:

$$\begin{aligned} i\delta(\sigma-\sigma') &= [X^{25}(\tau,\sigma),\Pi^{25}(\tau,\sigma)] \\ &= -\frac{i}{l}\sum_{n,n'}\frac{1}{n}[\alpha_n^{25},\alpha_{n'}^{25}]\exp\left[-\frac{i\pi(n+n')c\tau}{l}\right]\sin\frac{\pi n\sigma}{l}\sin\frac{\pi n'\sigma'}{l}. \end{aligned} \tag{22}$$

Since the LHS does not depend on τ, the coefficient of $\exp[-i\pi mc\tau/l]$ on the RHS must vanish for $m \neq 0$:

$$\boxed{\frac{1}{l}\sum_n \frac{1}{n}[\alpha_n^{25},\alpha_{m-n}^{25}]\sin\frac{\pi n\sigma}{l}\sin\frac{\pi(n-m)\sigma'}{l} = \delta(\sigma-\sigma')\delta_{m,0}.} \tag{23}$$

Solution to exercise 1.2 (2)

Multiplying both sides by $\sin[\pi n'\sigma/l]$ and integrating over σ now yields,

$$\frac{1}{2n}\left([\alpha_n^{25},\alpha_{m-n}^{25}]\sin\frac{\pi(n-m)\sigma'}{l}+[\alpha_{n+m}^{25},\alpha_{-n}^{25}]\sin\frac{\pi(n+m)\sigma'}{l}\right) = \sin\frac{\pi n\sigma'}{l}\delta_{m,0},$$

or,

$$\boxed{[\alpha_n^{25},\alpha_{m-n}^{25}] = n\delta_{m,0},}$$

as advertised.

The part of the Hamiltonian (1.3.19) contributed by the X^{25} oscillators is

$$\begin{aligned}
\frac{l}{4\pi\alpha' p^+}\int_0^l d\sigma\left(2\pi\alpha'\left(\Pi^{25}\right)^2+\frac{1}{2\pi\alpha'}\left(\partial_\sigma X^{25}\right)^2\right)
&= \frac{1}{4\alpha' p^+ l}\sum_{n,n'}\alpha_n^{25}\alpha_{n'}^{25}\exp\left[-\frac{i\pi(n+n')c\tau}{l}\right] \\
&\quad\times\int_0^l d\sigma\left(-\sin\frac{\pi n\sigma}{l}\sin\frac{\pi n'\sigma}{l}+\cos\frac{\pi n\sigma}{l}\cos\frac{\pi n'\sigma}{l}\right) \\
&= \frac{1}{4\alpha' p^+}\sum_n\alpha_n^{25}\alpha_{-n}^{25} \\
&= \frac{1}{4\alpha' p^+}\sum_{n=1/2}^{\infty}\left(\alpha_n^{25}\alpha_{-n}^{25}+\alpha_{-n}^{25}\alpha_n^{25}\right)
\end{aligned}$$

Solution to exercise 1.2 (3)

$$
\begin{aligned}
&= \frac{1}{4\alpha' p^+} \sum_n \alpha_n^{25} \alpha_{-n}^{25} \\
&= \frac{1}{4\alpha' p^+} \sum_{n=1/2}^{\infty} \left(\alpha_n^{25} \alpha_{-n}^{25} + \alpha_{-n}^{25} \alpha_n^{25} \right) \\
&= \frac{1}{2\alpha' p^+} \sum_{n=1/2}^{\infty} \left(\alpha_{-n}^{25} \alpha_n^{25} + \frac{n}{2} \right) \\
&= \frac{1}{2\alpha' p^+} \left(\sum_{n=1/2}^{\infty} \alpha_{-n}^{25} \alpha_n^{25} + \frac{1}{48} \right),
\end{aligned}
$$

where we have used (19) and (25). Thus the mass spectrum (1.3.36) becomes

$$
\begin{aligned}
m^2 &= 2p^+ H - p^i p^i \qquad (i = 2, \ldots, 24) \\
&= \frac{1}{\alpha'} \left(N - \frac{15}{16} \right),
\end{aligned}
$$

where the level spectrum is given in terms of the occupation numbers by

$$
N = \sum_{i=2}^{24} \sum_{n=1}^{\infty} n N_{in} + \sum_{n=1/2}^{\infty} n N_{25,n}.
$$

The ground state is still a tachyon,

$$
\boxed{m^2 = -\frac{15}{16\alpha'}.}
$$

Solution to exercise 1.2 (4)

The first excited state has the lowest X^{25} oscillator excited ($N_{25,1/2} = 1$), and is also tachyonic:

$$\boxed{m^2 = -\frac{7}{16\alpha'}.} \tag{30}$$

There are no massless states, as the second excited state is already massive:

$$\boxed{m^2 = \frac{1}{16\alpha'}.} \tag{31}$$

This state is 24-fold degenerate, as it can be reached either by $N_{i,1} = 1$ or by $N_{25,1/2} = 2$. Thus it is a massive vector with respect to the SO(24,1) Lorentz symmetry preserved by the D-brane. The third excited state, with

$$\boxed{m^2 = \frac{9}{16\alpha'},} \tag{32}$$

is 25-fold degenerate and corresponds to a vector plus a scalar on the D-brane—it can be reached by $N_{25,1/2} = 1$, by $N_{25,1/2} = 3$, or by $N_{i,1} = 1$, $N_{25,1/2} = 1$.

Exercise 2.1

2.7 (a) By computing the relevant OPEs, confirm the weights stated in eq. (2.4.17) and determine which operators are tensors.
(b) Do this for the same operators in the linear dilaton theory.

$\longrightarrow$ **Blackboard**

Thanks to

Martin Ammon (UCLA)
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John Fuini (UW)
and the graduate students at UW