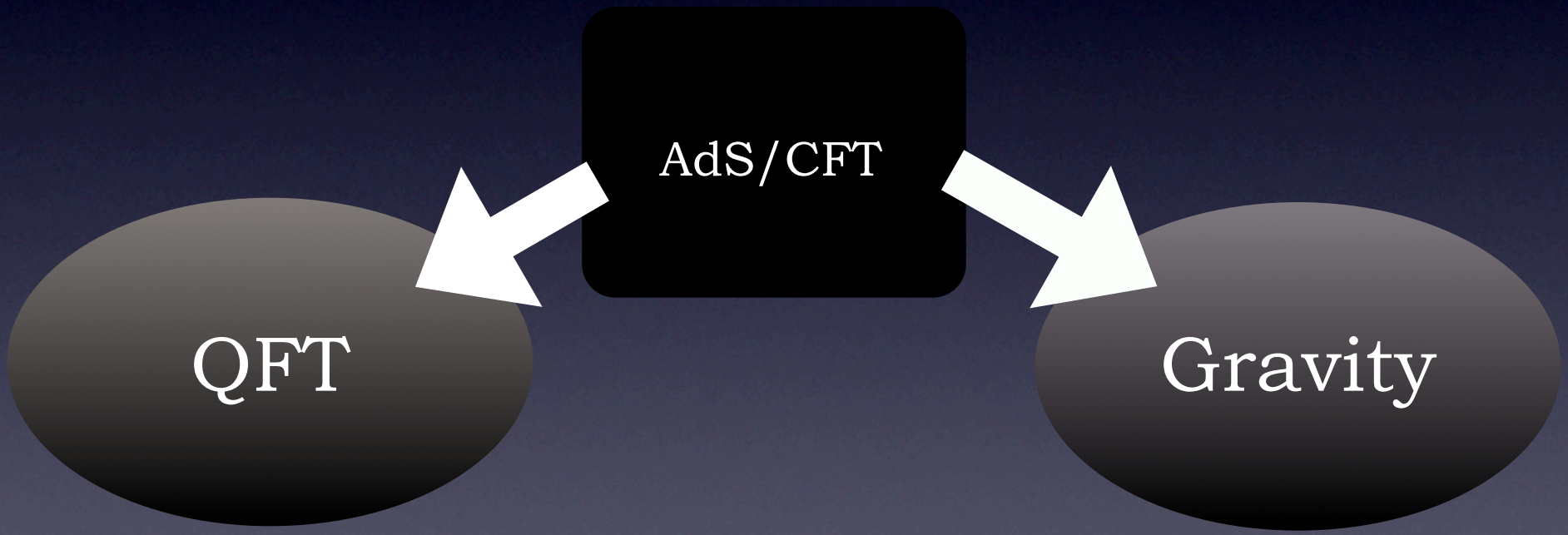
Introduction to Gauge/Gravity Correspondence & Heavy-Ion-Applications

FIAS Frankfurt, HGS-HIRe Powerweek, June 27-30th 2011



by Matthias Kaminski
(Princeton University)

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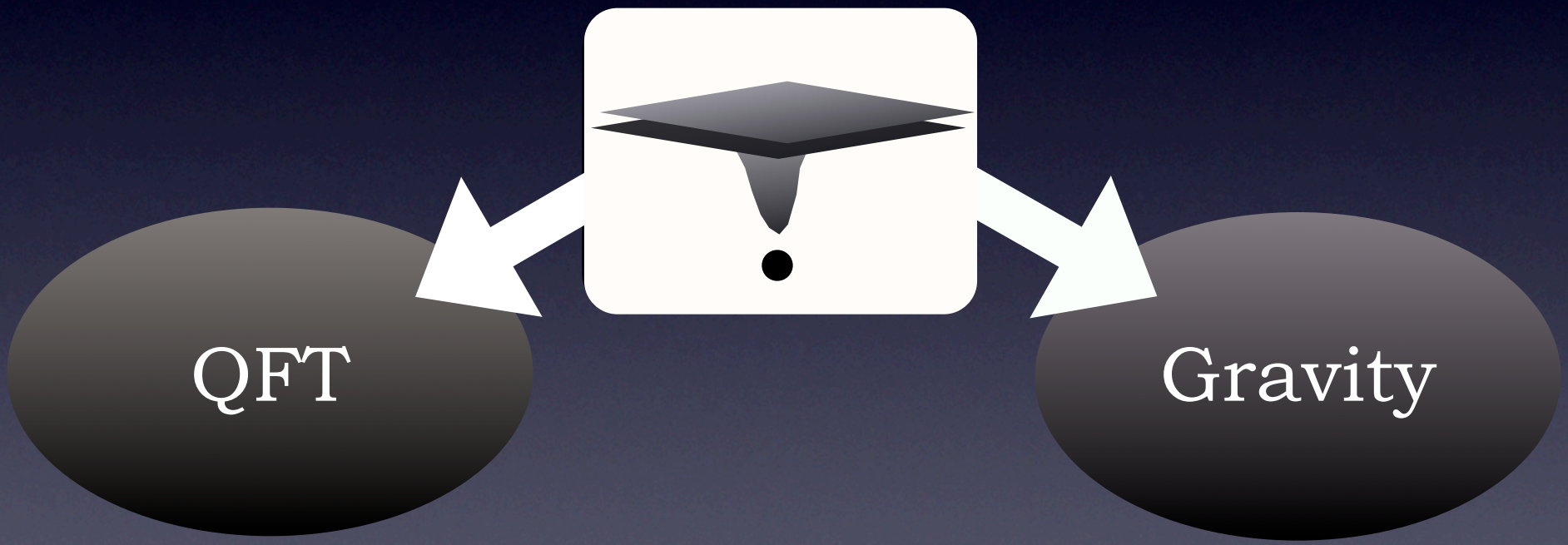

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Lecture I & II: Introduction to Gauge/Gravity

Invitation: Gauge/Gravity Correpondence

-Reasons to ignore the correspondence

1. String Theory may not describe our nature.

2. Gauge/Gravity Correspondence may be wrong. It is a conjecture, which is not proven in general, only in special cases.

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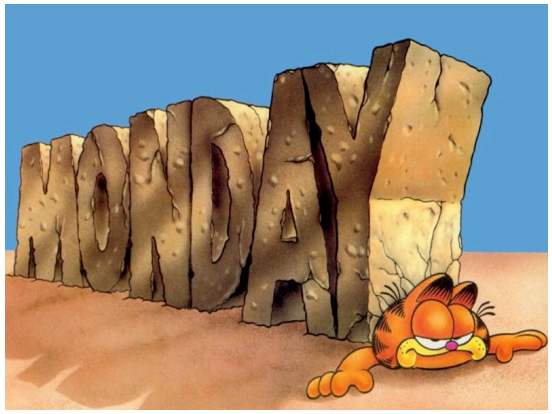
1. String Theory may not describe our nature.

2. Gauge/Gravity Correspondence may be wrong. It is a conjecture, which is not proven in general, only in special cases.

➡ Foundations of Gauge/Gravity Correspondence may be unphysical since it arises in the context of String Theory, and in addition it may be mathematically wrong.

So, why am I here at 9 a.m. on a Monday morning?



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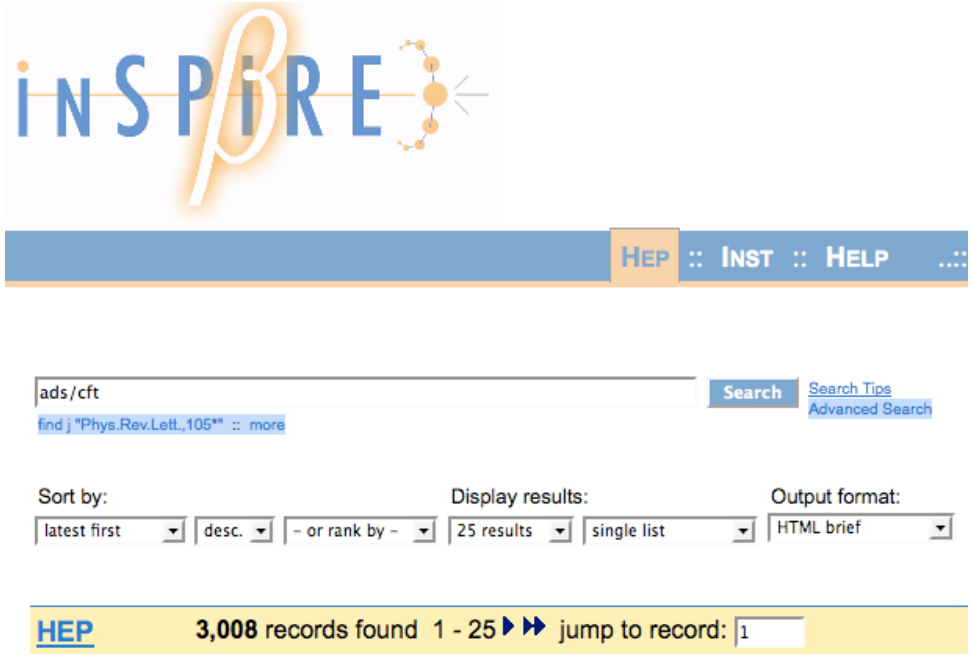
-Strange calculations with publications



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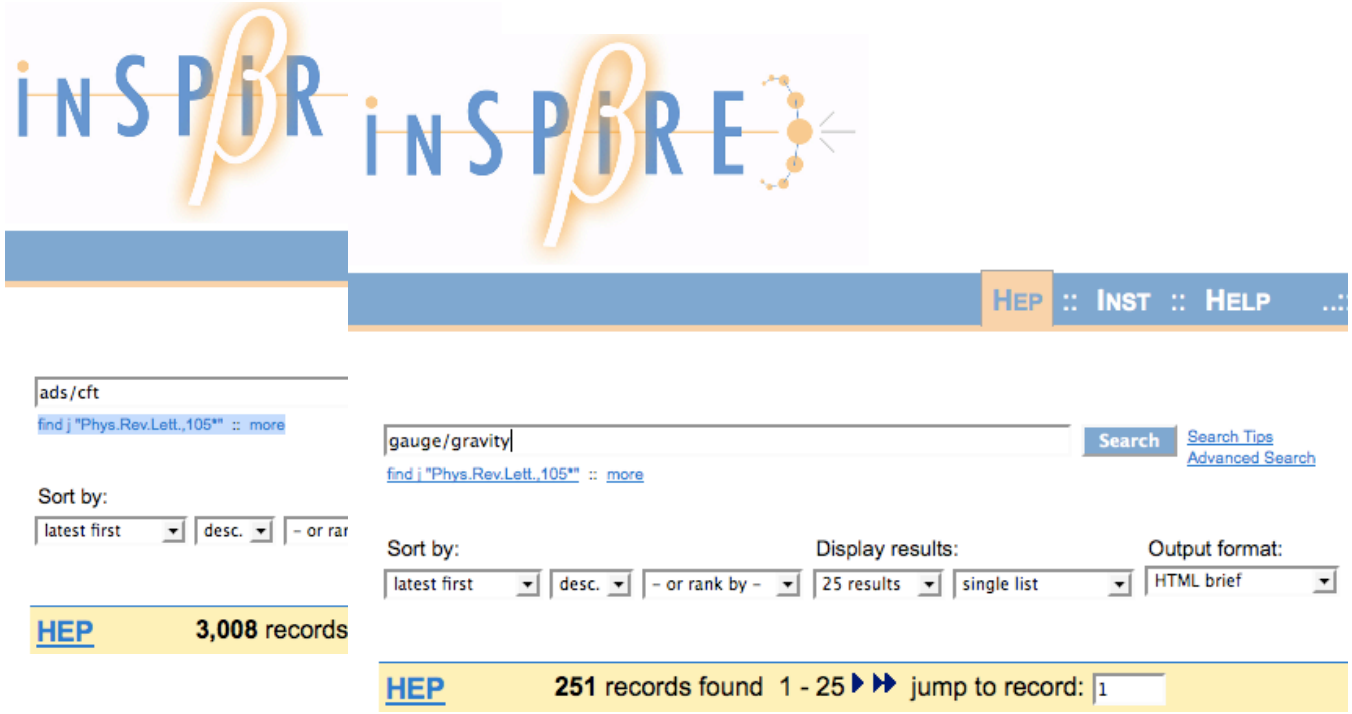

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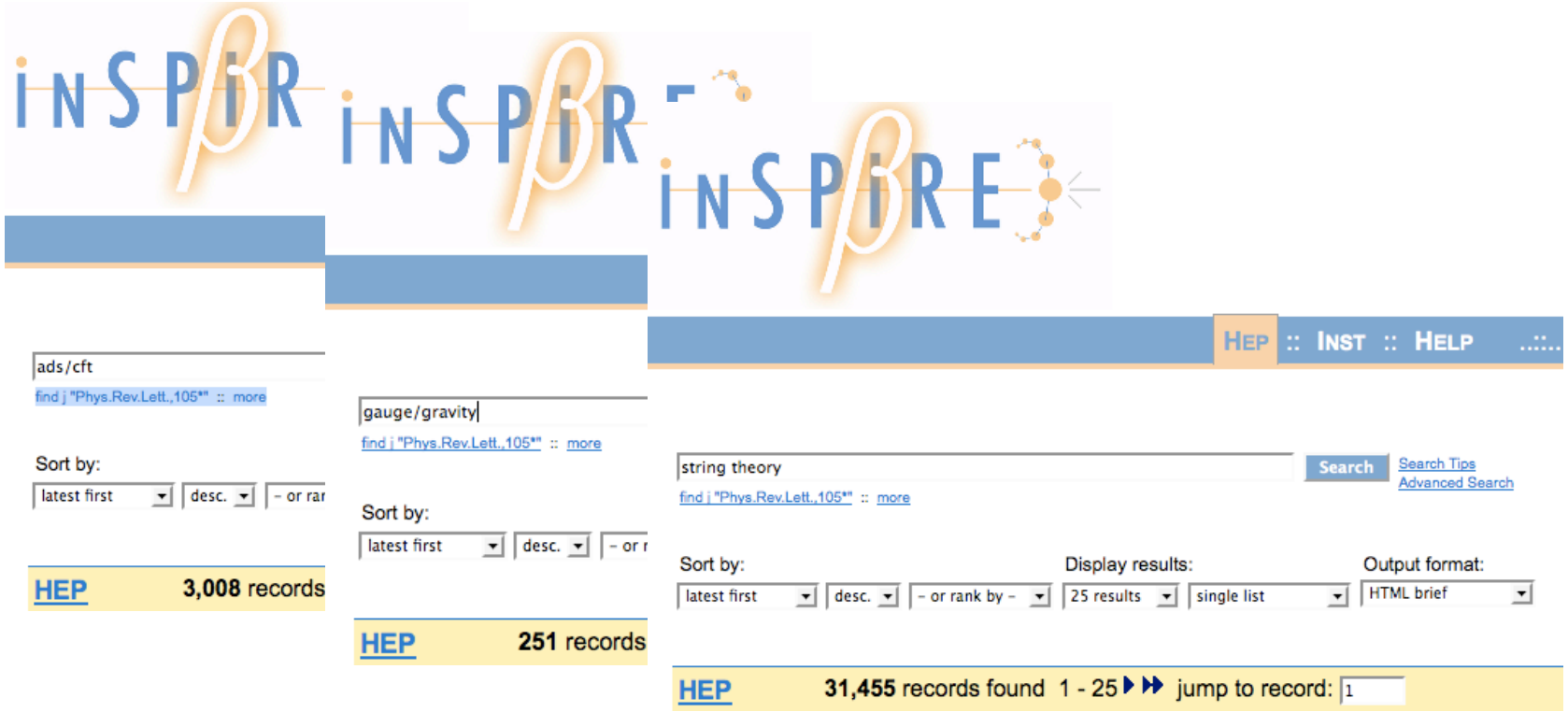

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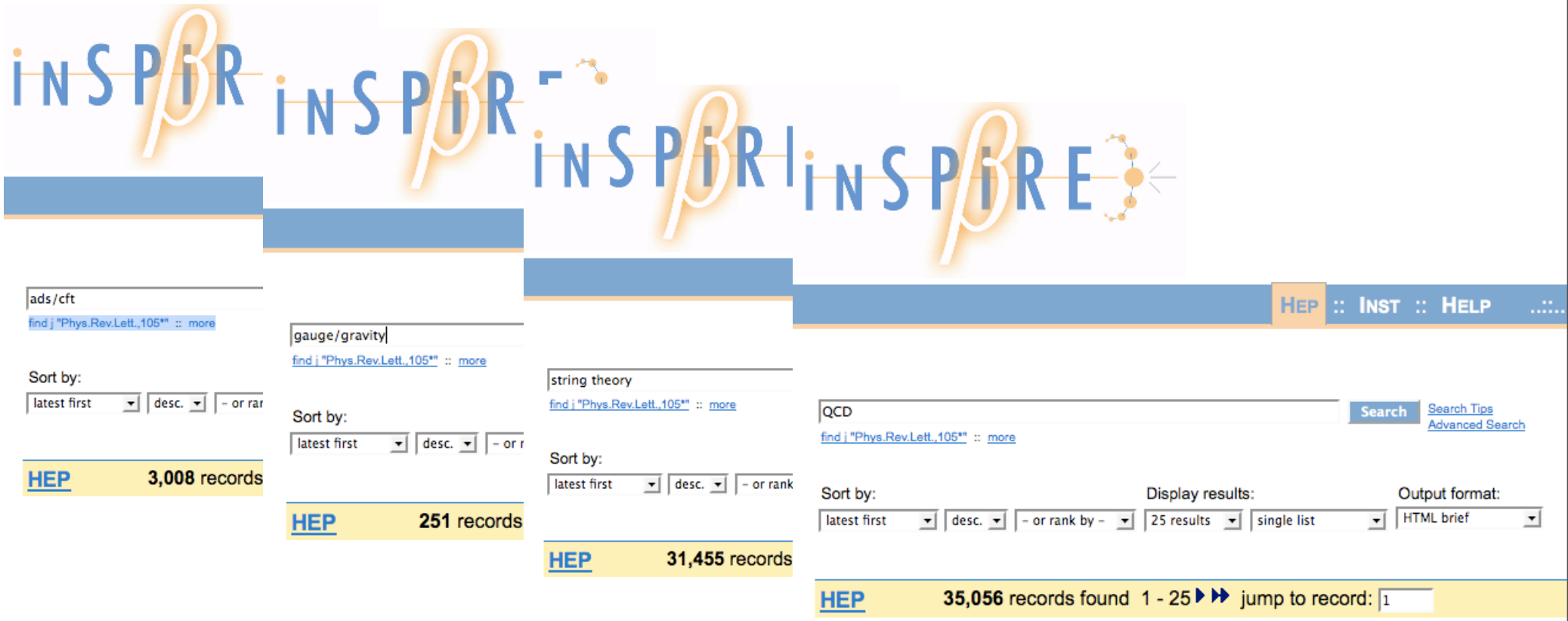

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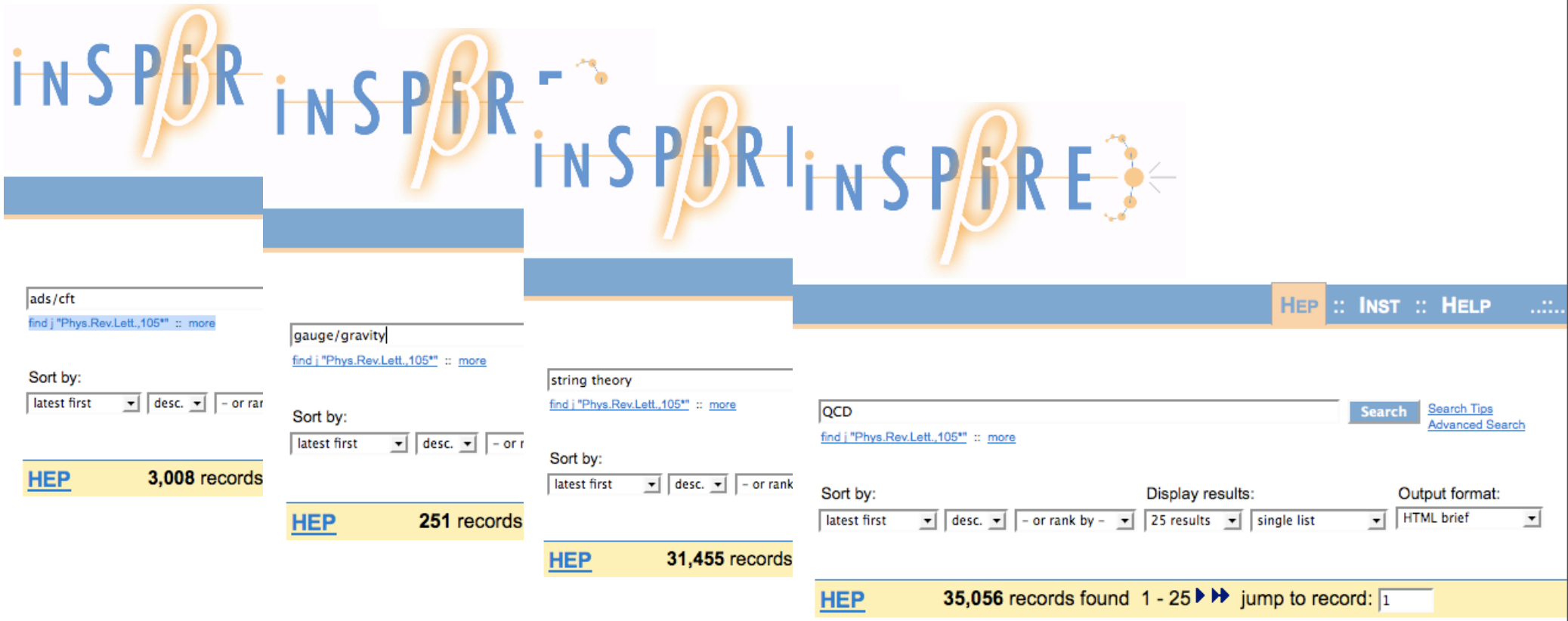

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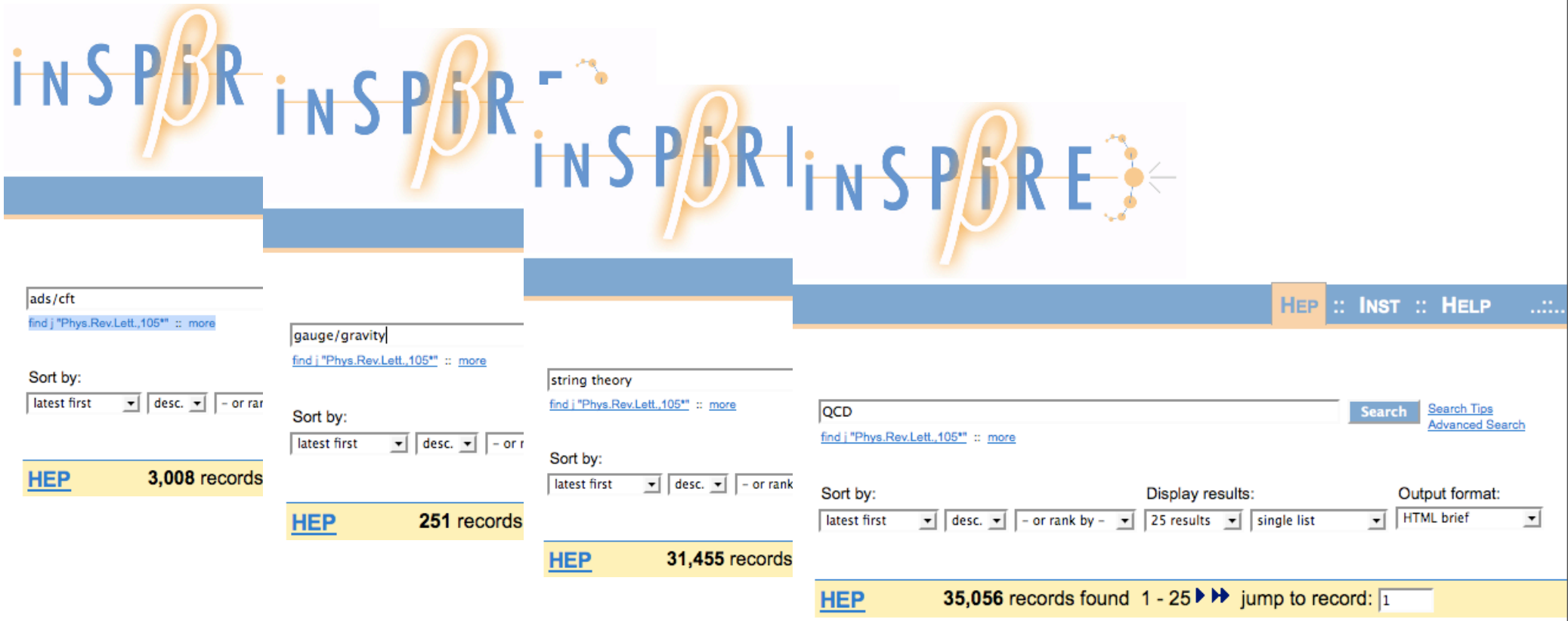

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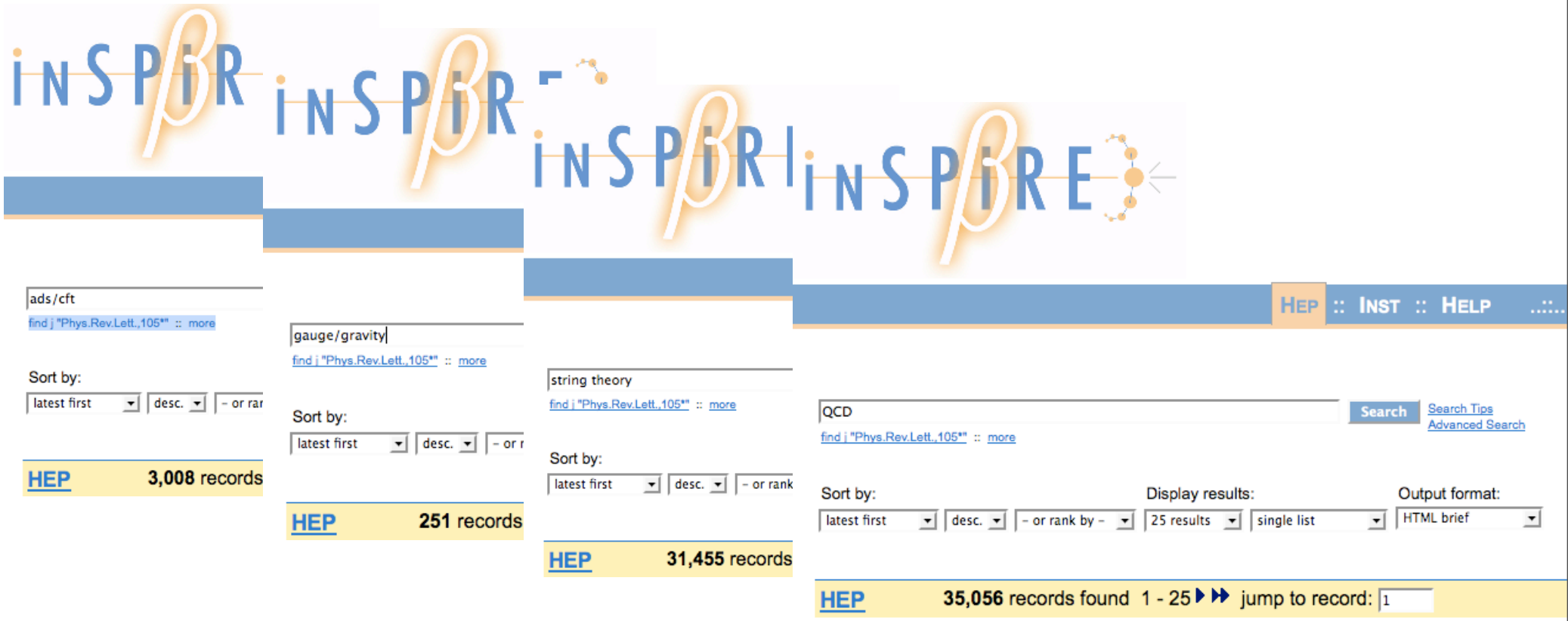

- ➡ ~10% of String Theory *"are"* AdS/CFT
- ➡ AdS/CFT is a factor 10 less *"important"* than QCD
- ➡ String Theory is almost as *"important"* as QCD

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14. **The Large N limit of superconformal field theories and supergravity.**
Juan Martin Maldacena (Harvard U.). HUTP-98-A097. Nov 1997. 19 pp.
Published in **Adv.Theor.Math.Phys. 2 (1998) 231-252**
Talk given at SPIRES Conference C98/11/28 (Conference information coming soon)
e-Print: **hep-th/9711200**

References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Abstract and Postscript and PDF from arXiv.org
Journal Server
Journal Server
ATMP Server
Mathematical Reviews

Detailed record - Similar records - Cited by 7553 records

String Theory is almost as *important* as QCD

Proofs of Gauge/Gravity Correspondences

-Some examples

- Three-point functions of N=4 Super-Yang-Mills theory
- Conformal anomaly of the same theory
- RG flows away from most symmetric case
- ... many other symmetric instances of the correspondence

Evidence for Gauge/Gravity

-Reasonable example results from Gauge/Gravity!

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- [AdS/QCD (bottom-up approach) distinct from string constr.]

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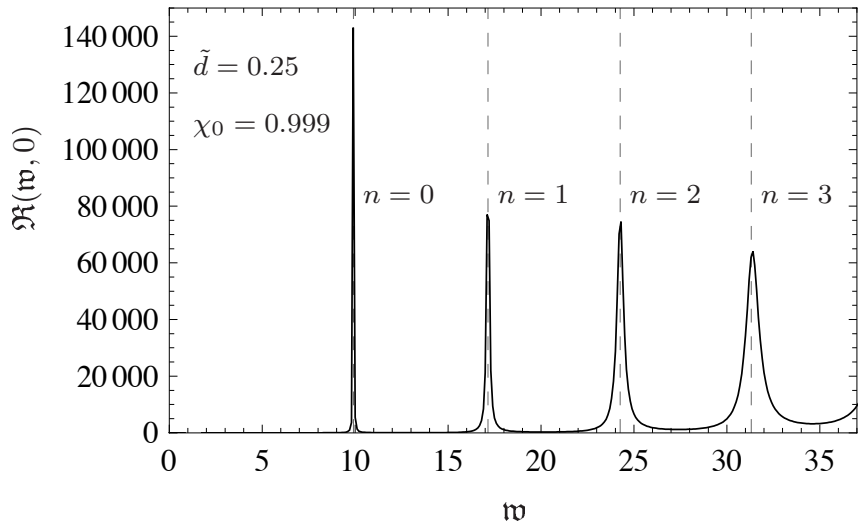
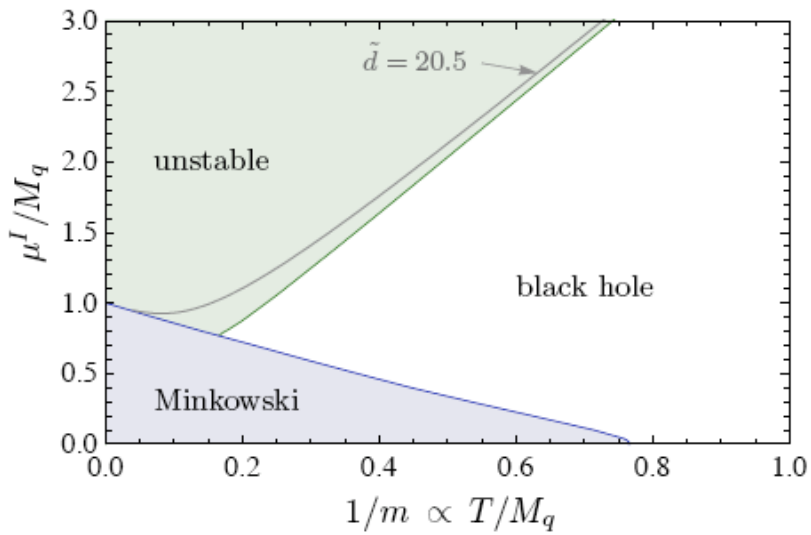


e.g. phase diagrams of fundamental matter (quarks) in a hot and dense plasma carrying isospin charge



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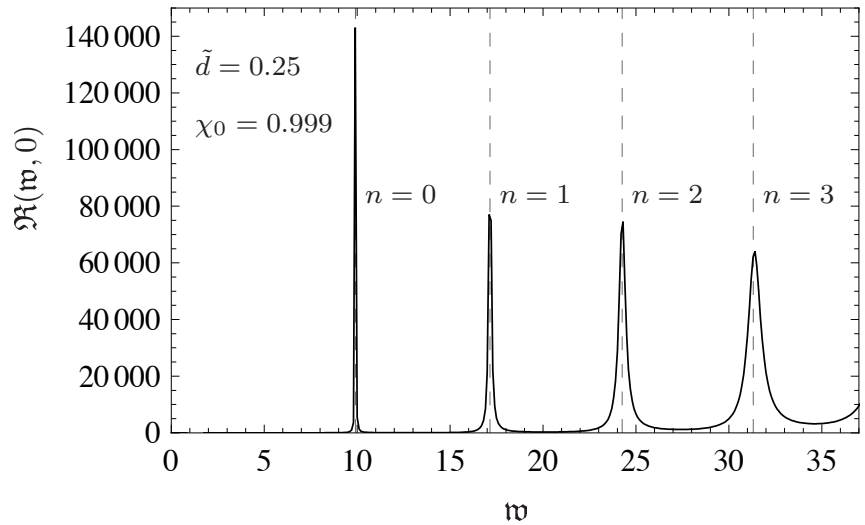

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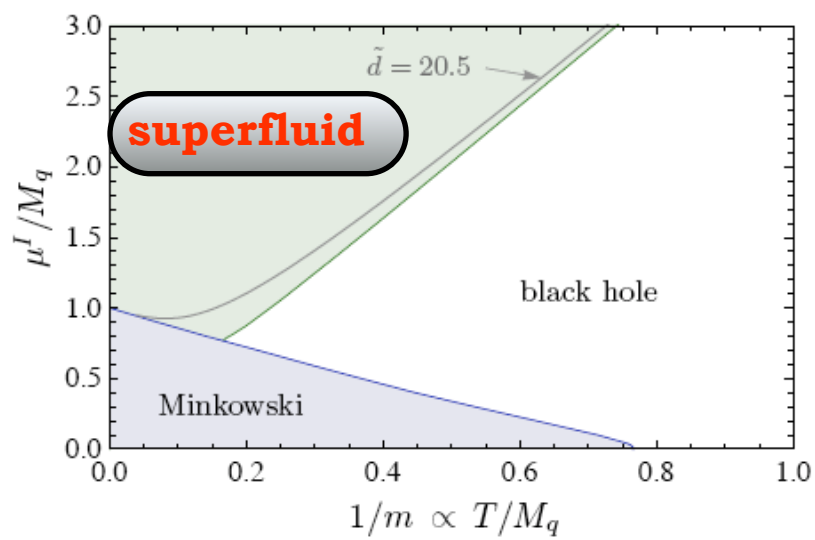

Low shear viscosity

Theory/Model	η/s	Reference
Lattice QCD	$0.134(33)$	*[Meyer, 2007]*
Hydro (Glauber)	0.19	*[Drescher et al., 2007]*
Hydro (CGC)	0.11	*[Drescher et al., 2007]*
Viscous Hydro (Glauber)	$0.08\,, 0.16,\ \{0.03\}$	*[Romatschke et al.,2007]*

Gauge/Gravity: $\frac{\eta}{s} \geq \frac{1}{4\pi} \approx 0.08$ *[Policastro, Son, Starinets, 2001]*

✓ Correct prediction!

Gauge/Gravity is a Powerful Tool

- non-perturbative results, strong coupling
- treat many-body systems
- direct computations in real-time thermal QFT (transport)
- no sign-problem at finite charge densities
- methods often just require solving ODEs in classical gravity
- quick numerical computations (~few seconds on a laptop)
- (turn around: study strongly coupled gravity)

Invitation: Gauge/Gravity Correpondence

-Less pessimistic view:

1. String Theory may not describe our nature uniquely. But it is mathematically correct.

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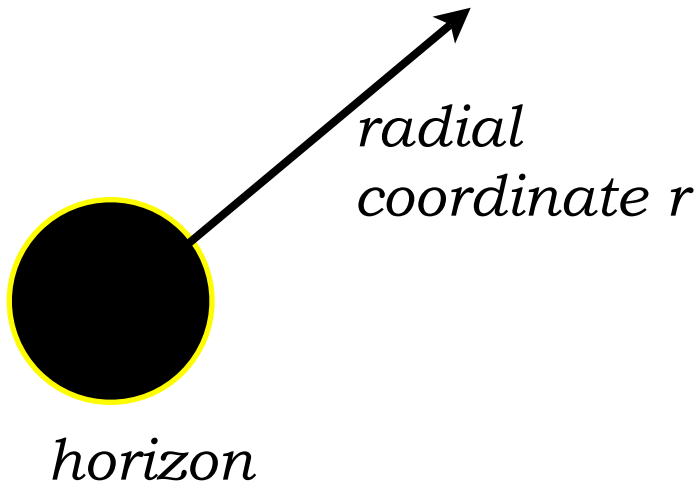
   ➡ Gauge/Gravity Correspondence *may be* a mathematically correct map, which relates particular quantum field theories to particular gravity theories (assuming the conjecture can be proven).

   ➡ Gauge/Gravity *may be* used as a mathematical tool to map effective field theories to gravity theories, even if string theory is not describing our nature.

An example: Correct math, wrong physics?

-Perturbations near a classical black hole:



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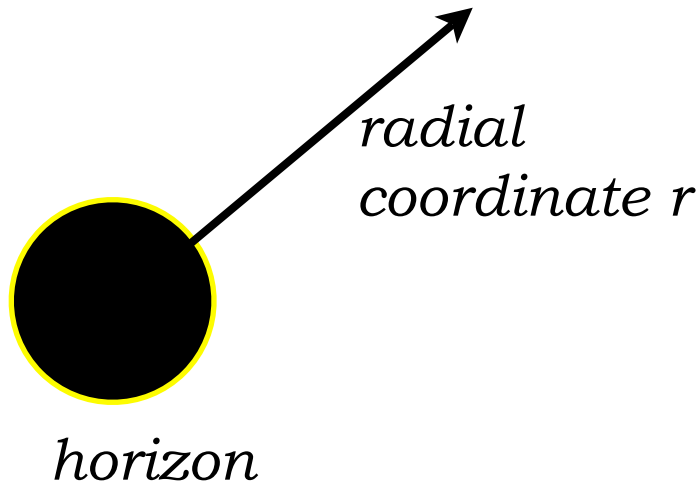

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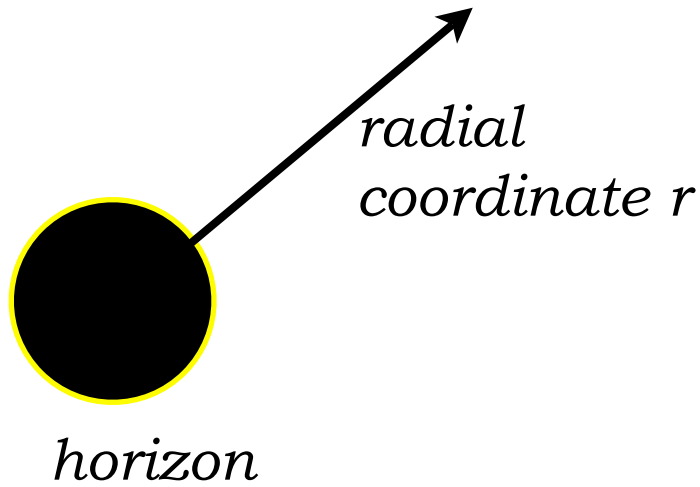

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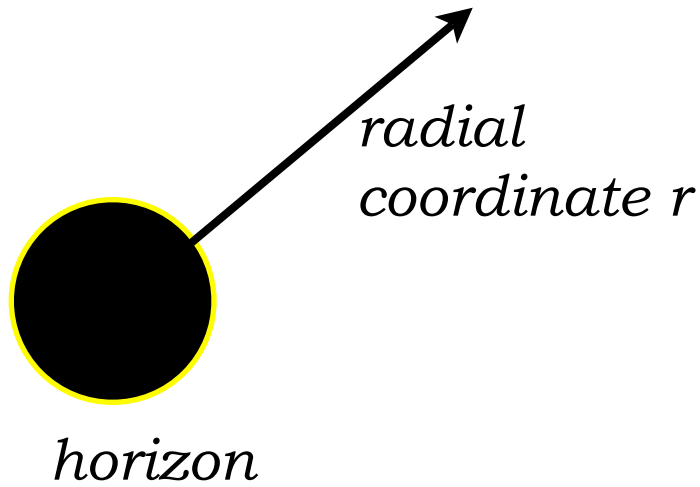

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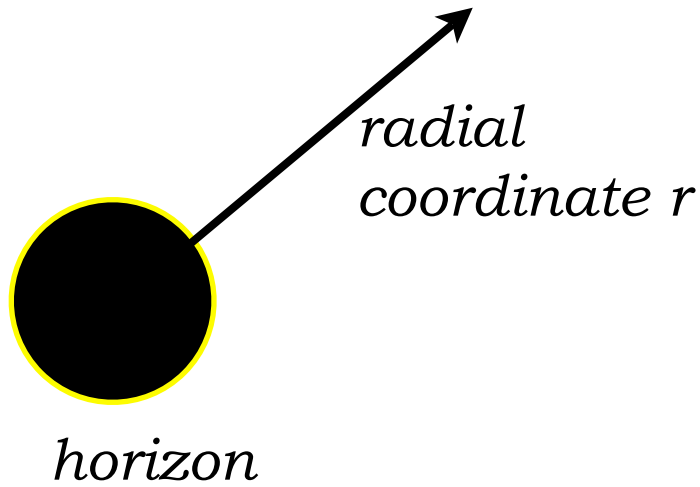

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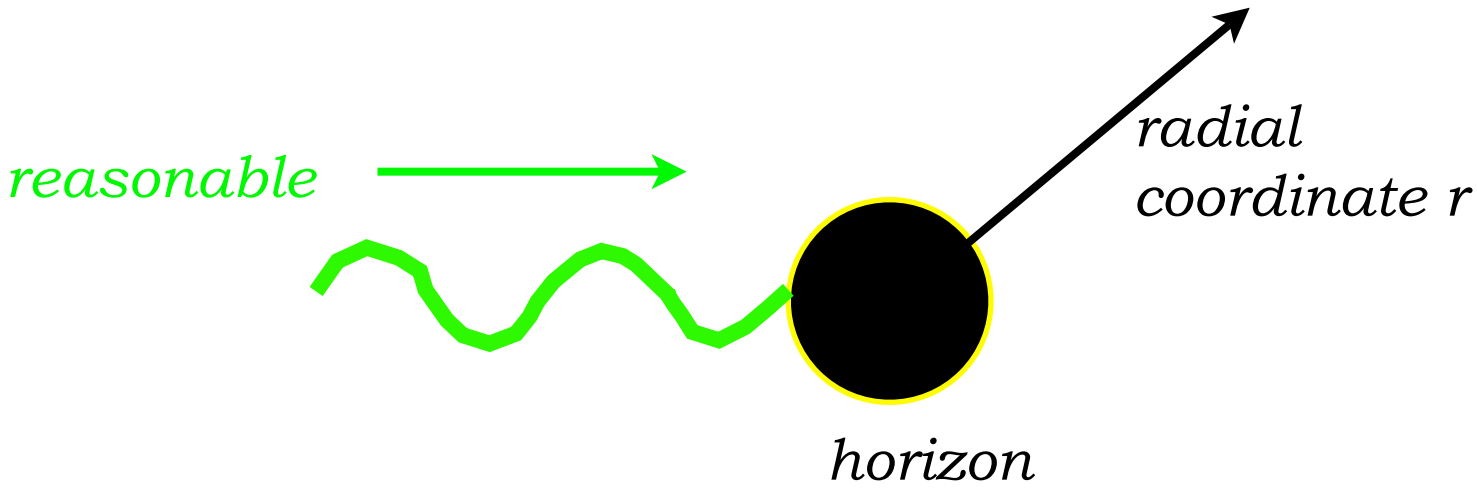

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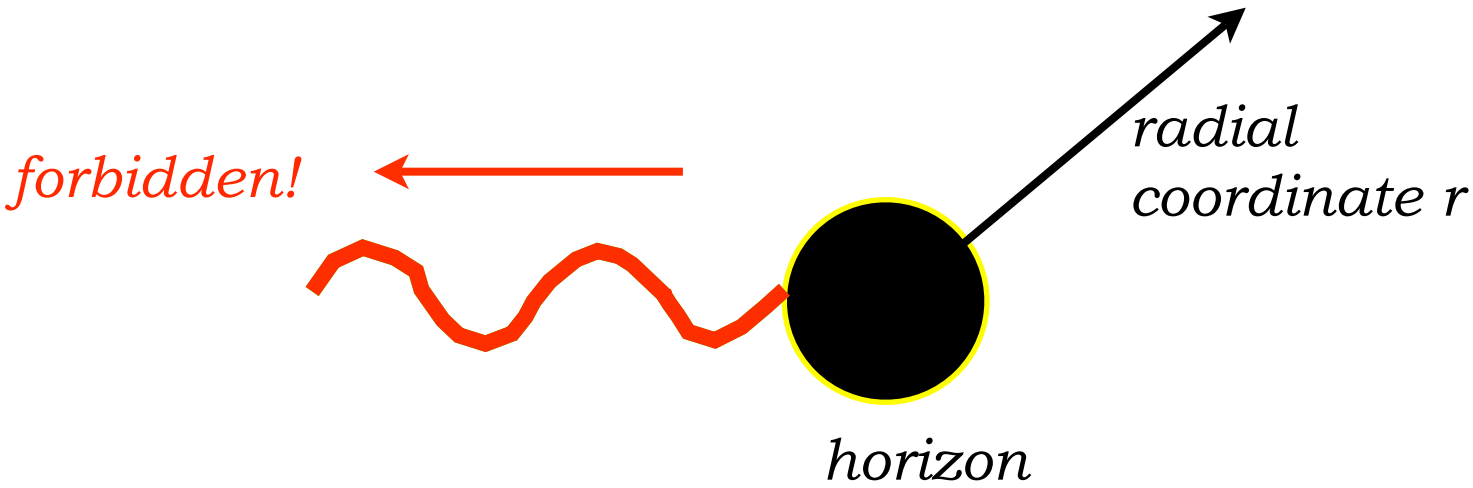

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Why Anti-deSitter, isn't our world deSitter ?

Invitation: Gauge/Gravity Correpondence

-Optimistic view:

1. Some “String Theory” *may describe* our nature uniquely.

2. Gauge/Gravity Correspondence *may be* an exact duality between two ways of describing this one unique theory.

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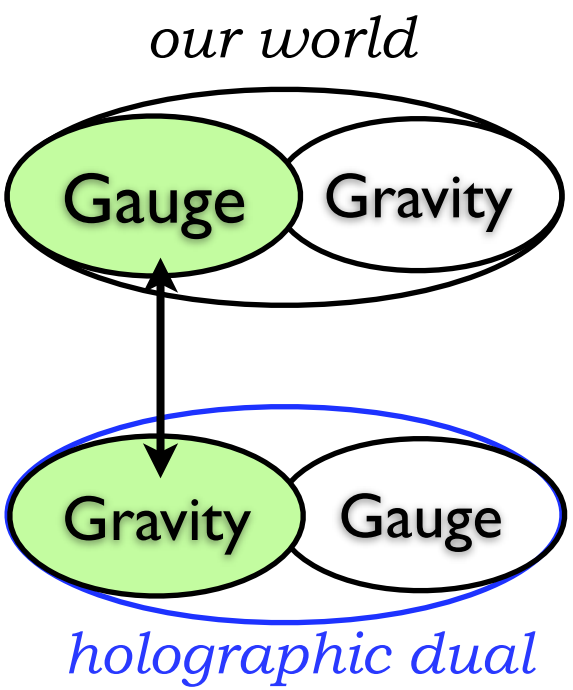

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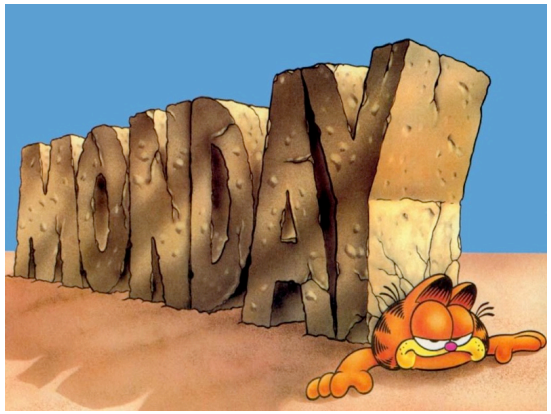


Two answers:

1. Gain a geometric (a dual) understanding of strongly coupled dynamics.

2. Studying examples of gauge/gravity dualities, we learn about the particular quantum field theory and the particular gravity, but possibly also about quantum gravity in general!

Lectures

Tuesday-Thursday 9:00 - 10:30, FIAS ground floor

Lecture I: Introduction to Gauge/Gravity & Applications I

Lecture II: Introduction to Gauge/Gravity & Applications II

Lecture III: Thermal Spectral Functions

Lecture IV: Beyond Hydrodynamics

Lecture V: Phase Transitions

Exercises

Monday-Thursday 13:30 - 17:00, on the roof of FIAS

Exercise I: AdS Coordinates & Brane Embeddings

Exercise II: Thermal Green's Functions & Viscosity

Exercise III: More than Hydrodynamics from Gravity

Exercise IV: Superfluid Phase Transition & Conductivity

Goals

Participants of the powerweek will be able to

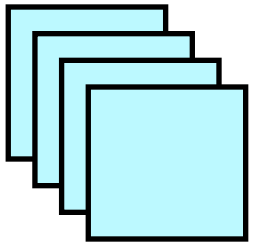
- carry out computations in classical (super)gravity which are state-of-the-art in gauge/gravity research

- translate gravity results into gauge theory expressions (at least for the subset of examples presented)

- judge the relevance of gauge/gravity to their own work

Invitation: Gauge/gravity correspondence

-Why does it work?

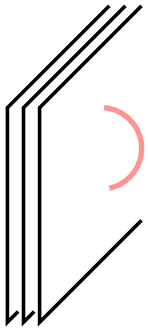


Stack of Nc **D3-branes** (coincident)
in 10 dimensions

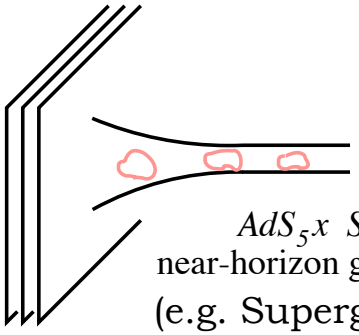
Two distinct ways to describe this stack:

4-dimensional worldvolume
theory on the D3-branes
(e.g. $\mathcal{N} = 4$ Super-Yang-Mills)

gauge side



duality



$AdS_5 x\ S^5$
near-horizon geometry
(e.g. Supergravity)

gravity side

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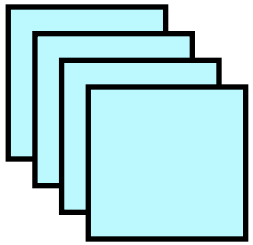

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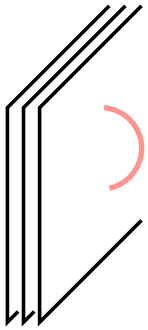

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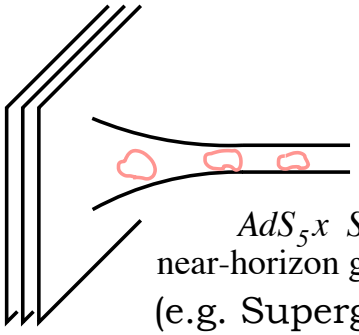

$AdS_5 x\ S^5$ near-horizon geometry (e.g. Supergravity)

gravity side

-How does it work?

Add/change geometric objects on 'gravity side':

Geometric setup: Strings/Branes → Find solution configuration → Field Theory result

Example: Schwarzschild radius corresponds to temperature

Lecture III: Thermal Spectral Functions

Example: Gauge field fluctuations

[Erdmenger, M.K., Rust 0710.0334]

Effective action: $$S_{\mathrm{D7}} = \int \mathrm{d}^8 x \sqrt{\left|\det\{[g+F]+\tilde{F}\}\right|}\ , \quad F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$$

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Fluctuations

Equation of motion: $$0 = \tilde{A}'' + \frac{\partial_\rho[\sqrt{|\det G|}G^{22}G^{44}]}{\sqrt{|\det G|}G^{22}G^{44}}\tilde{A}' - \frac{G^{00}}{G^{44}}\varrho_H{}^2\omega^2\tilde{A}$$

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Fluctuations

Equation of motion:

`Curved' Maxwell equations:

$$\partial_\mu F^{\mu\nu} = 0$$

$$\partial_\mu \left(\sqrt{-G} G^{\mu\nu} G^{\rho\sigma} F_{\nu\sigma}\right) = 0$$

$$\partial_\mu \left(\sqrt{-G} G^{\mu\nu} G^{\rho\sigma} \partial_{[\nu} \tilde{A}_{\sigma]}\right) = 0$$

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Translation to Gauge Theory by duality: $$\underset{\text{(source)}}{A_\mu} \overset{\text{AdS/CFT}}{\longleftrightarrow} J^\mu$$

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→ shooting from horizon

Translation to Gauge Theory by duality: $$\underset{\text{(source)}}{A_\mu} \overset{\text{AdS/CFT}}{\longleftrightarrow} J^\mu$$

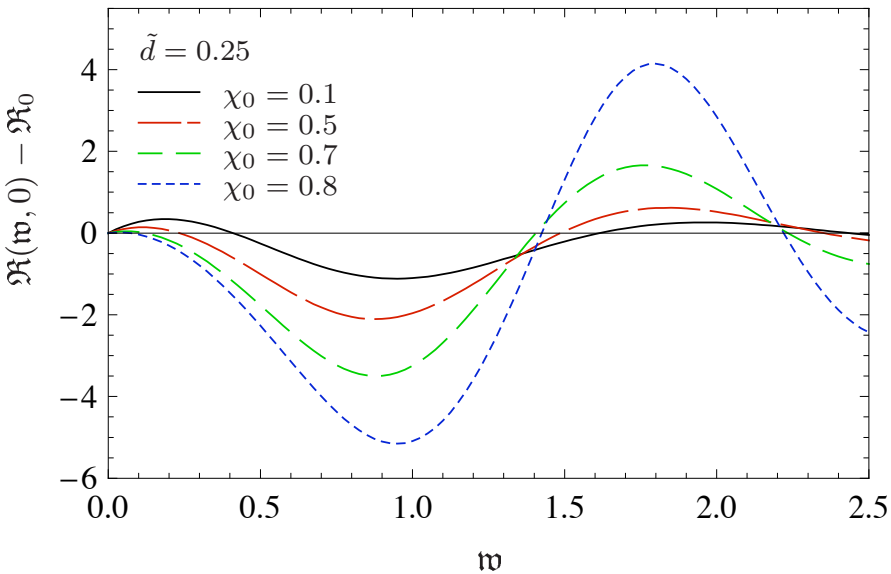
Gauge Correlator:
[Son et al.'02]

$$G^{\mathrm{ret}} = \frac{N_f N_c T^2}{8} \lim_{\rho\to\rho_{\mathrm{bdy}}} \left(\rho^3 \frac{\partial_\rho \tilde{A}(\rho)}{\tilde{A}(\rho)}\right)$$

Gauge theory results: spectral functions

[Erdmenger, M.K., Rust 0710.0334]

Finite baryon density



$$L(\varrho) = \varrho\,\chi(\varrho)$$

$$\chi_0 = \chi(\rho)\big|_{\rho\to\rho_H} \sim \frac{m_{\text{quark}}}{T}$$

$$\chi = \chi(\tilde{d}, \rho)$$

Gauge theory results: spectral functions

[Erdmenger, M.K., Rust 0710.0334]

Finite baryon density

Lower temperature

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Gauge theory results: spectral functions

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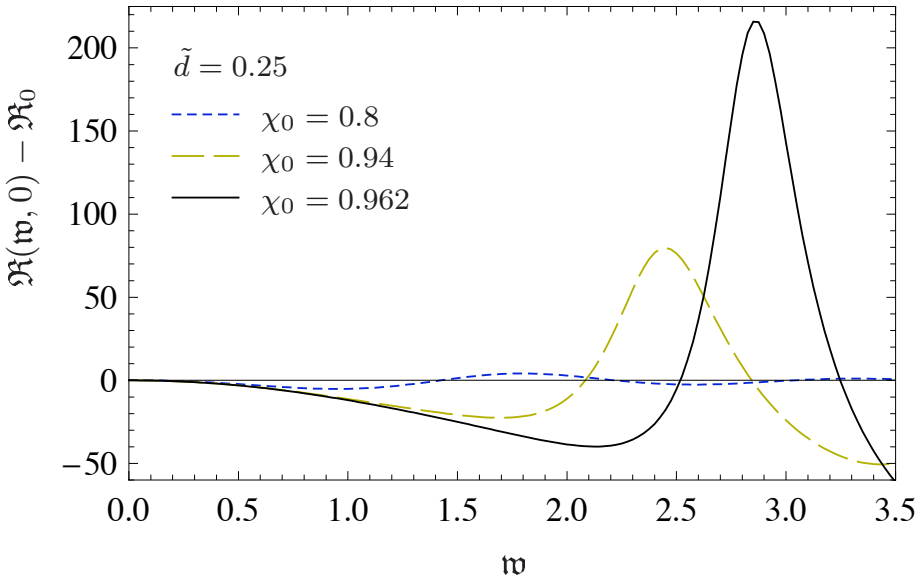

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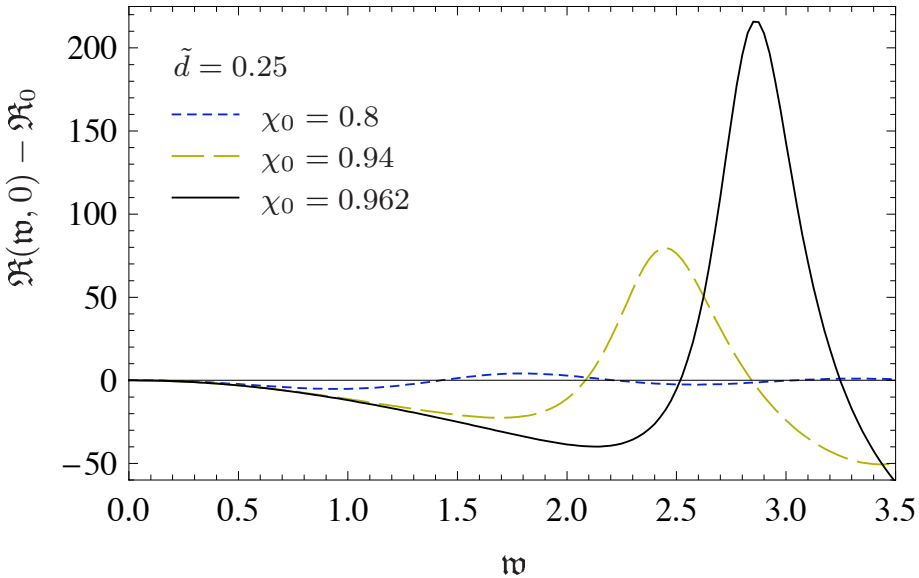

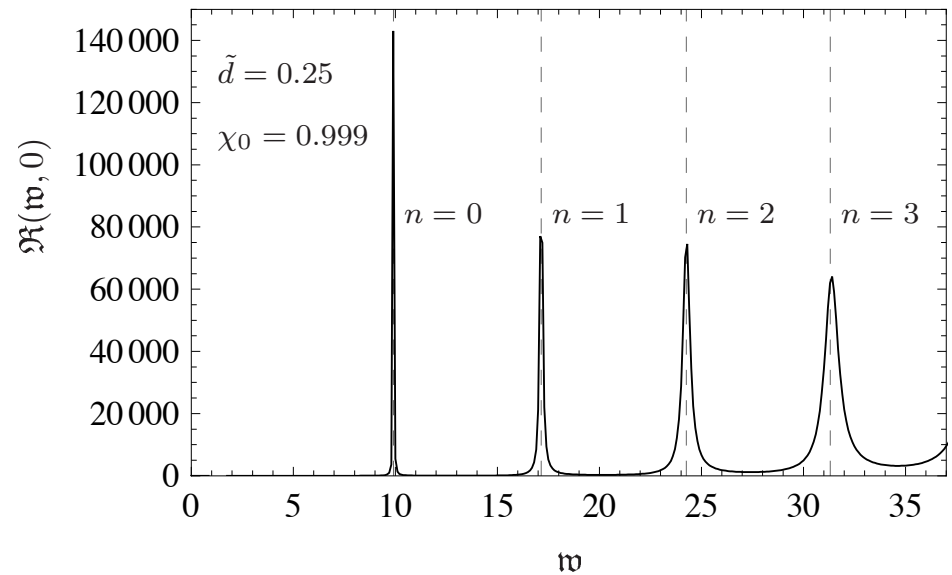

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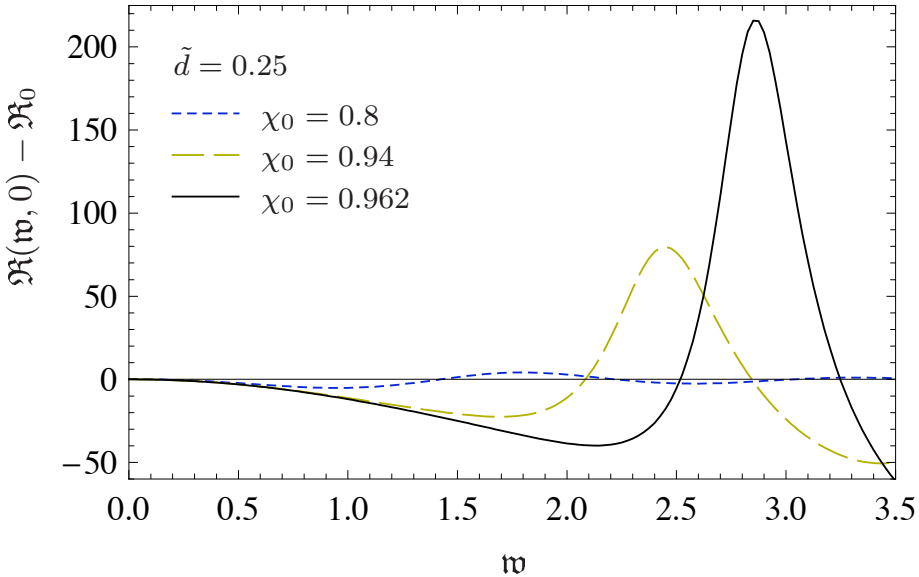
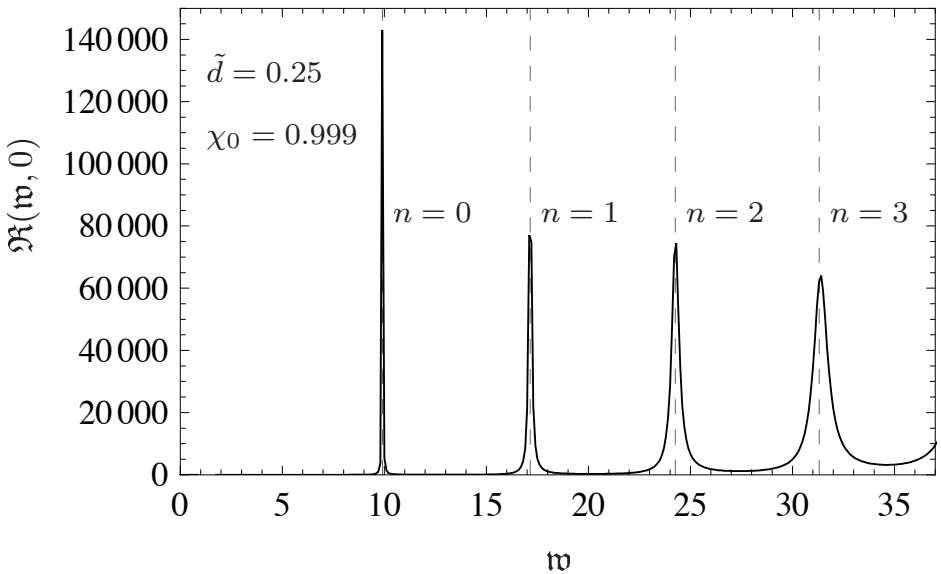
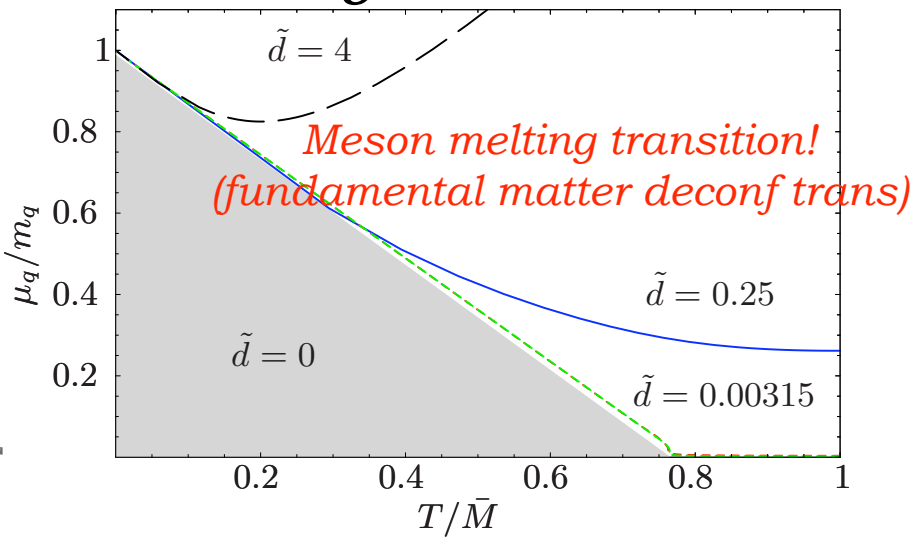




Phase diagram

[Karch, O'Bannon '07]



Gauge theory results: spectral functions

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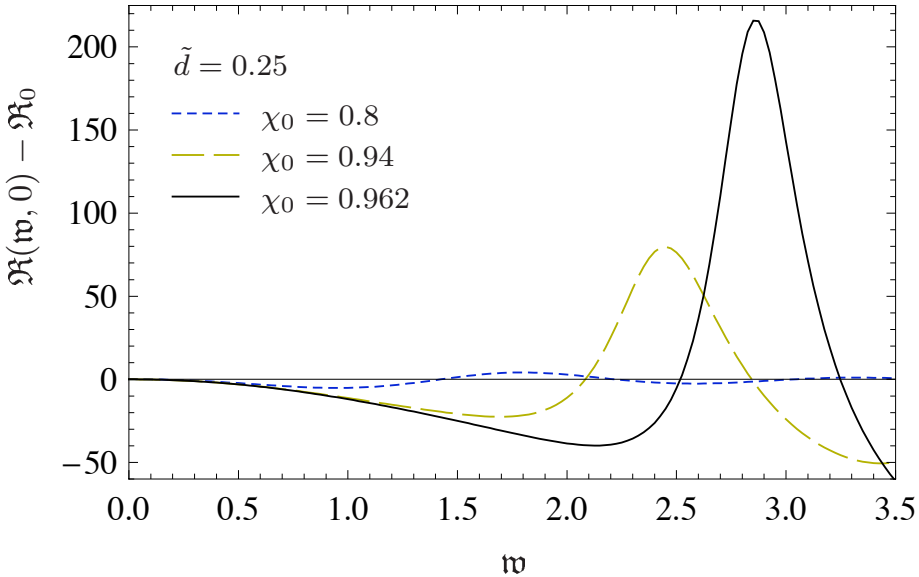

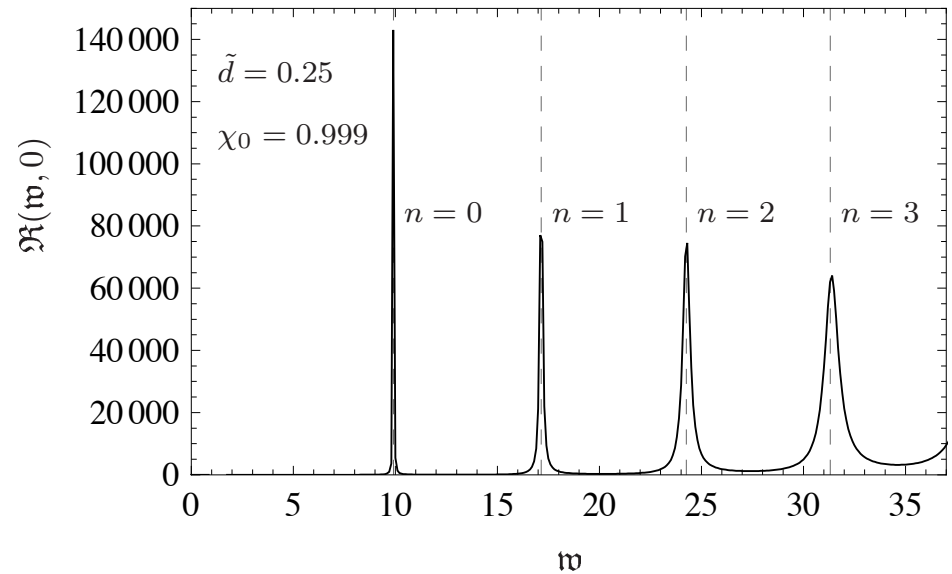

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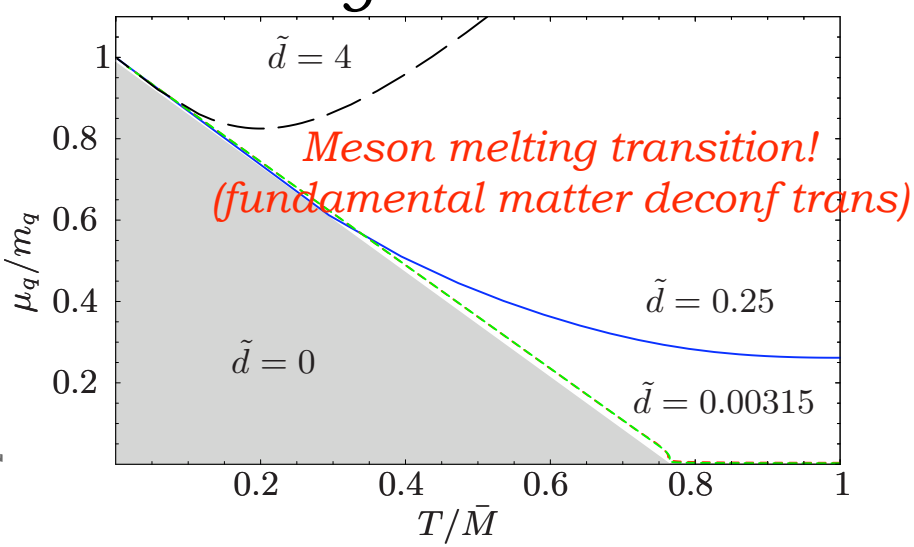
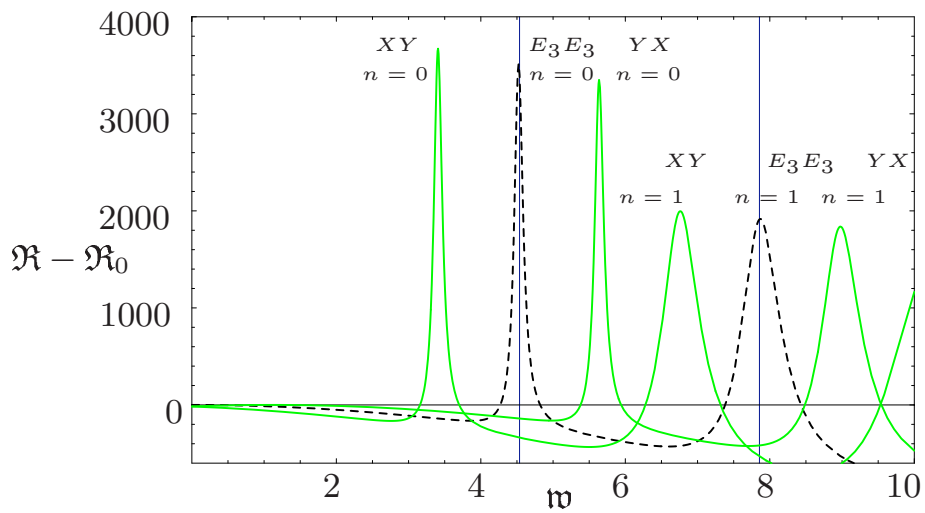


Finite isospin density



Gauge theory results: spectral functions

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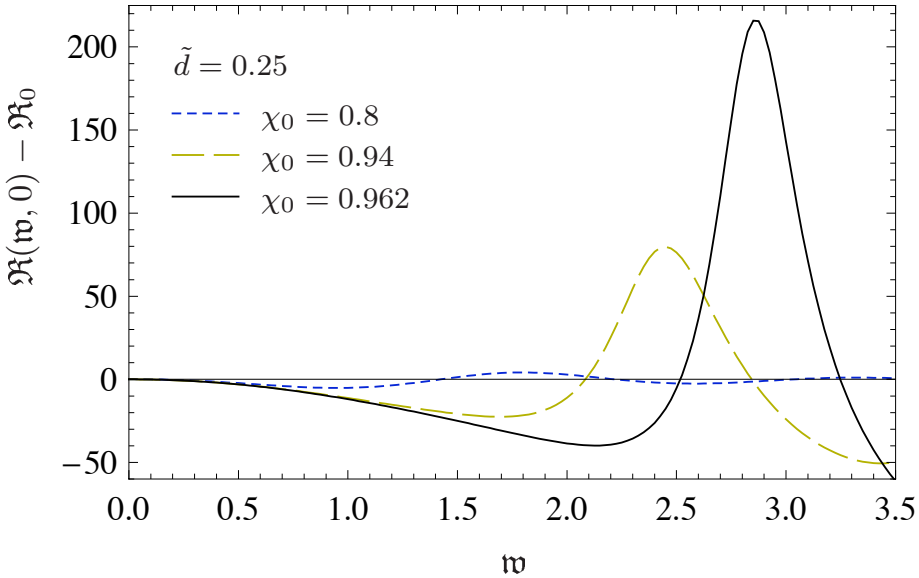

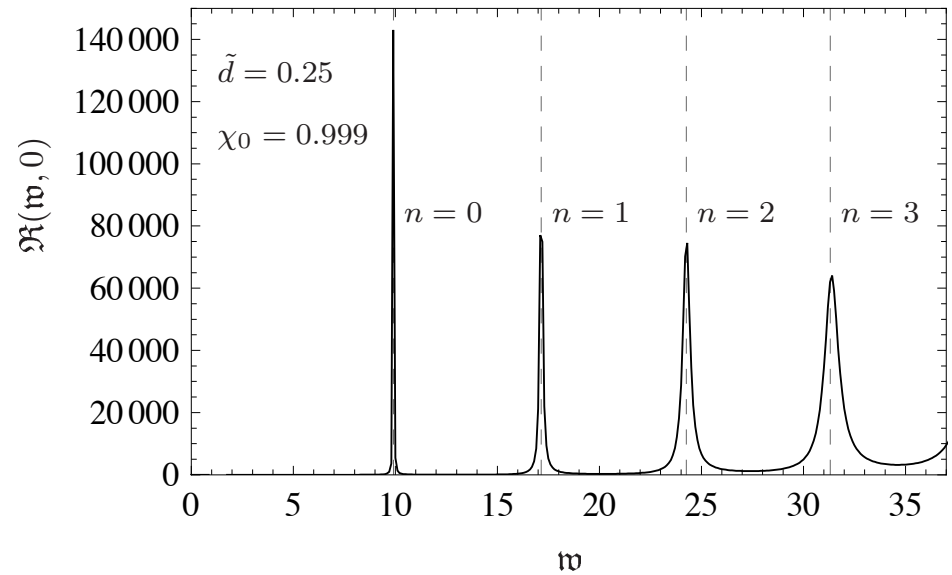

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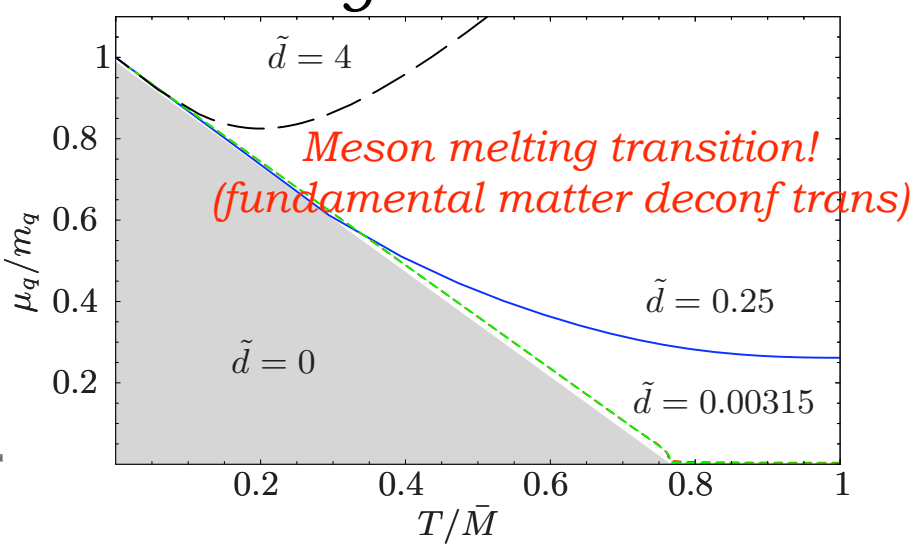

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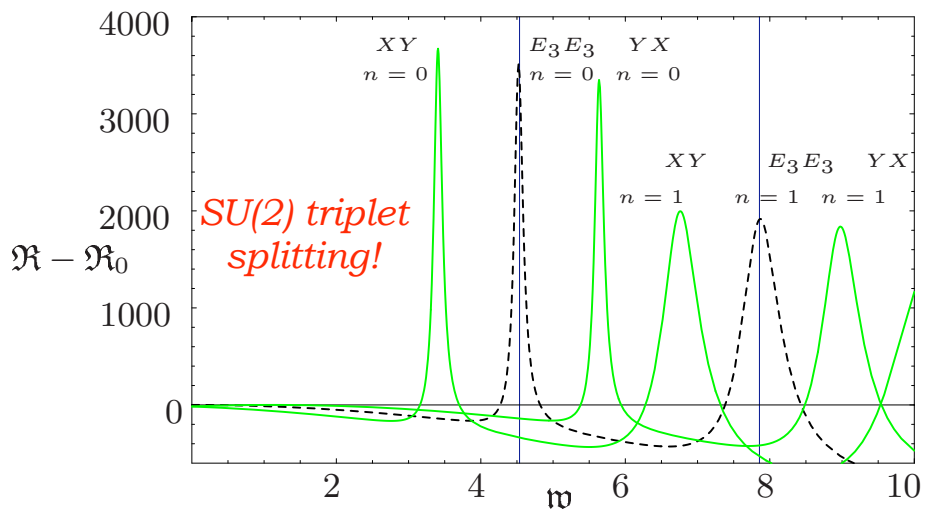
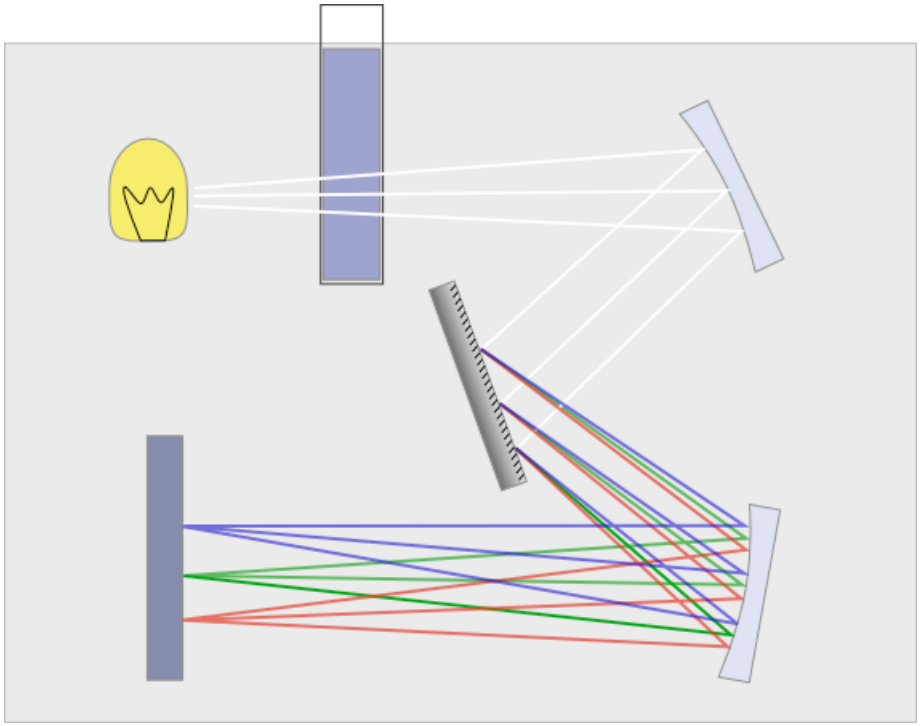


Analytically: [PhD thesis '08]

Lecture IV: Beyond Hydrodynamics

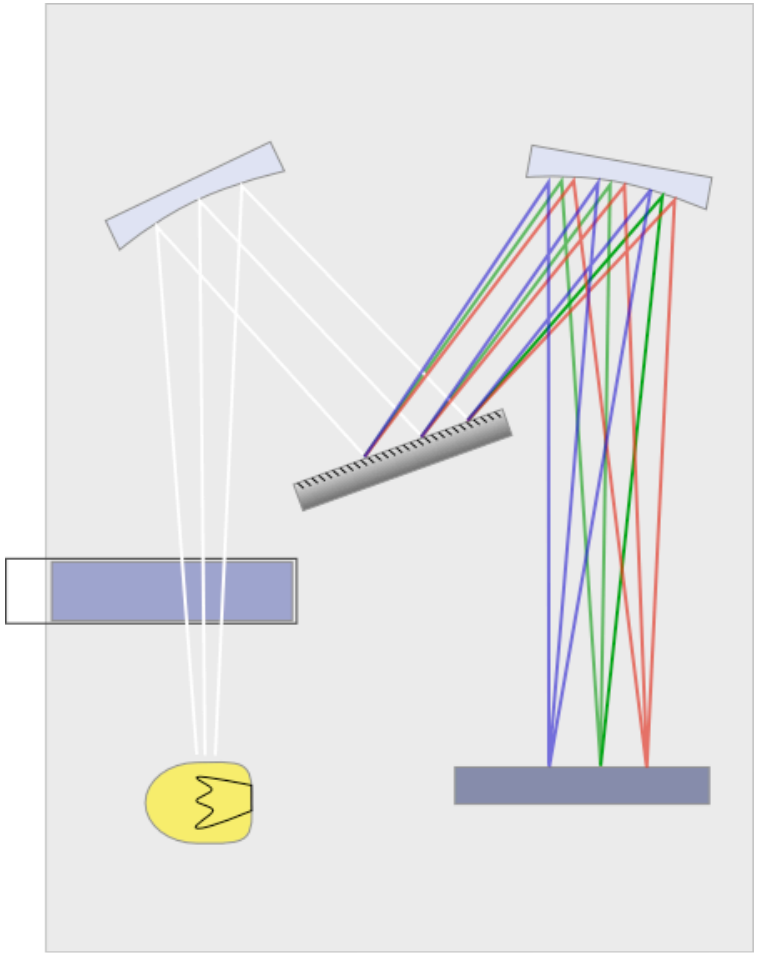
IV. Thermal Spectral Function



Thermal spectral function $\mathfrak{R}$ contains all information about diffusion and quasiparticle resonances in QG-plasma.

$$\mathfrak{R}(\omega,\, \mathbf{q}) = -2\,\mathrm{Im}G^{\mathrm{ret}}(\omega,\, \mathbf{q})$$

IV. Thermal Spectral Function



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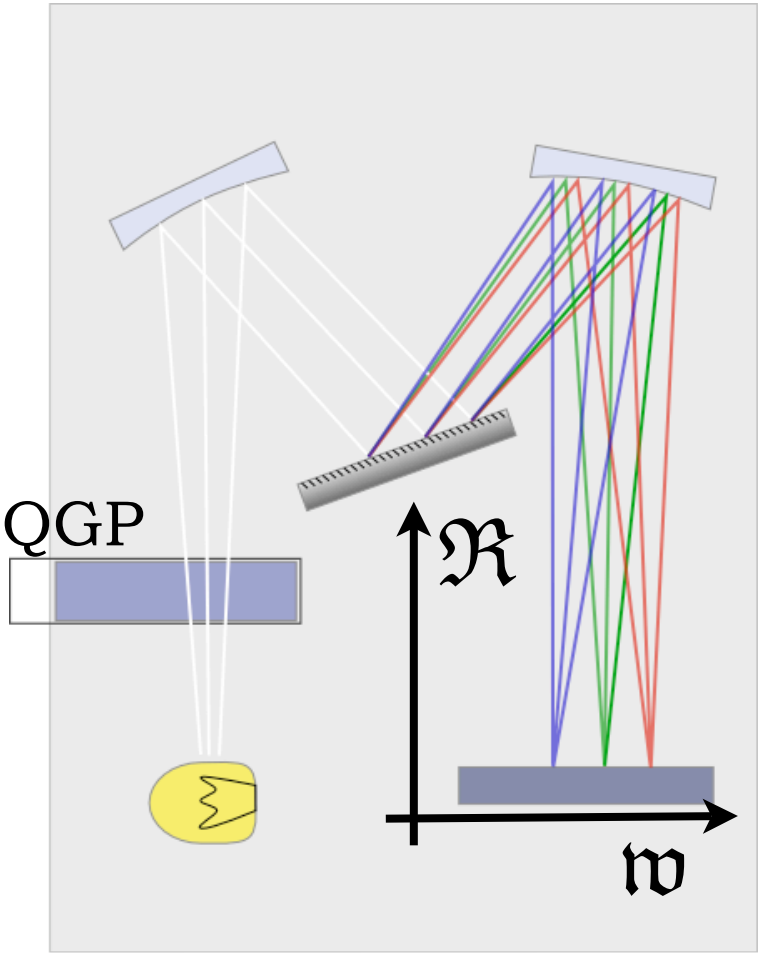

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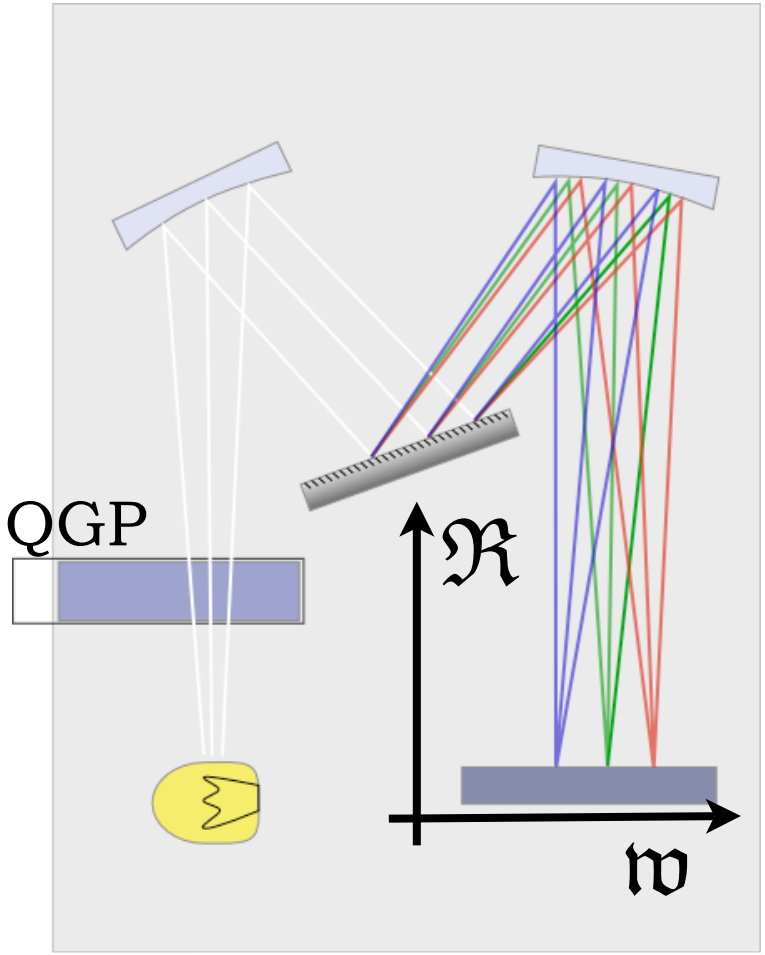

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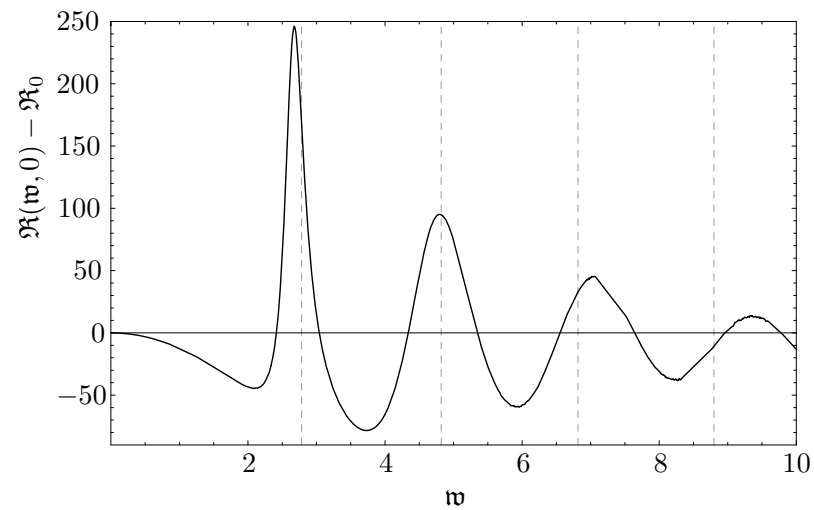

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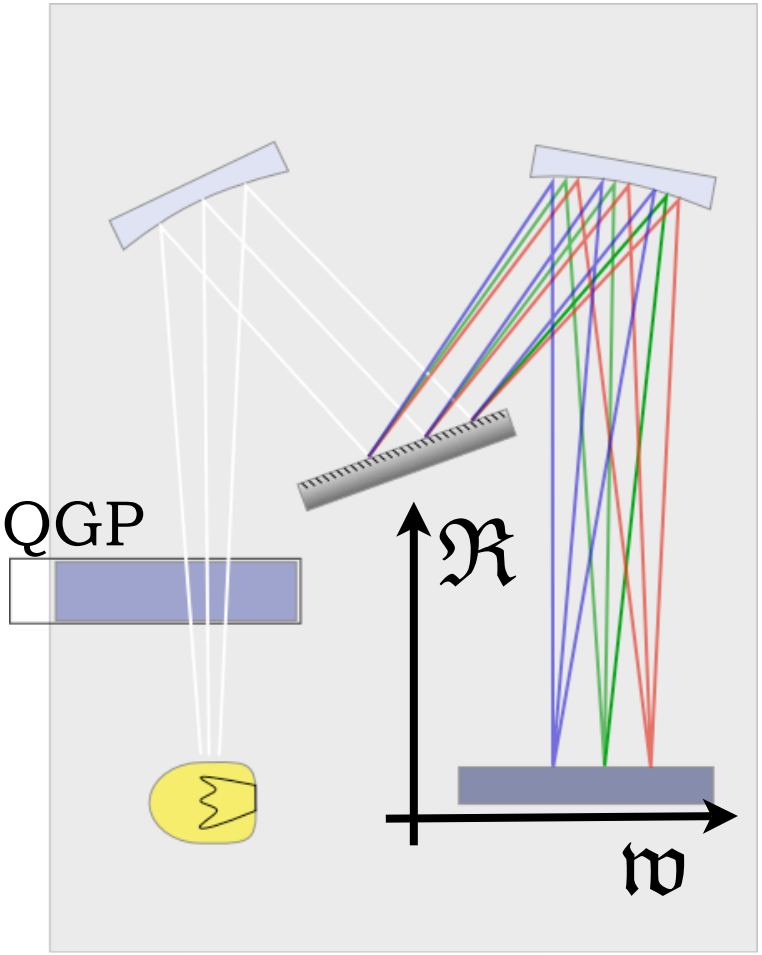

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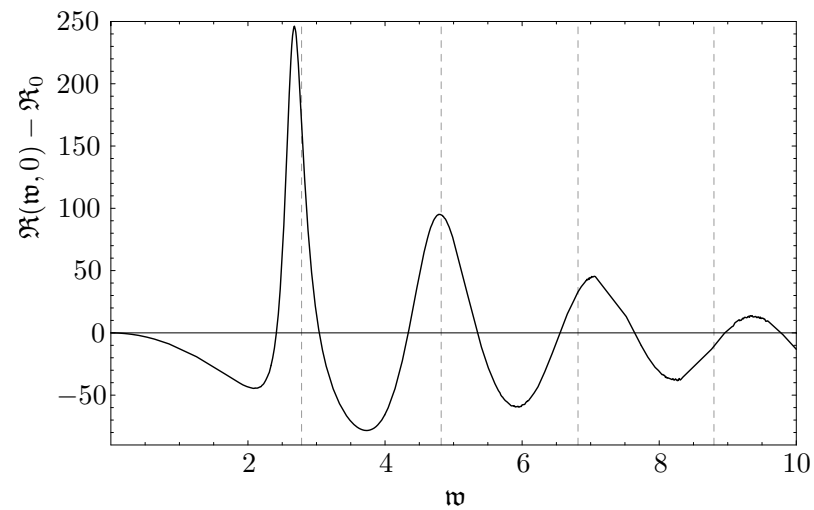

Transport coefficients using Kubo formulae, e.g.

$$\sigma \sim \lim_{\omega\to 0} \frac{1}{\omega}\langle [J^t, J^t]\rangle$$

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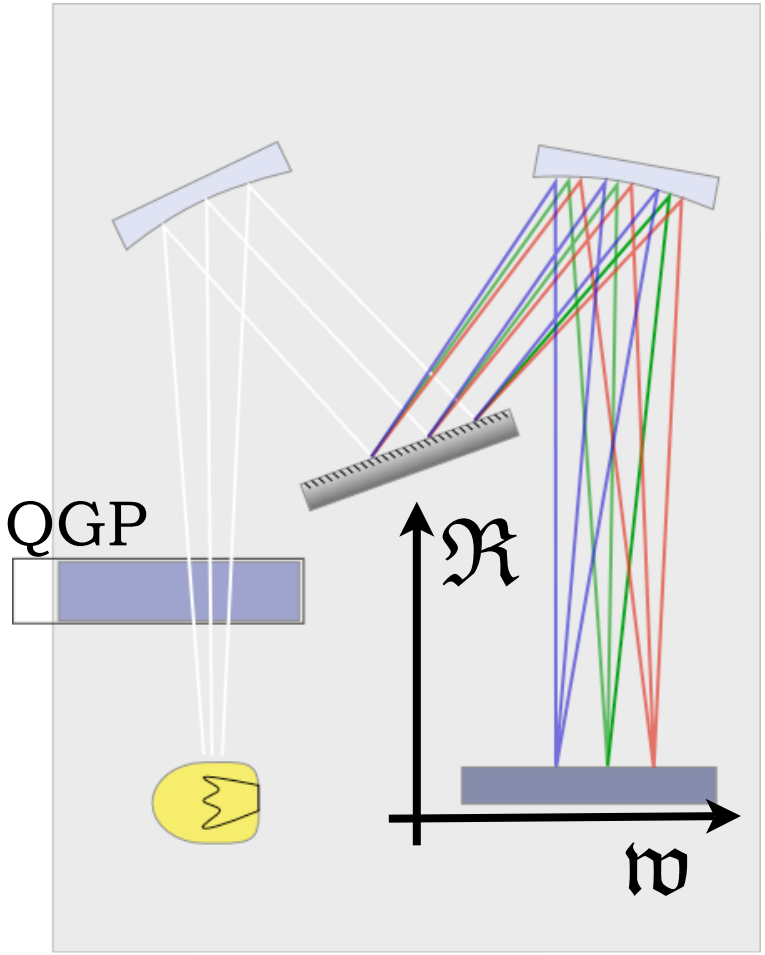

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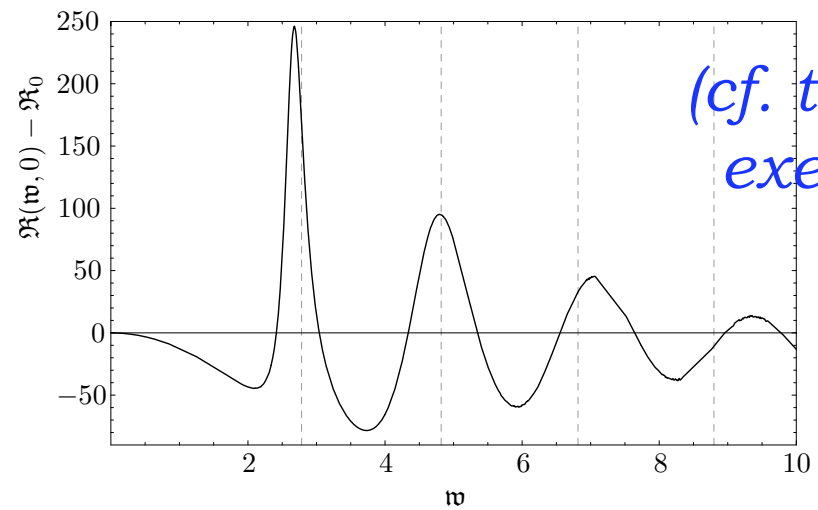

(cf. today's exercise)

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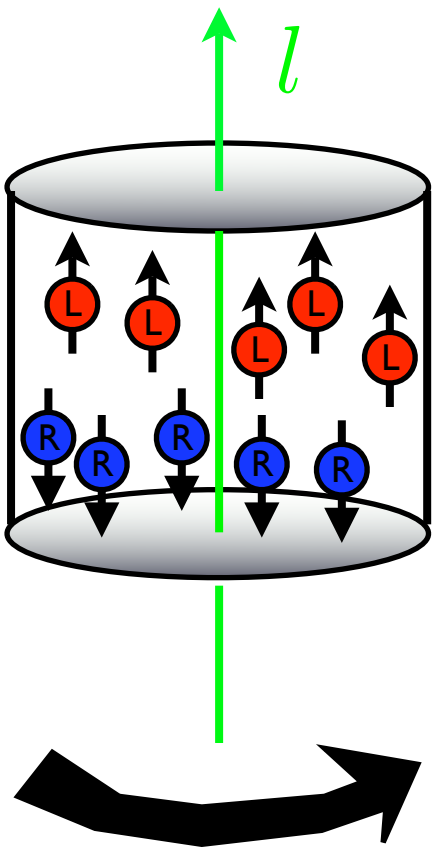
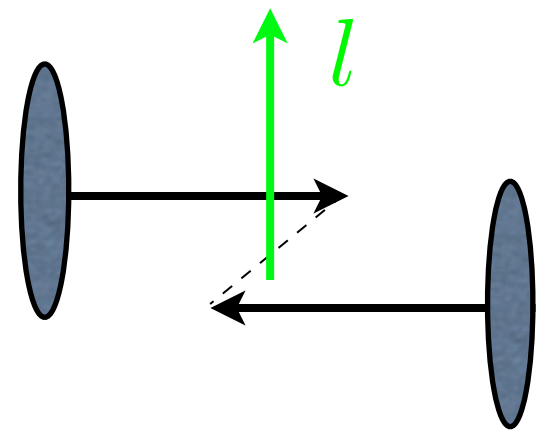
IV. Chiral transport effects in QGP

Chiral vortex effect



Heavy-ion-collision



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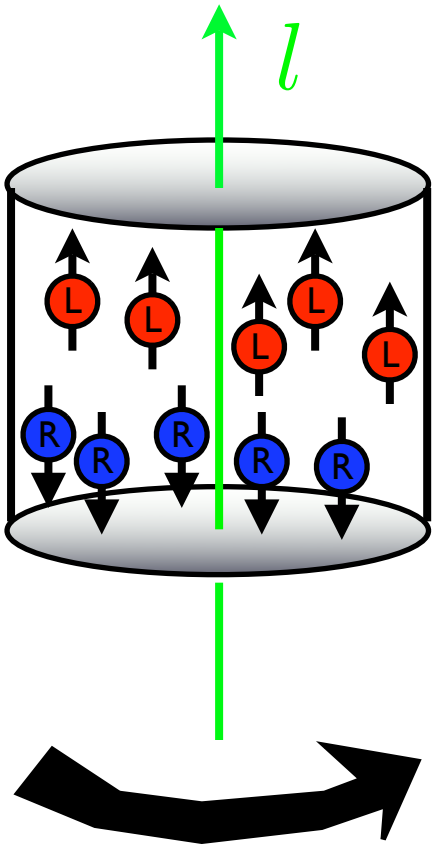

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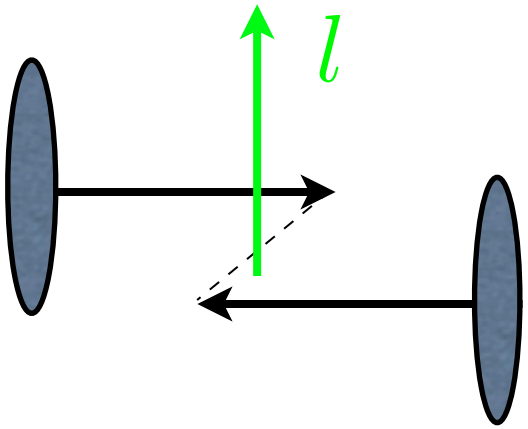
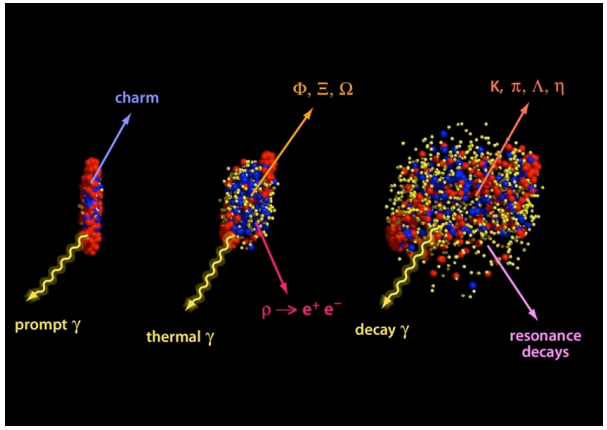
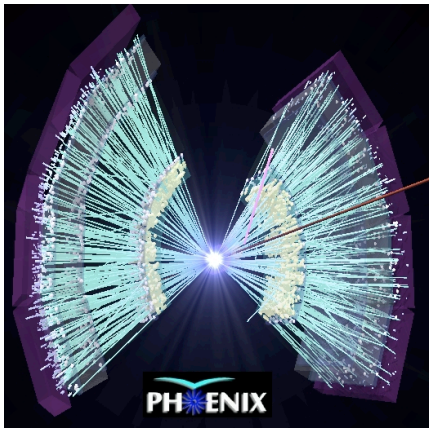






(similar: chiral magnetic effect)

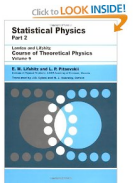
IV. Chiral transport

-First order hydrodynamics

Relativistic fluids with one conserved charge, with an anomaly (chiral or parity)

Conservation equations

$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \qquad \nabla_\mu j^\mu = C E^\mu B_\mu$$



Constitutive equations

$$T^{\mu\nu} = \frac{\epsilon}{3}(4u^\mu u^\nu + g^{\mu\nu}) + \Pi^{\mu\nu}$$

$$j^\mu = n u^\mu - \sigma T (g^{\mu\nu} + u^\mu u^\nu)\partial_\nu \left(\frac{\mu}{T}\right) + \xi \omega^\mu \qquad \omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$

[Erdmenger, Haack, M.K., Yarom 0809.2488]

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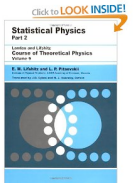

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from a gravity dual

$$S = -\frac{1}{16\pi G_5}\int \left[\sqrt{-g}\left(R + 12 - \frac{1}{4}F^2\right) - \frac{1}{12\sqrt{3}}\epsilon^{MNOPQ} A_M F_{NO} F_{PQ}\right] d^5x$$

[Erdmenger, Haack, M.K., Yarom 0809.2488]

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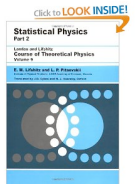

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[Erdmenger, Haack, M.K., Yarom 0809.2488]

New vorticity term arises!
related to triangle anomaly

$$\partial_\mu j^\mu = -\frac{1}{8} C \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{\mu^3 n}{\epsilon + P}\right)$$

[Son,Surowka 0906.5044]

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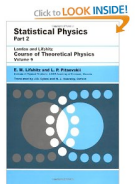

Constitutive equations

$$T^{\mu\nu} = \frac{\epsilon}{3}(4u^\mu u^\nu + g^{\mu\nu}) + \Pi^{\mu\nu}$$

$$j^\mu = n u^\mu - \sigma T (g^{\mu\nu} + u^\mu u^\nu)\partial_\nu \left(\frac{\mu}{T}\right) + \xi \omega^\mu \qquad \omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$

from a gravity dual

$$S = -\frac{1}{16\pi G_5}\int \left[\sqrt{-g}\left(R + 12 - \frac{1}{4}F^2\right) - \frac{1}{12\sqrt{3}}\epsilon^{MNOPQ} A_M F_{NO} F_{PQ}\right] d^5x$$

[Erdmenger, Haack, M.K., Yarom 0809.2488]

New vorticity term arises!
related to triangle anomaly

$$\partial_\mu j^\mu = -\frac{1}{8} C \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{\mu^3 n}{\epsilon + P}\right)$$

[Son,Surowka 0906.5044]

Fixed by anomaly coefficient!

Remarks & ideas

ξ

ξ

Remarks & ideas

- New coefficient at first order hydrodynamics (~viscosity)

ξ

ξ

Remarks & ideas

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- ξ completely determined by C and equation of state

$$\xi$$

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→ Relativistic hydrodynamics needs to be completed. *[Baier et al, Minwalla et al 2008]*

→ Effects measured in heavy-ion-collisions? *[Kharzeev, Son]*

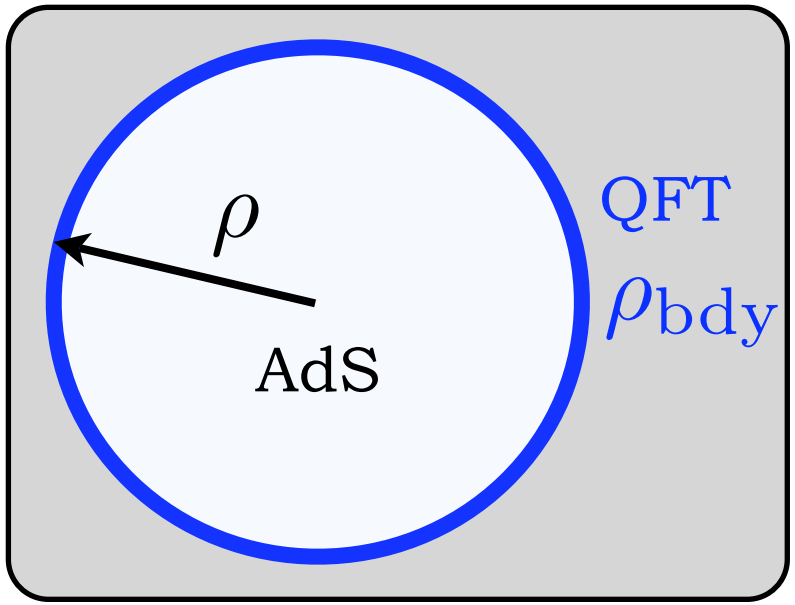
→ Not in non-relativistic setups, so repeat for 2+1 dimensional QFT, with condensed matter applications in mind (parity anomaly?) *[Nicolis & Son,1103.2137]* *[1106.xxxx]*

IV. Concepts -the dictionary

gravity theory (weak)	$\longleftrightarrow$	gauge theory (strong)

Near AdS-boundary $(\rho \to \infty)$

$$A = \underbrace{A^{(0)}}_{\text{non-normalizable (source)}} + \frac{\overbrace{A^{(2)}}^{\text{normalizable (vev)}}}{\rho^2} + \cdots$$



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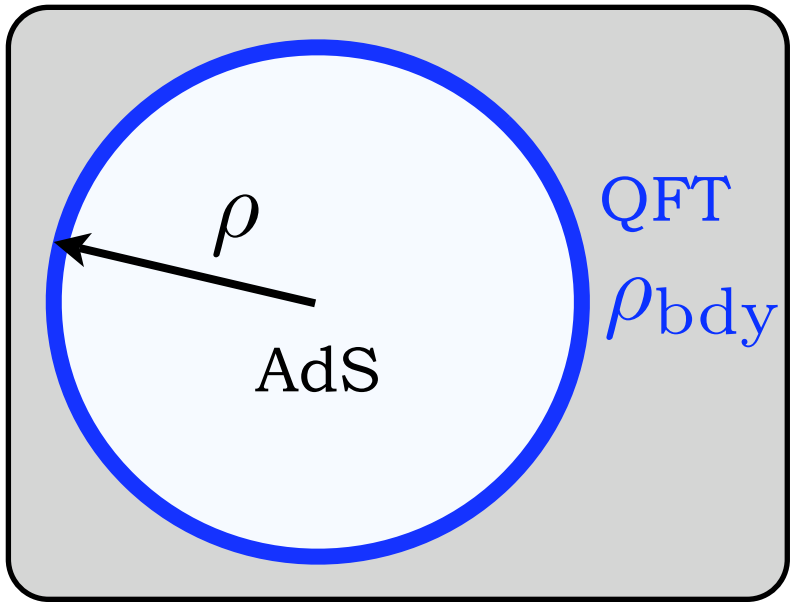

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Dictionary

QFT FEATURE (energy scale) $\longleftrightarrow$ GEOMETRY (radial coord. ρ)

operator J_μ $\longleftrightarrow$ field A_μ (gauge)

vev $\longleftrightarrow$ $A^{(2)}$ (charge)

source $\longleftrightarrow$ $A^{(0)}$ (chem. pot.)

IV. Quasinormal modes

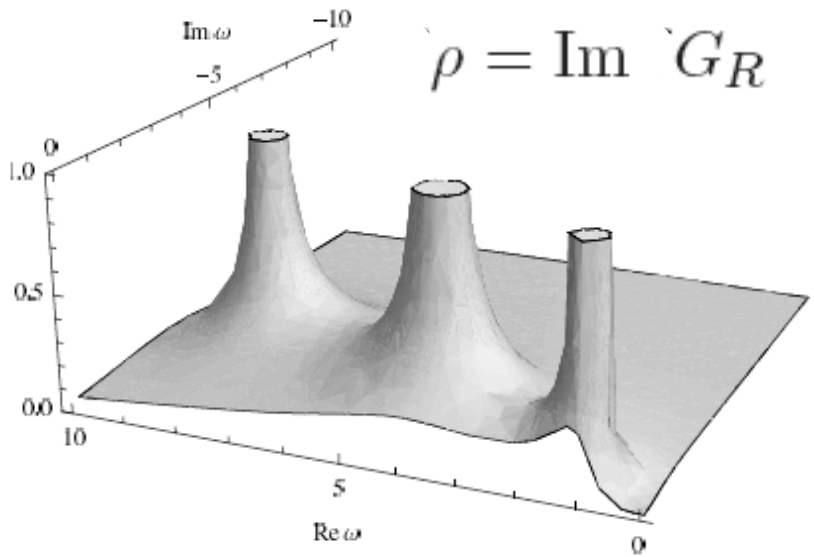
e.g. [Berti et al. '09]

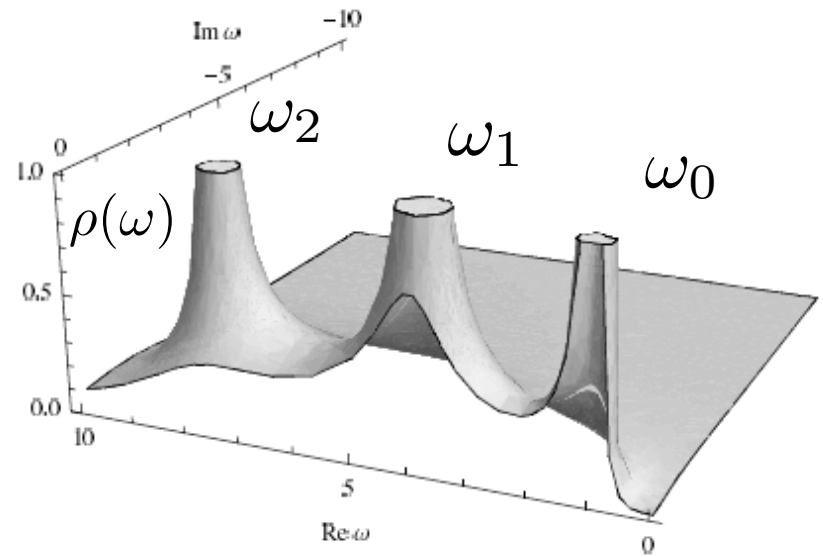
Special frequencies: (quasinormal) $\omega_{\rm n} \in \mathbb{C}\,;$ $\lim\limits_{\rho\to\rho_{\rm bdy}} |\tilde{A}(\omega_{\rm n})|^2 = 0$

$$e^{-i\omega r} = e^{-i{\rm Re}\{\omega\}r} e^{{\rm Im}\{\omega\}r}$$

Example:

$$G^{\rm ret} = \frac{N_f N_c T^2}{8} \lim_{\rho\to\rho_{\rm bdy}} \left(\rho^3 \frac{\partial_\rho \tilde{A}(\rho)}{\tilde{A}(\rho)}\right)$$





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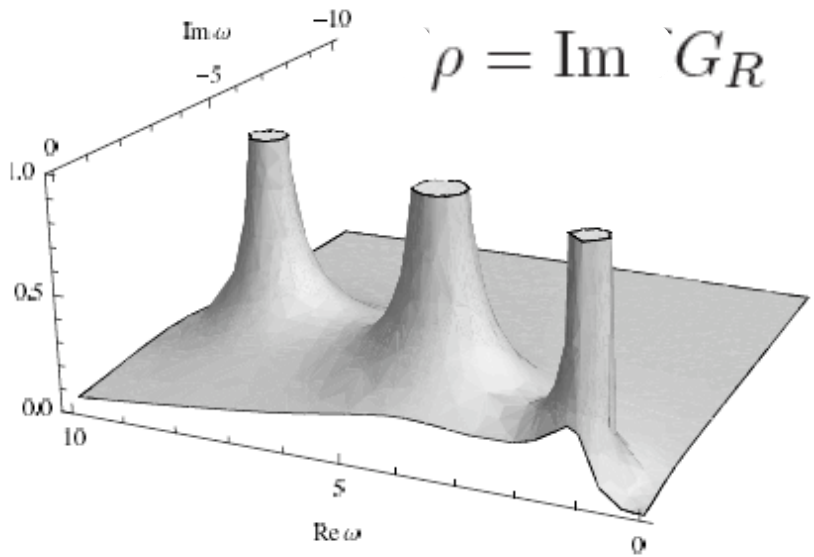
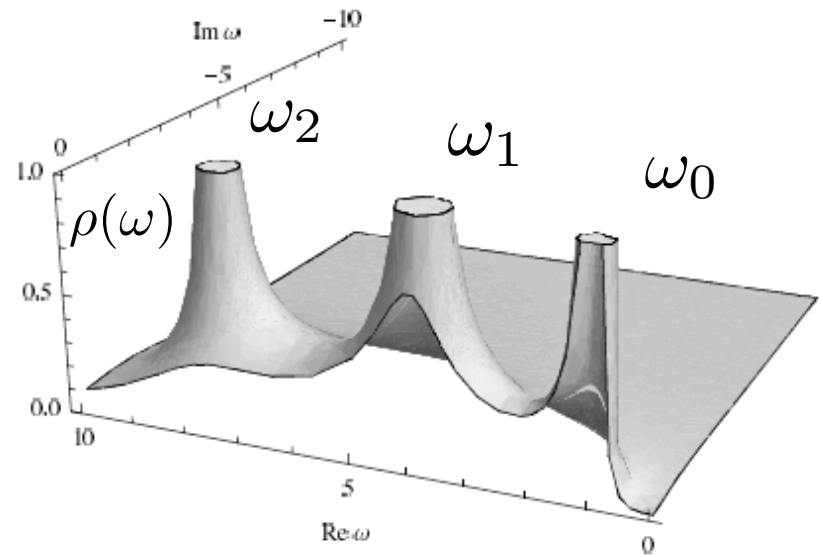
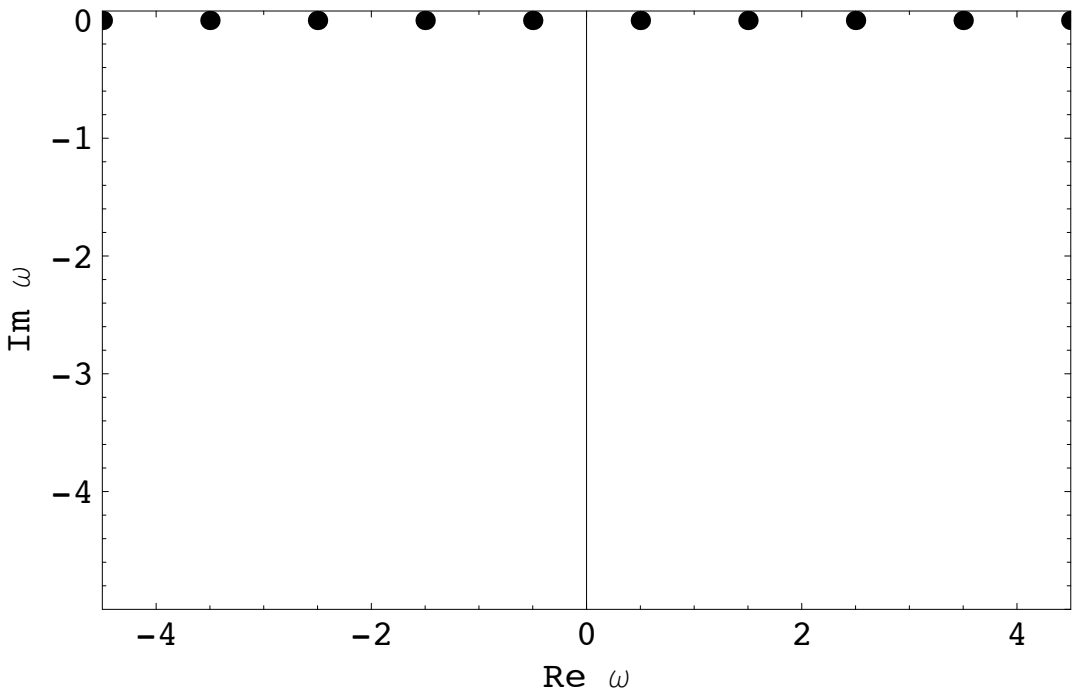




Gravity: quasinormal frequencies $\longleftrightarrow$ Gauge theory: poles of correlator (energy, damping, stability of mesonic excitations)

IV. Quasi Normal Modes (QNMs)

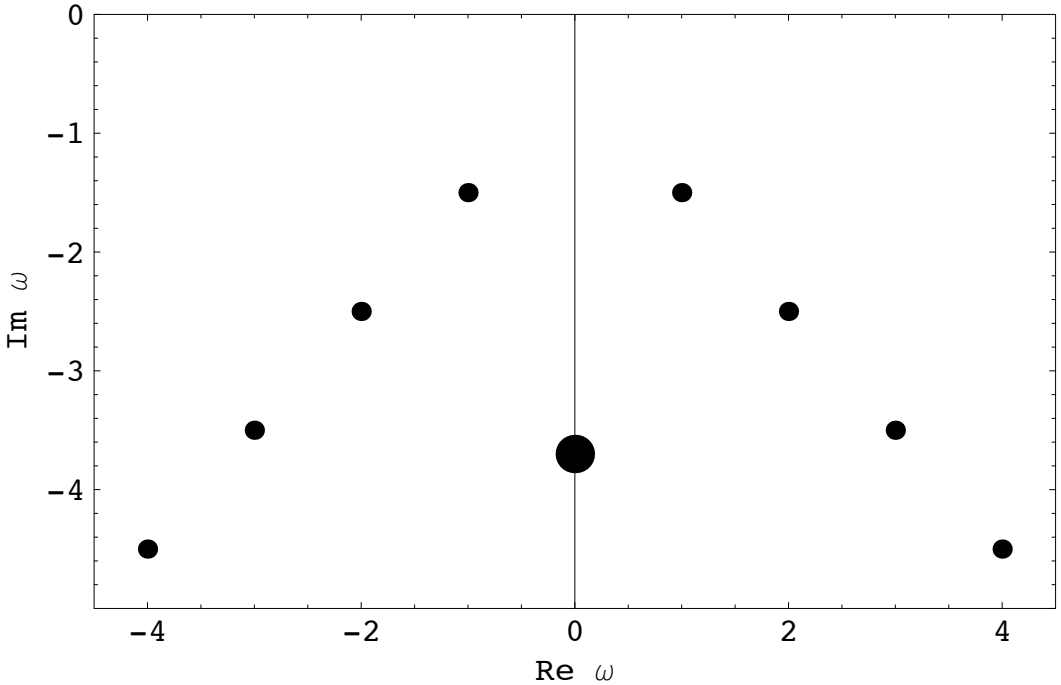


quasinormal frequencies

Simple example:
Eigenfrequencies / normal modes
of the quantum mechanical
harmonic oscillator
(no damping)

$$\omega_n = \frac{1}{2} + n$$

IV. Quasi Normal Modes (QNMs)



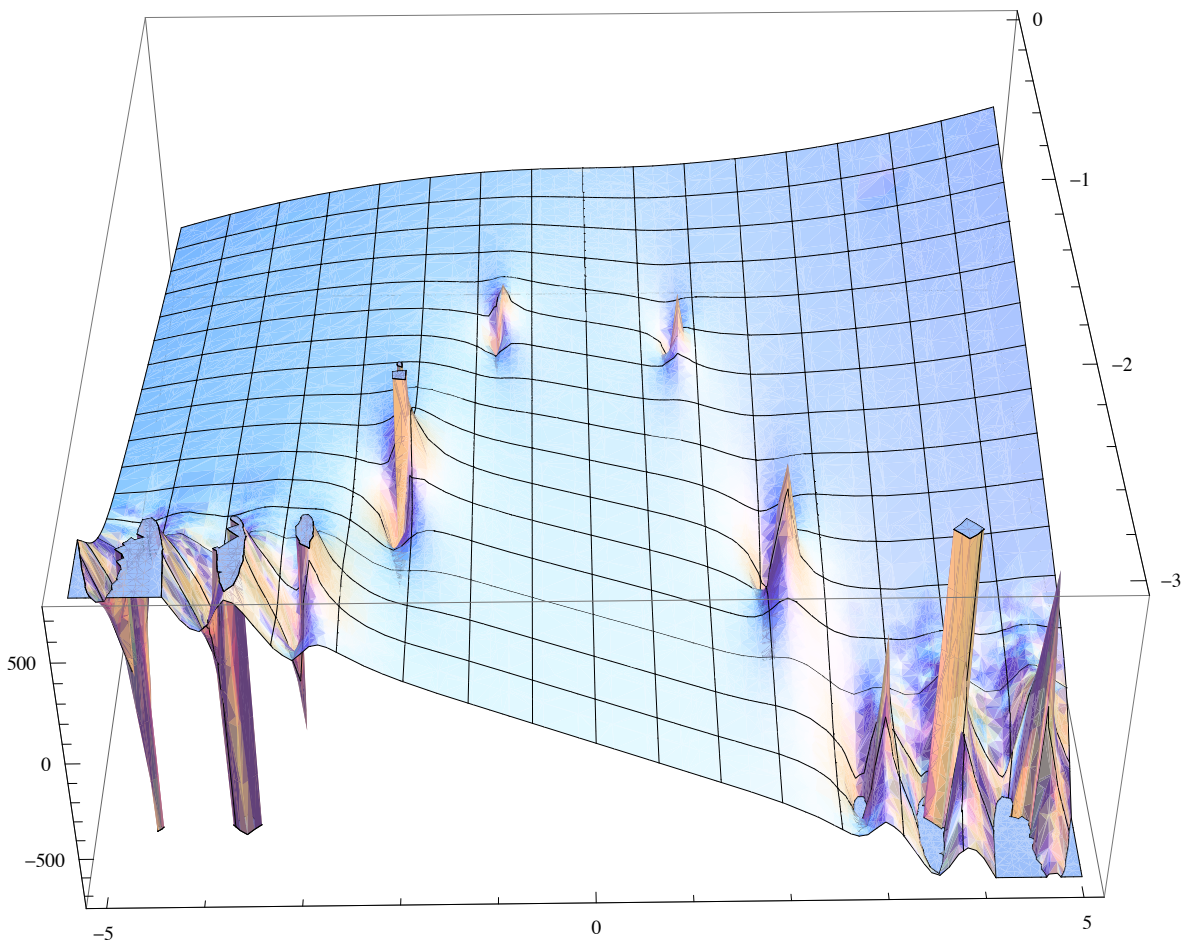
Example: Poles of charge current correlator

- QNMs are the quasi-eigenmodes of gauge field
- Dual QFT: lowest QNM identified with hydrodynamic diffusion pole (not propagating)
- Higher QN modes: gravity field waves propagate through curved b.h. background while decaying (dual gauge currents analogously)

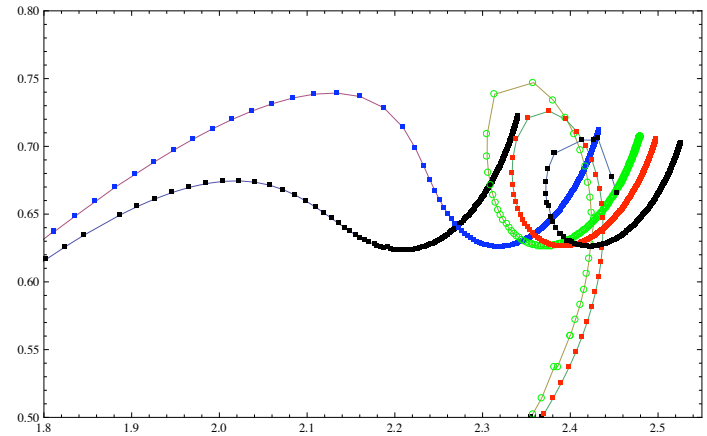
$$G_{ret} \propto \frac{1}{i\omega - Dq^2}$$

IV. Quasi Normal Modes (QNMs)

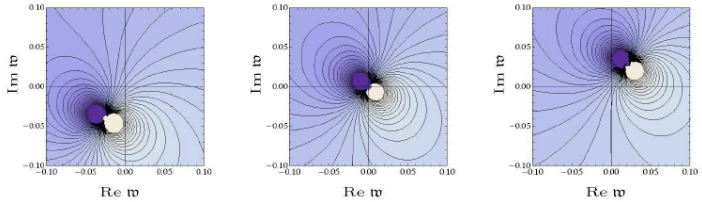
Complex frequency plane



Trajectories (dial k)



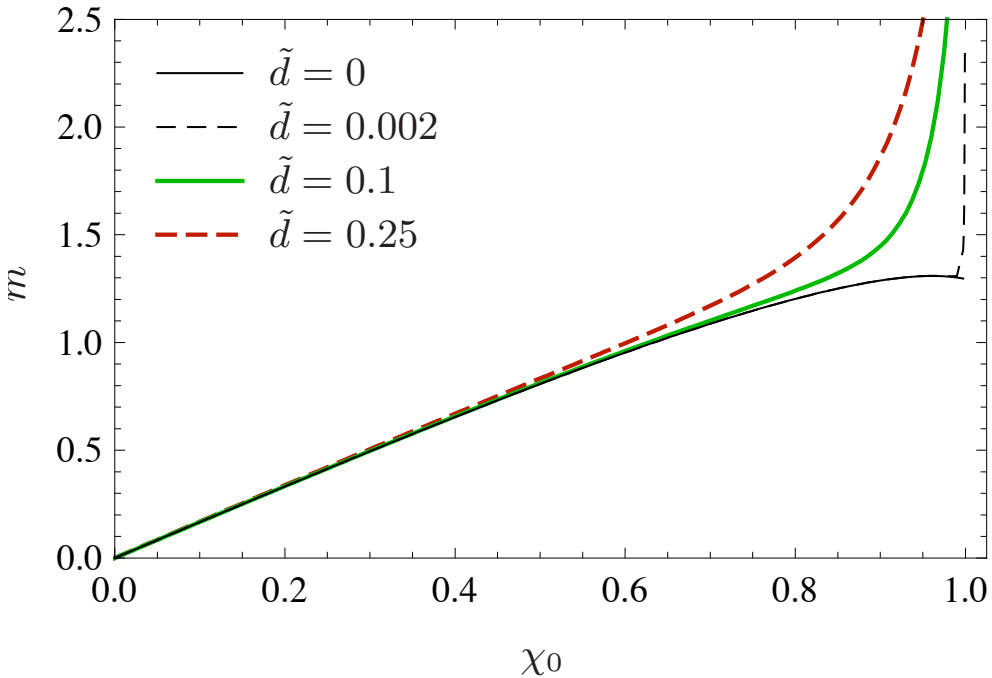
Instabilities



Lecture V: Phase Transitions

Result to 3.2 d) from exercise III

The mass parameter m depending on the parameter χ_0.



$$\chi_0 = \chi(\rho)\big|_{\rho \to \rho_H}$$

$$m = \lim_{\rho \to \rho_{\rm bdy}} \rho\, \chi(\rho) = \frac{2 m_{\rm quark}}{\sqrt{\lambda} T}$$

Near-boundary expansions:

$$\chi(\rho) = \frac{m}{\rho} + \frac{c}{\rho^3} + \ldots$$

$$A_0 = \mu - \frac{1}{\rho^2} \frac{\tilde{d}}{2\pi\alpha'} + \ldots$$

Other relations:

$$L(\varrho) = \varrho\, \chi(\varrho)\,, \quad \rho = \frac{\varrho}{\varrho_H}$$

Meson melting transition

- N_c D3-branes
  dual to $\mathcal{N} = 4$ SYM with $SU(N_c)$

Meson melting transition

N_f D7-branes

dual to $\mathcal{N}=2 \quad SU(N_f) \quad \text{flavor}$

[Karch,Katz hep-th/0205236]

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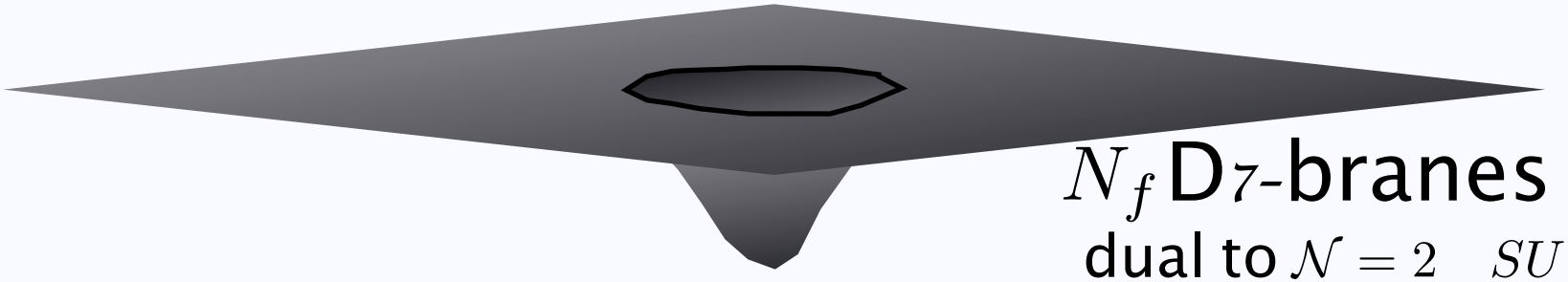

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dual to $\mathcal{N}=2$ $SU(N_f)$ flavor

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N_c D3-branes (black)

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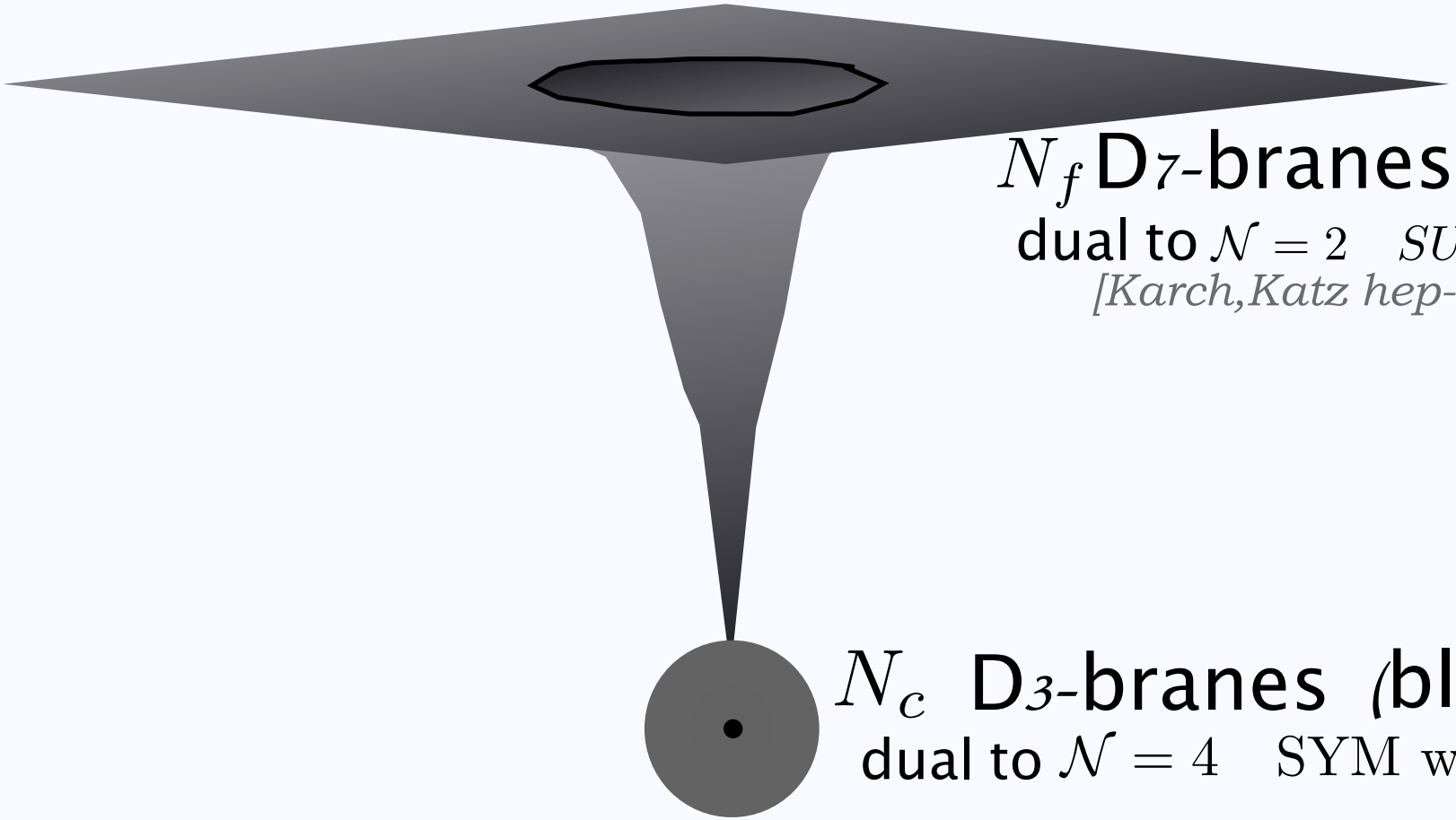

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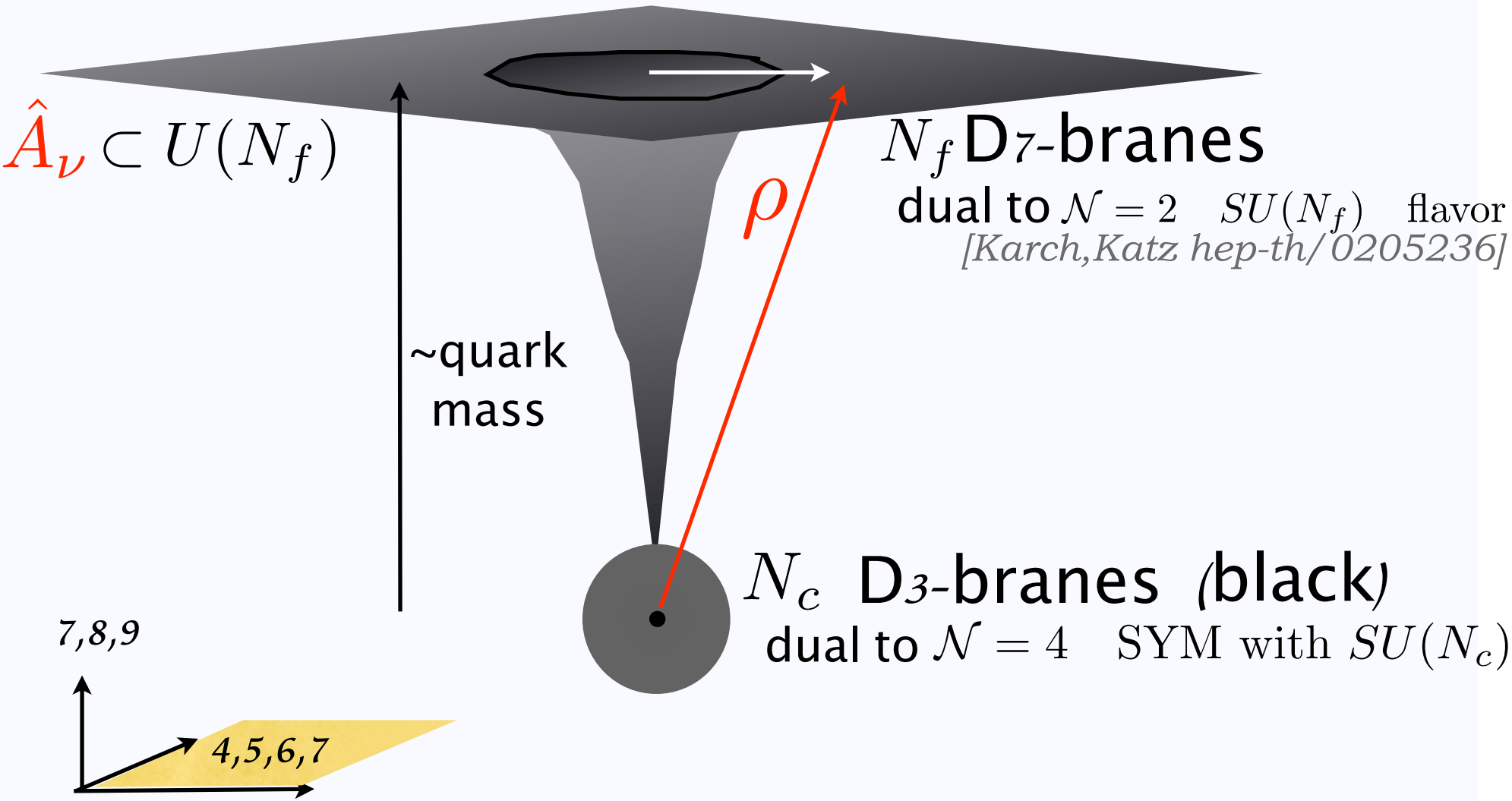

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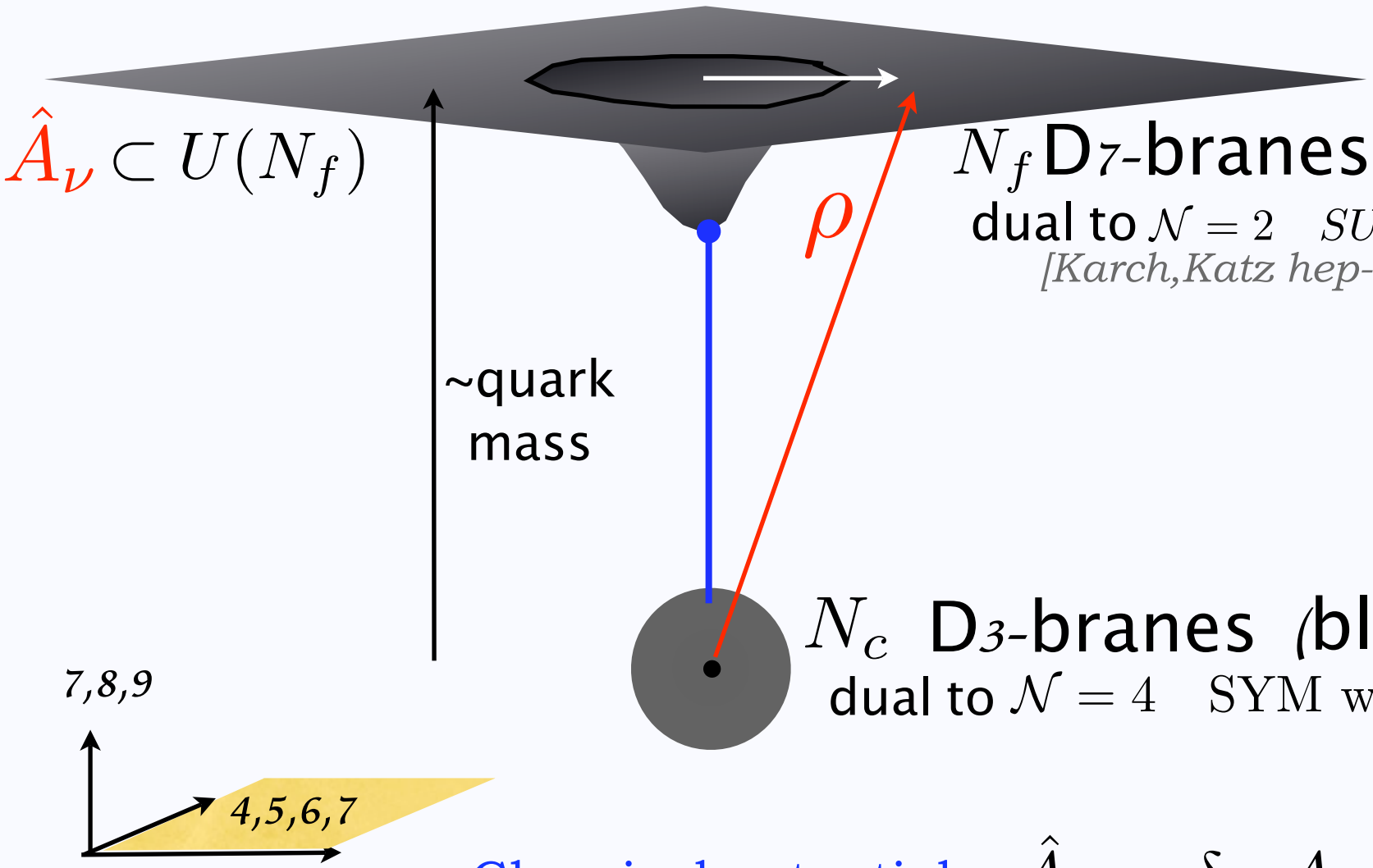

Chemical potential: $\hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu$

(cf. therm. FT)

[Myers et al., hep-th/0611099]
[Mateos et al. 0709.1225]

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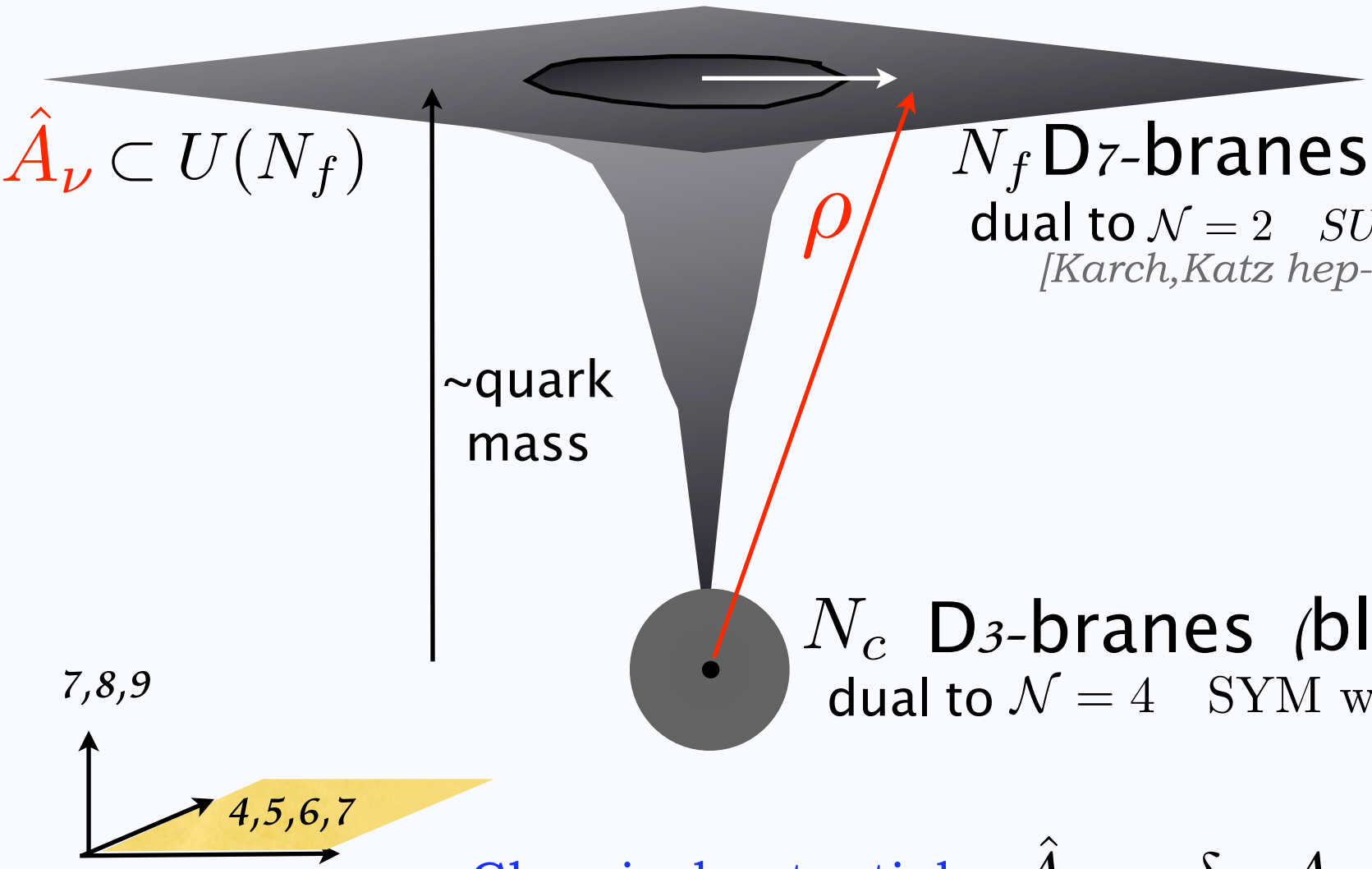

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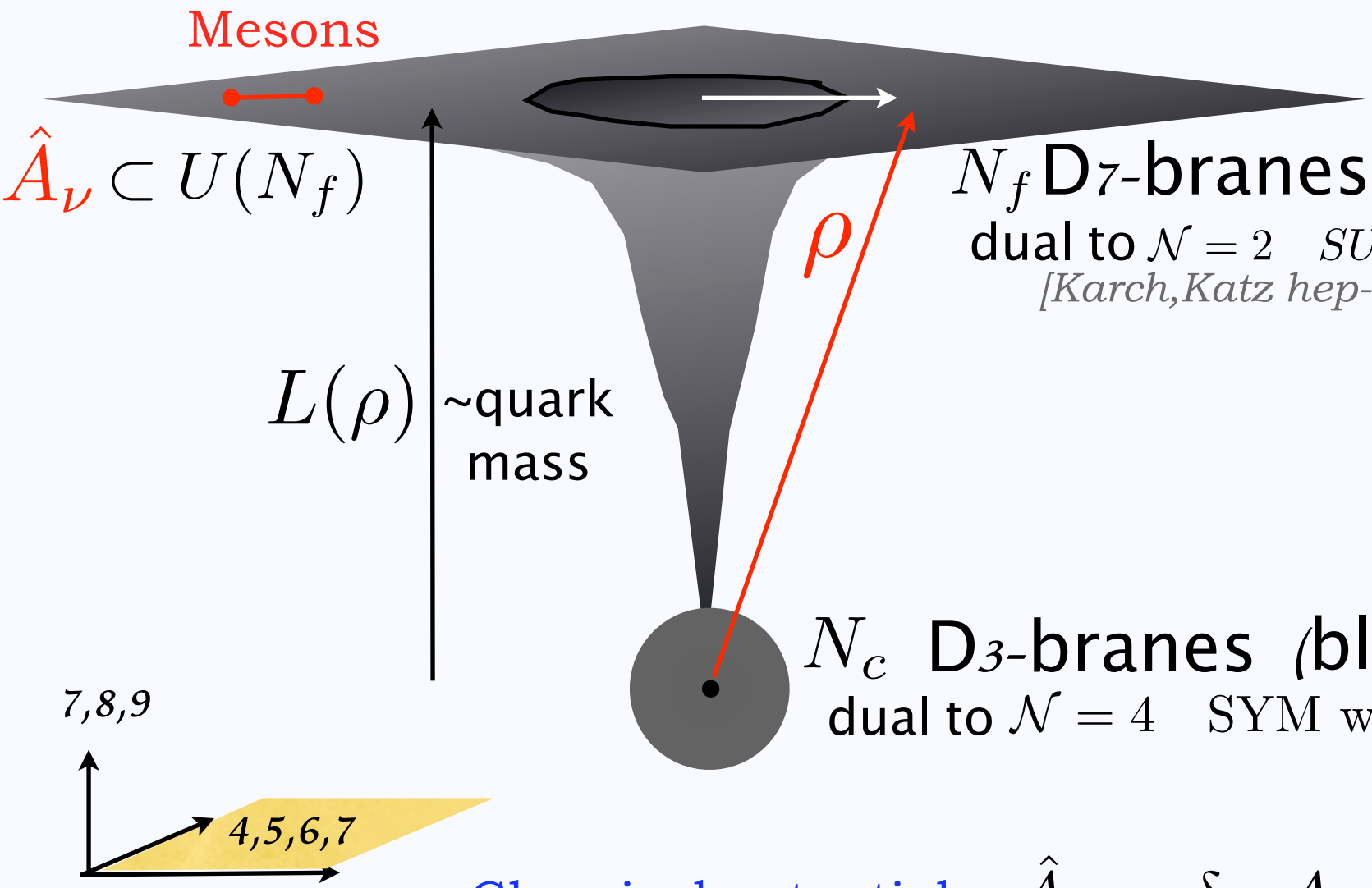

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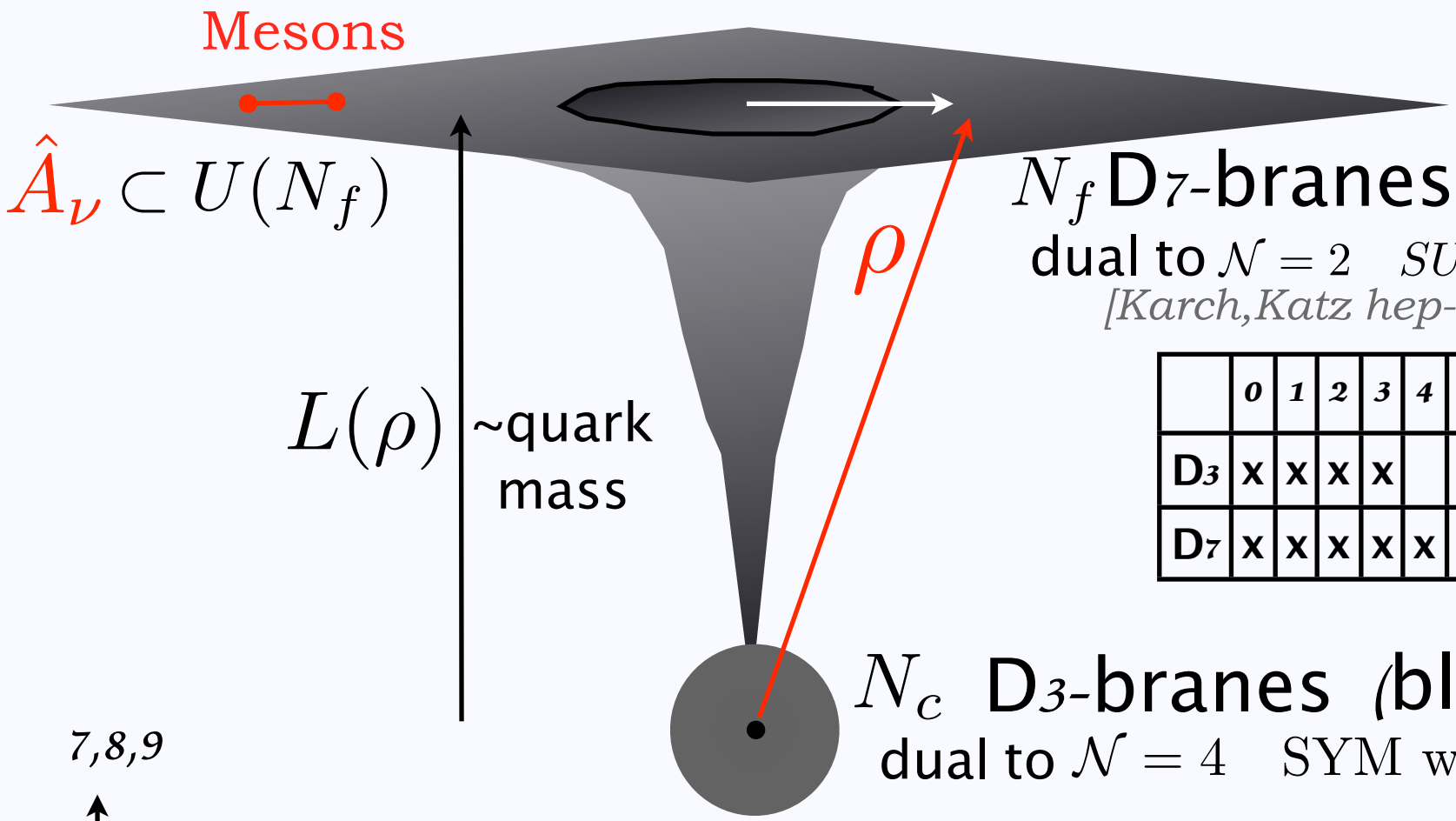

	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x						
D7	x	x	x	x	x	x	x	x		

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Gravity solution & translation

[Erdmenger, M.K., Rust 0710.0334]

Effective action: $$S_{\mathrm{D7}} = \int \mathrm{d}^8 x \sqrt{\left|\det\{[g+F]+\tilde{F}\}\right|}\ , \quad F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$$

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"Background": Brane embedding & gauge field

$$S_{\mathrm{DBI}} = -N_f T_{\mathrm{D7}} \varrho_H^3 \int \mathrm{d}^8\xi\, \frac{\rho^3}{4} f \tilde{f} (1-\chi^2)$$
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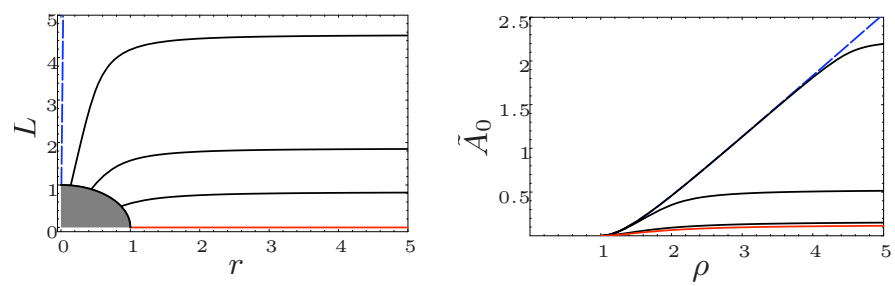
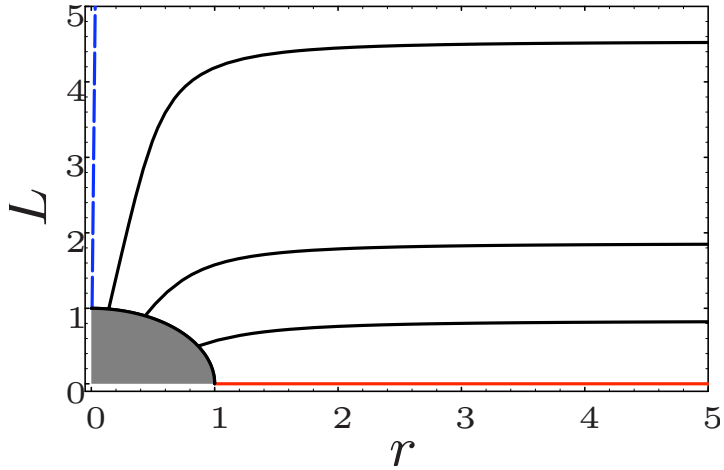
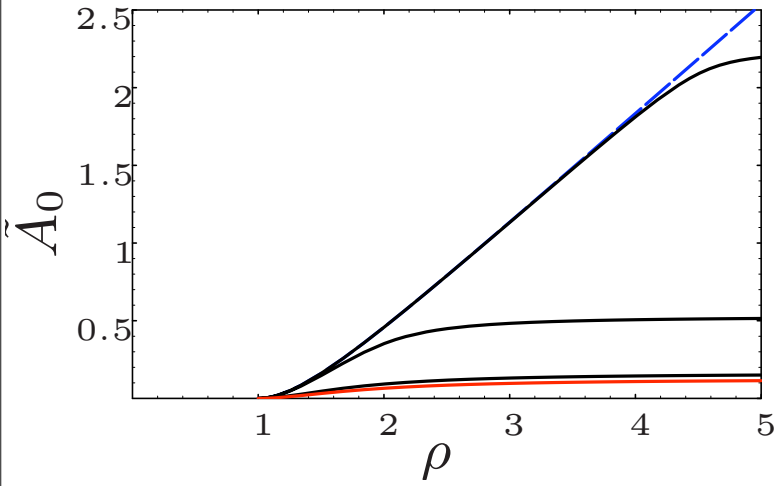
Solutions to Euler-Lagrange equations give "profiles" in radial direction

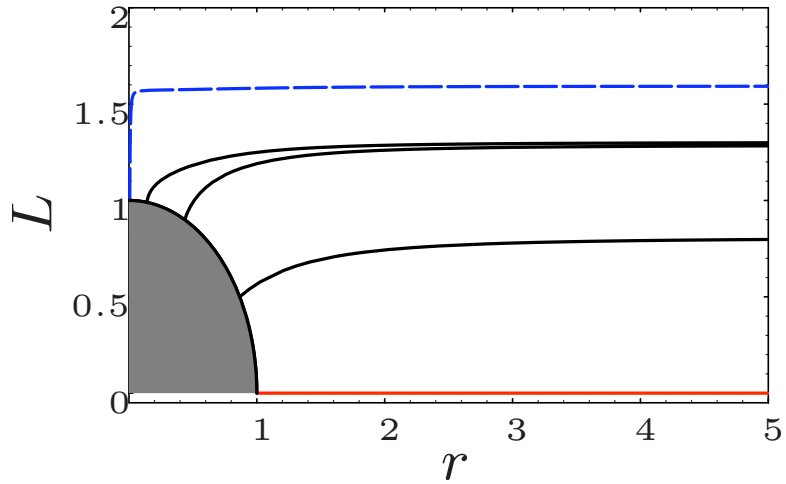


Flavor brane embeddings (D7-branes)

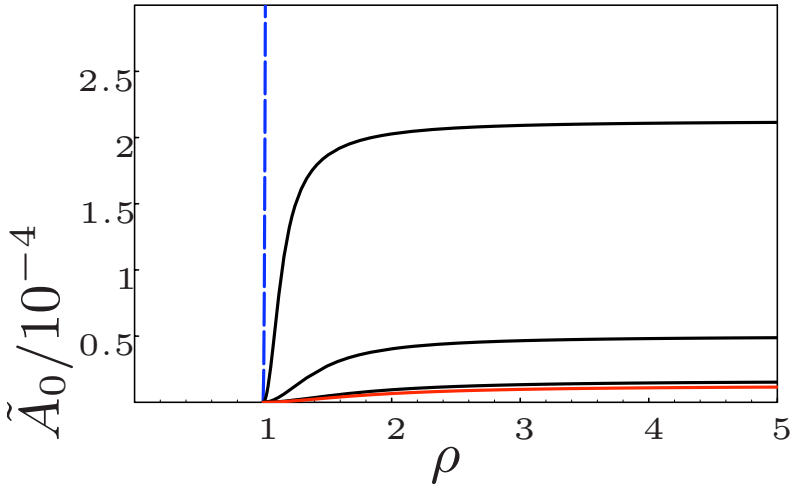


$$\tilde{d} = 0.25$$





$$\tilde{d} = \frac{10^{-4}}{4}$$



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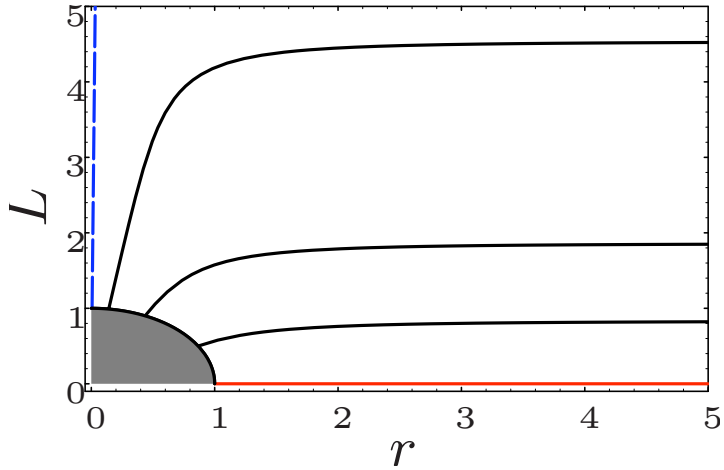

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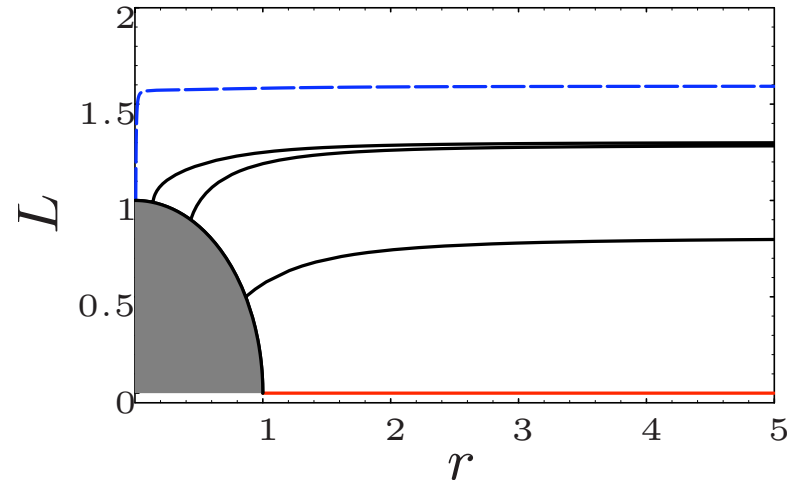

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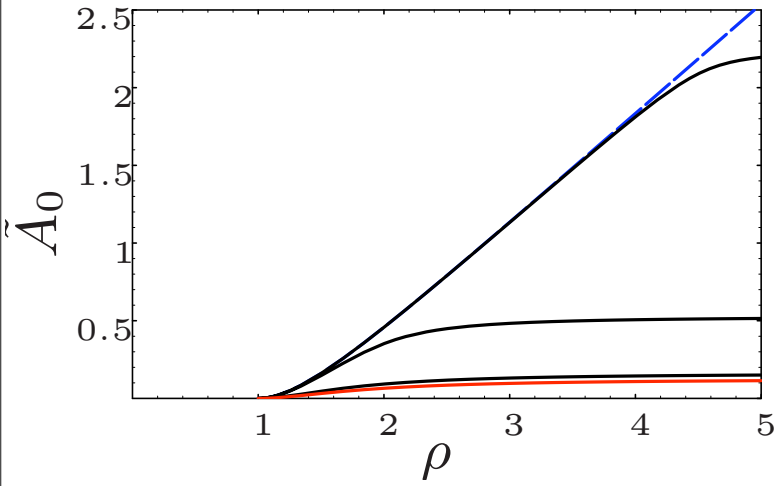

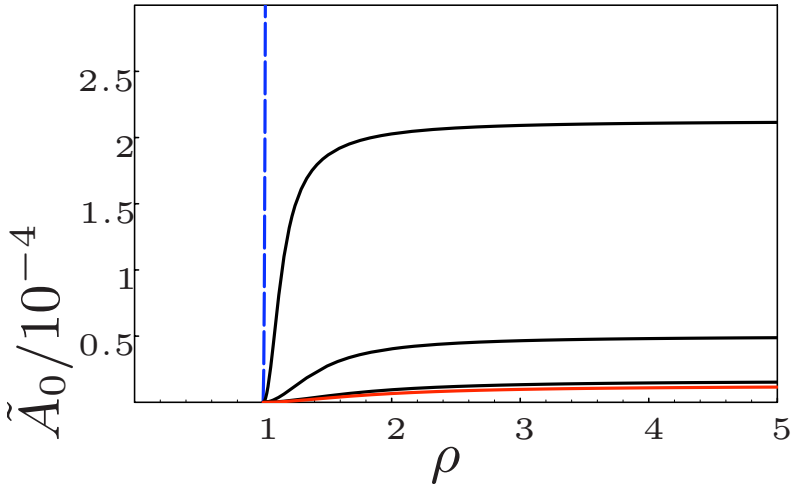

Numerical method: shooting from horizon

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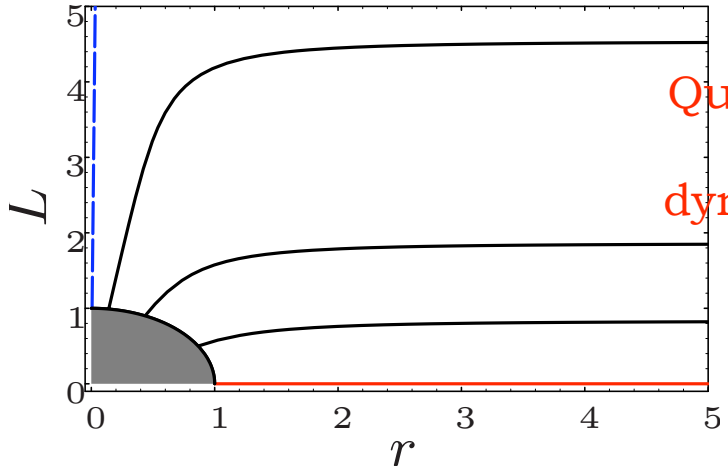

Quark mass fixed dynamically!

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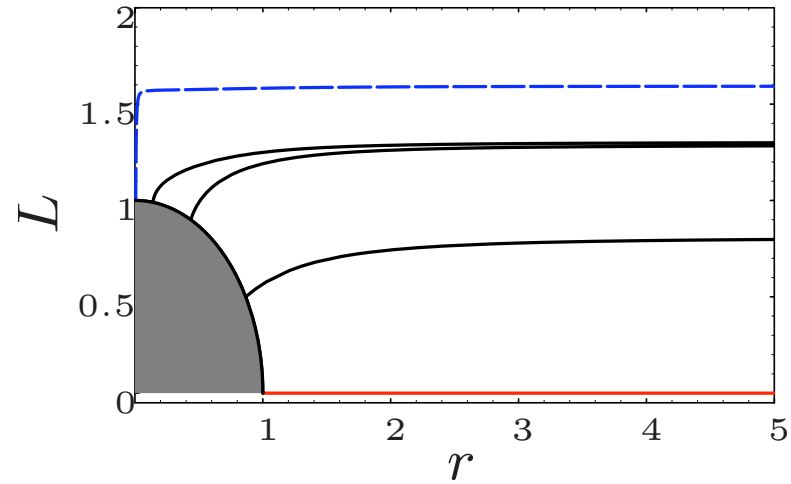
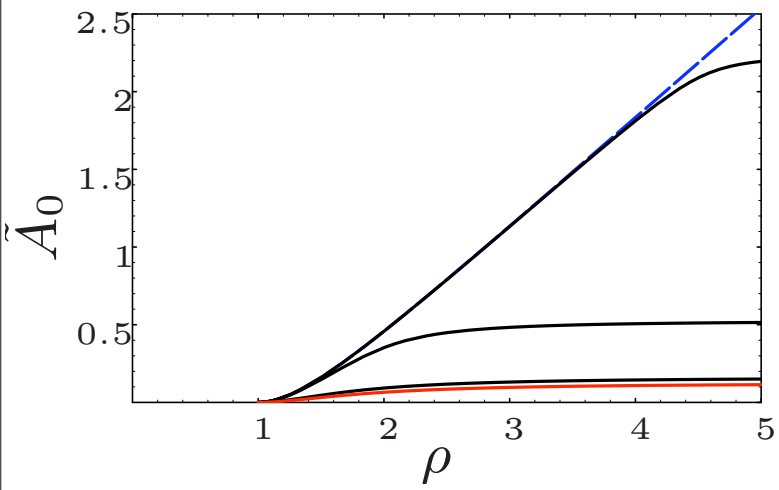
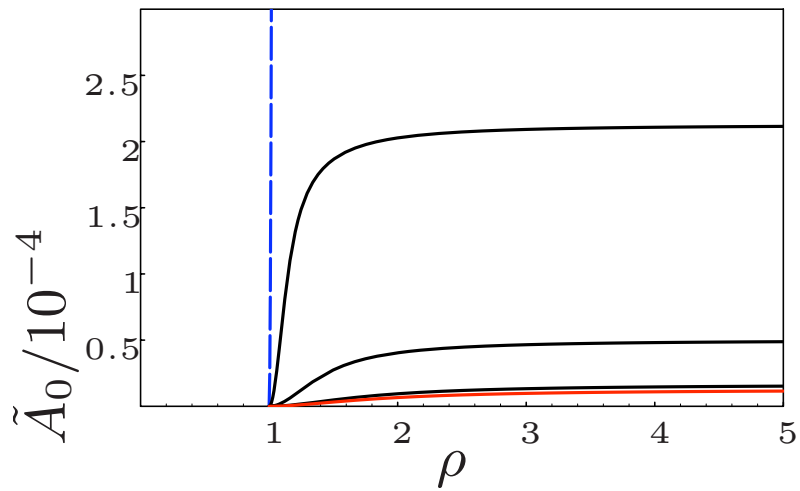


$$\tilde{d} = \frac{10^{-4}}{4}$$

... together with gauge field.





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[Erdmenger, M.K., Rust 0710.0334]

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Fluctuations

Equation of motion: $$0 = \tilde{A}'' + \frac{\partial_\rho[\sqrt{|\det G|}G^{22}G^{44}]}{\sqrt{|\det G|}G^{22}G^{44}}\tilde{A}' - \frac{G^{00}}{G^{44}}\varrho_H{}^2\omega^2\tilde{A}$$

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Fluctuations

Equation of motion:

`Curved' Maxwell equations:

$$\partial_\mu F^{\mu\nu} = 0$$

$$\partial_\mu \left(\sqrt{-G} G^{\mu\nu} G^{\rho\sigma} F_{\nu\sigma}\right) = 0$$

$$\partial_\mu \left(\sqrt{-G} G^{\mu\nu} G^{\rho\sigma} \partial_{[\nu} \tilde{A}_{\sigma]}\right) = 0$$

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Equation of motion: $$0 = \tilde{A}'' + \frac{\partial_\rho[\sqrt{|\det G|}G^{22}G^{44}]}{\sqrt{|\det G|}G^{22}G^{44}}\tilde{A}' - \frac{G^{00}}{G^{44}}\varrho_H{}^2\omega^2\tilde{A}$$

Boundary conditions: $$\tilde{A} = (\varrho - \varrho_H)^{-i\mathfrak{w}}[1 + \frac{i\mathfrak{w}}{2}(\varrho - \varrho_H) + \ldots]$$

Gravity solution & translation

[Erdmenger, M.K., Rust 0710.0334]

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→ shooting from horizon

Translation to Gauge Theory by duality: $$\underset{\text{(source)}}{A_\mu} \overset{\text{AdS/CFT}}{\longleftrightarrow} J^\mu$$

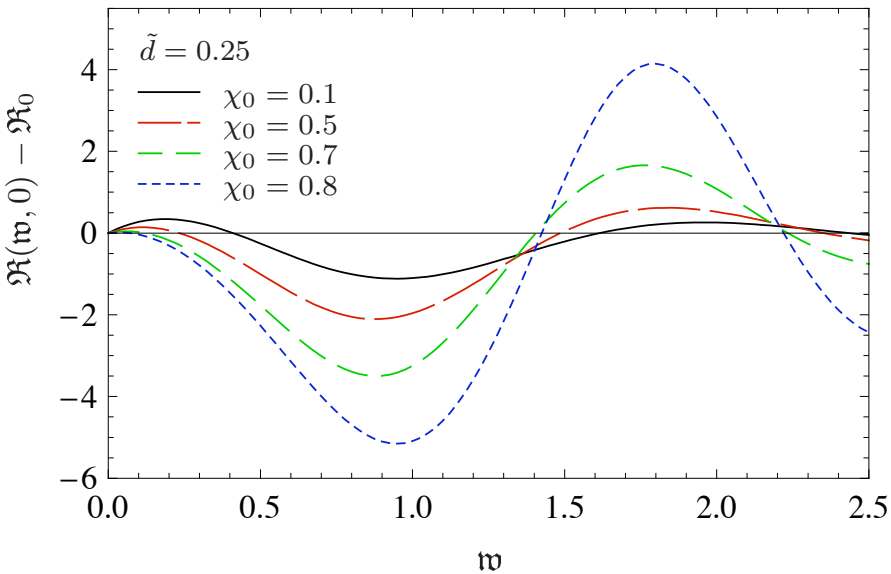
Gauge Correlator:
[Son et al.'02]

$$\boxed{G^{\mathrm{ret}} = \frac{N_f N_c T^2}{8} \lim_{\rho\to\rho_{\mathrm{bdy}}} \left(\rho^3 \frac{\partial_\rho \tilde{A}(\rho)}{\tilde{A}(\rho)}\right)}$$

Gauge theory results: spectral functions

[Erdmenger, M.K., Rust 0710.0334]

Finite baryon density



$$L(\varrho) = \varrho\,\chi(\varrho)$$

$$\chi_0 = \chi(\rho)\big|_{\rho\to\rho_H} \sim \frac{m_{\text{quark}}}{T}$$

$$\chi = \chi(\tilde{d}, \rho)$$

Gauge theory results: spectral functions

[Erdmenger, M.K., Rust 0710.0334]

Finite baryon density

Lower temperature

$$L(\varrho) = \varrho\,\chi(\varrho)$$

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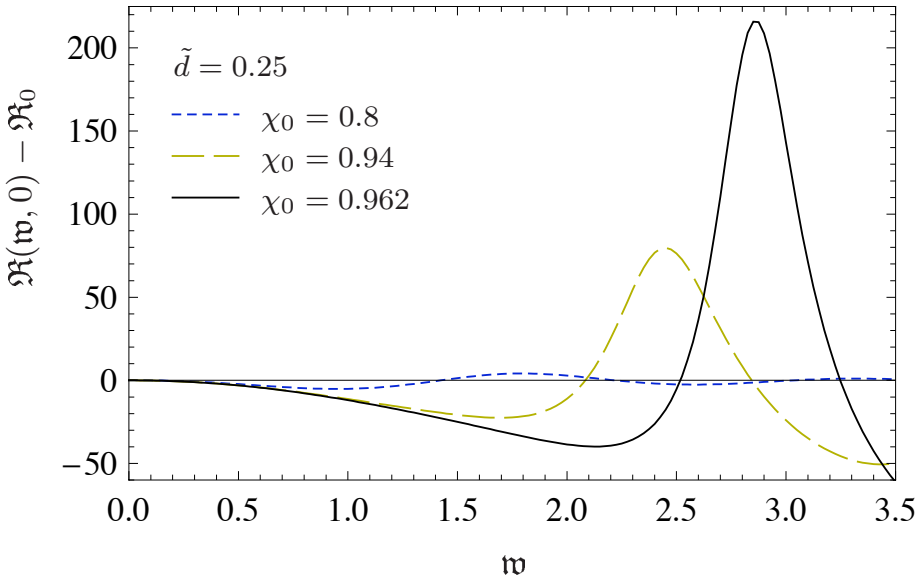

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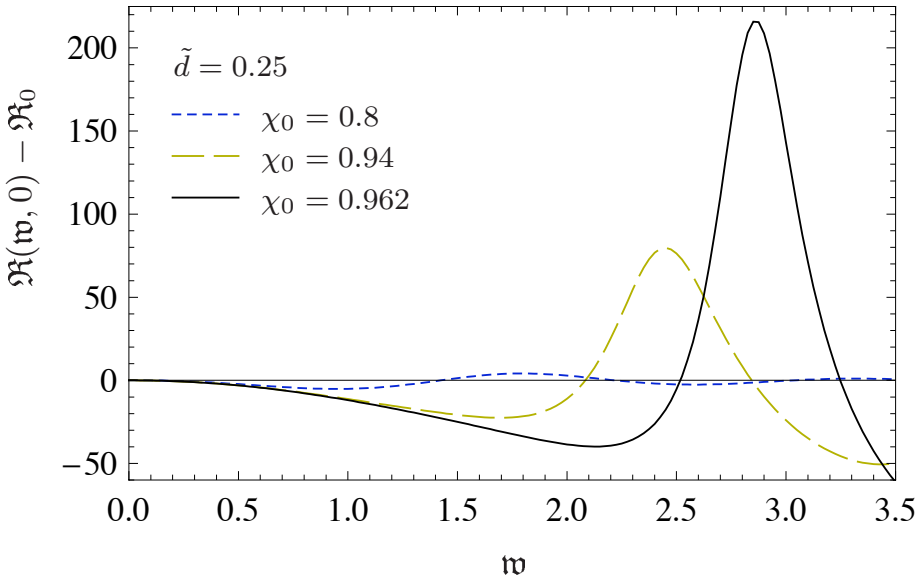

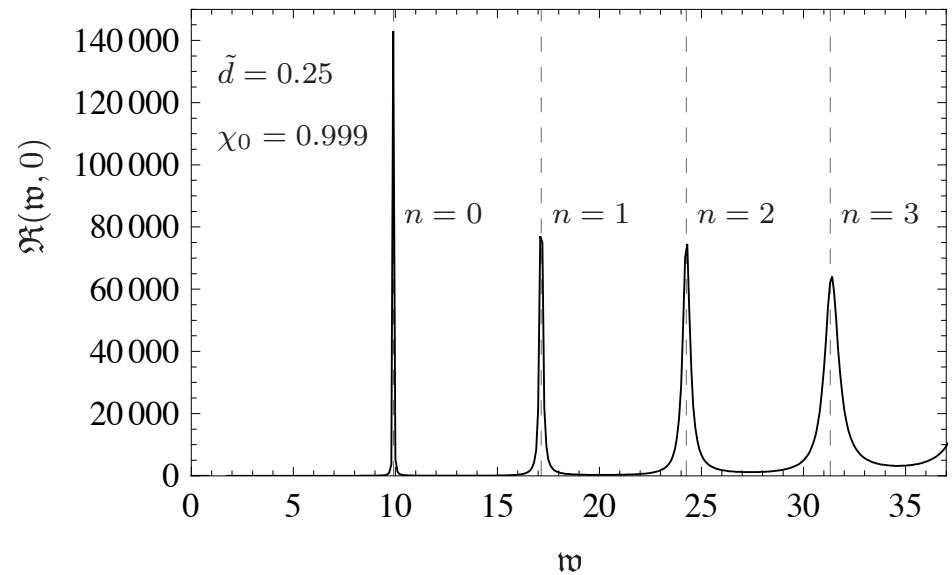

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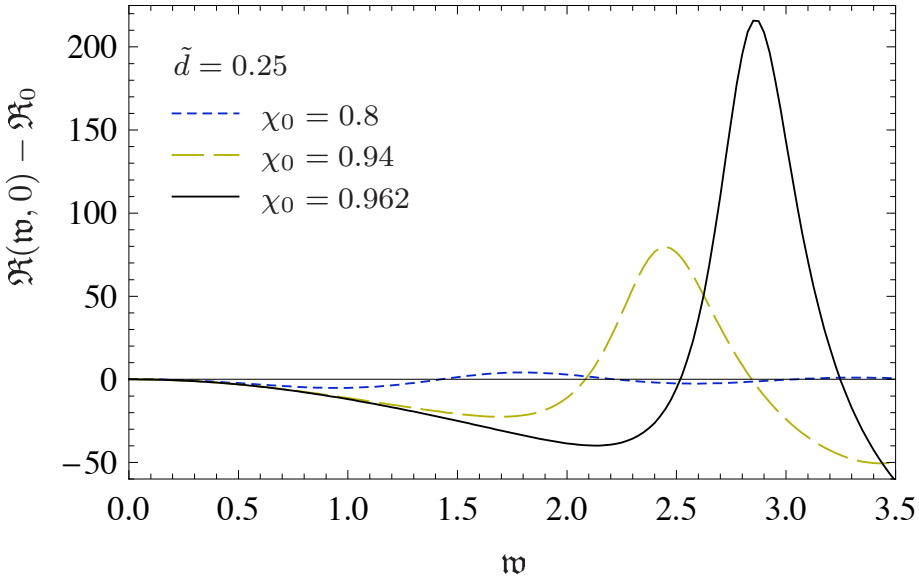
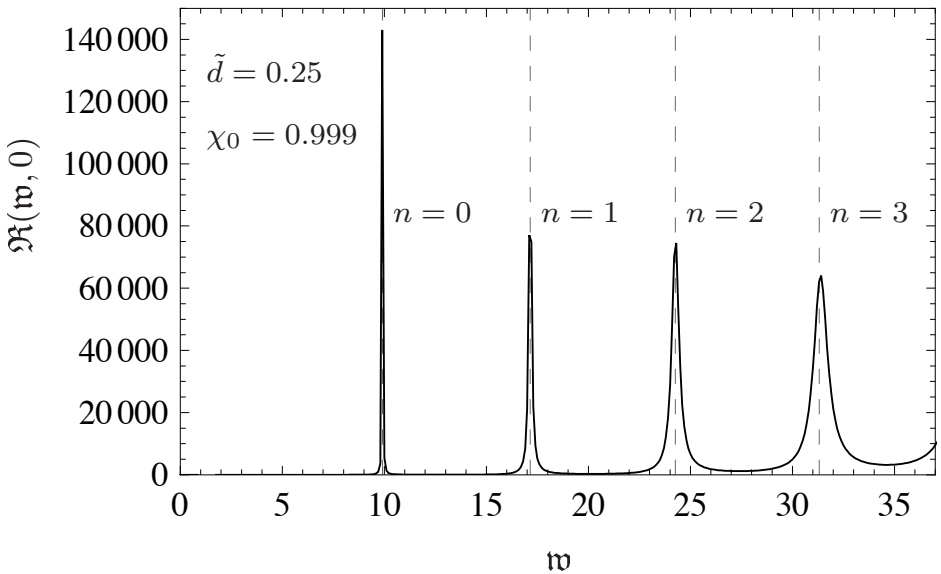
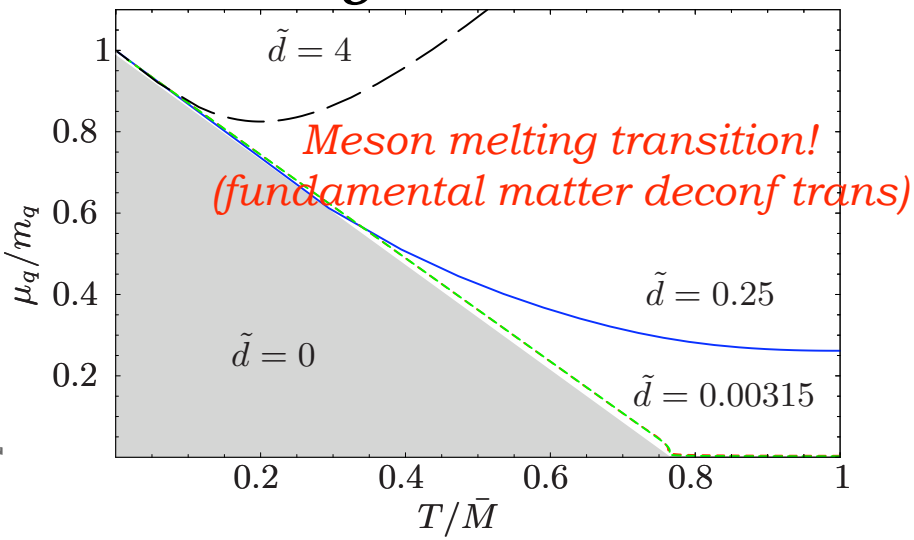




Phase diagram

[Karch, O'Bannon '07]



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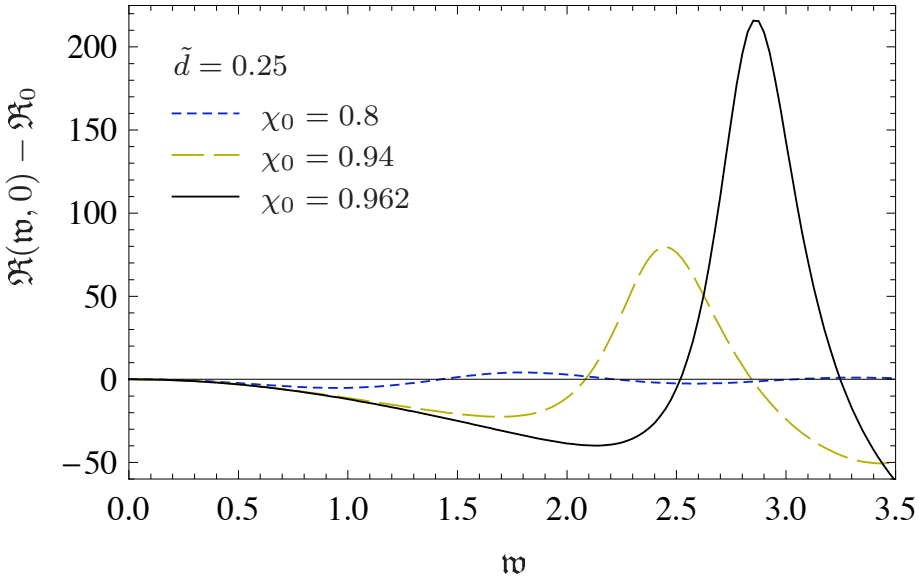

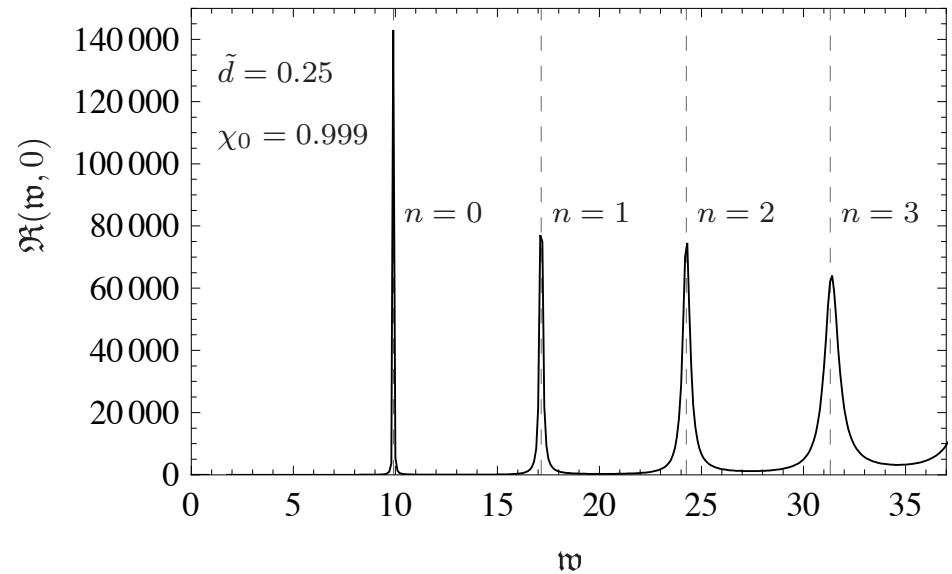

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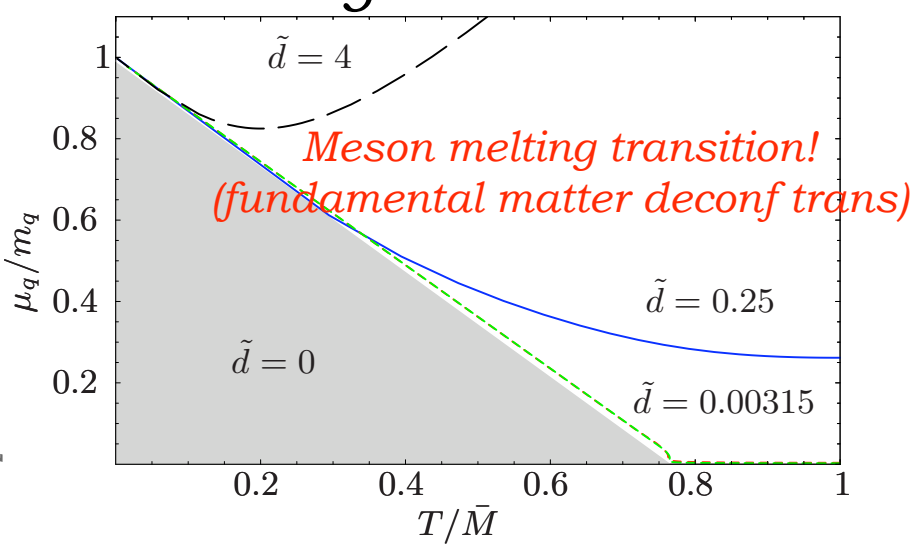
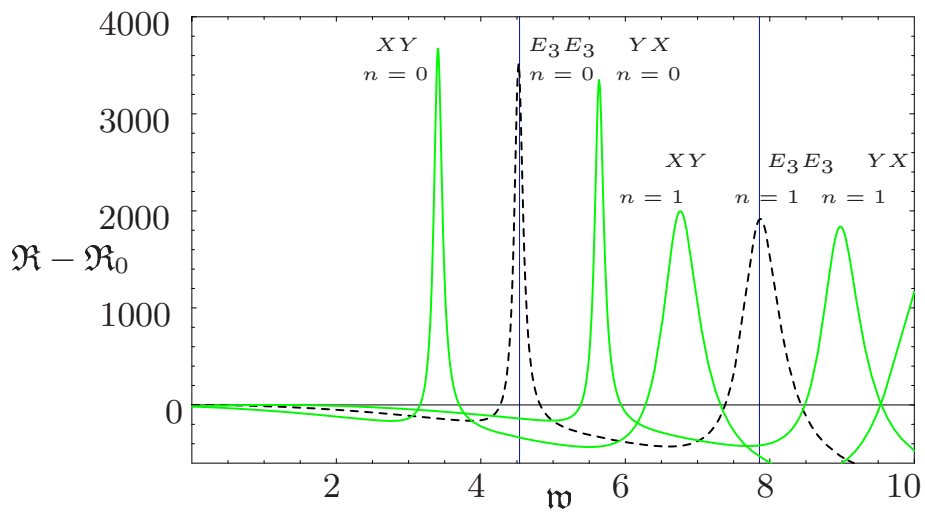


Finite isospin density



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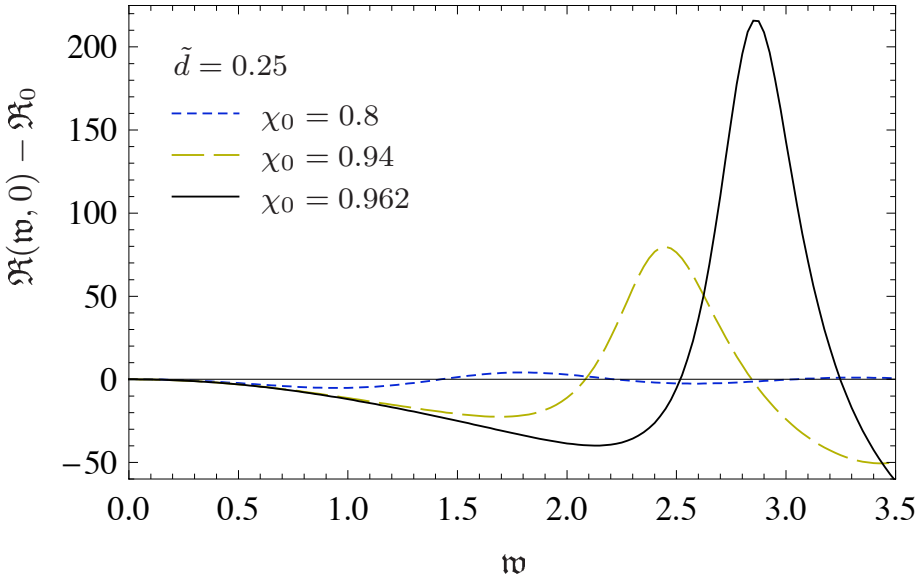

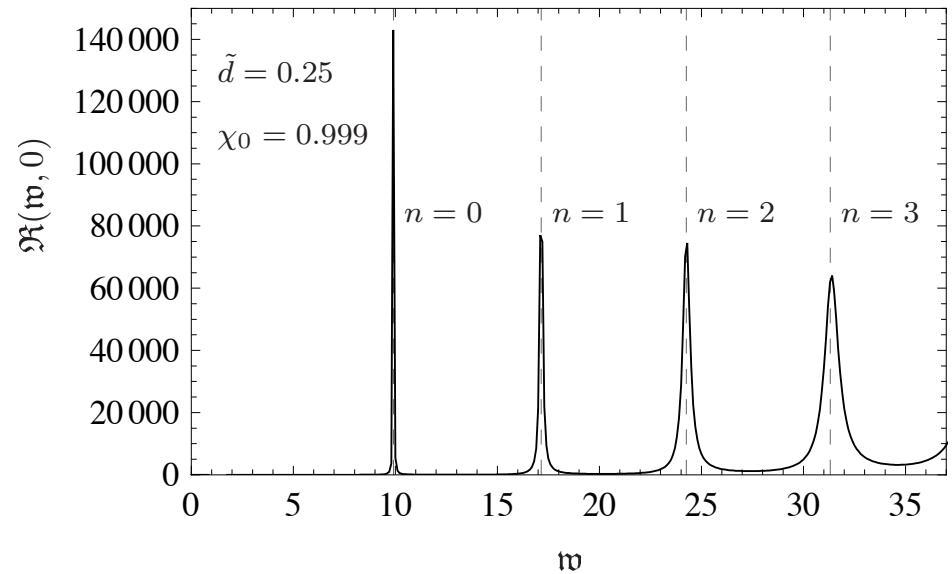

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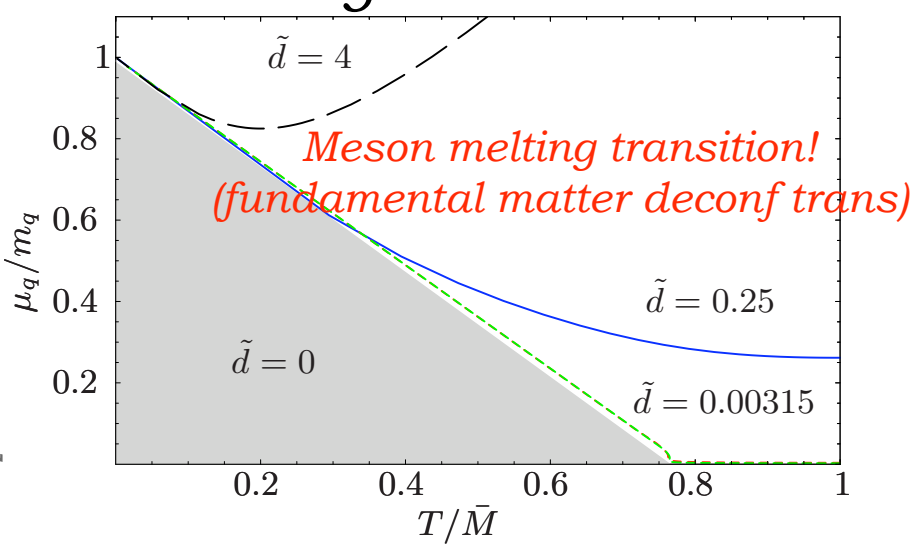

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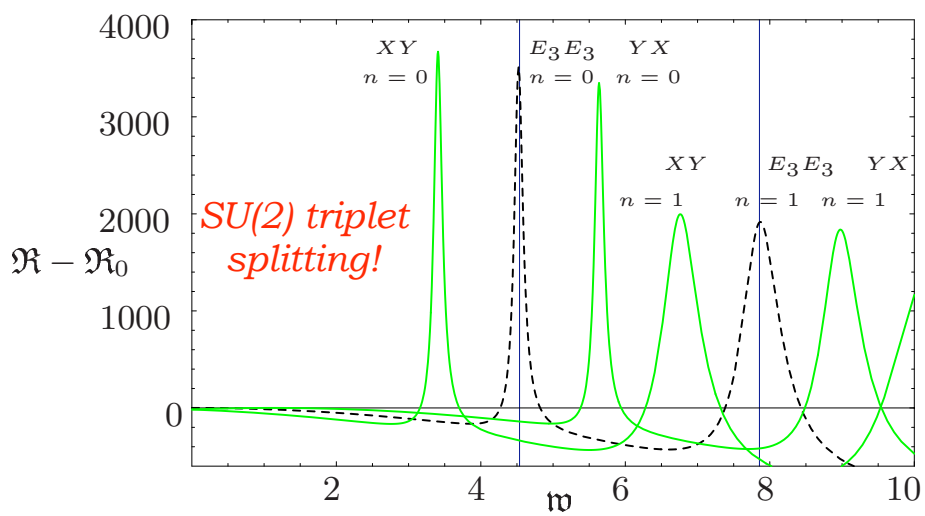

Analytically: [PhD thesis '08]

Flavor brane embeddings (T=0, d=0)

[Kruczenski et al, hep-th/0304032]

Background

Analytical solution for embeddings gives induced metric

$$ds^2 = \frac{\rho^2 + L^2}{R^2} ds^2(\mathbb{E}^{(1,3)}) + \frac{R^2}{\rho^2 + L^2} d\rho^2 + \frac{R^2 \rho^2}{\rho^2 + L^2} d\Omega_3^2$$

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Fluctuations

Analytic solutions with spherical harmonics (l)

Supersymmetric vector meson mass formula

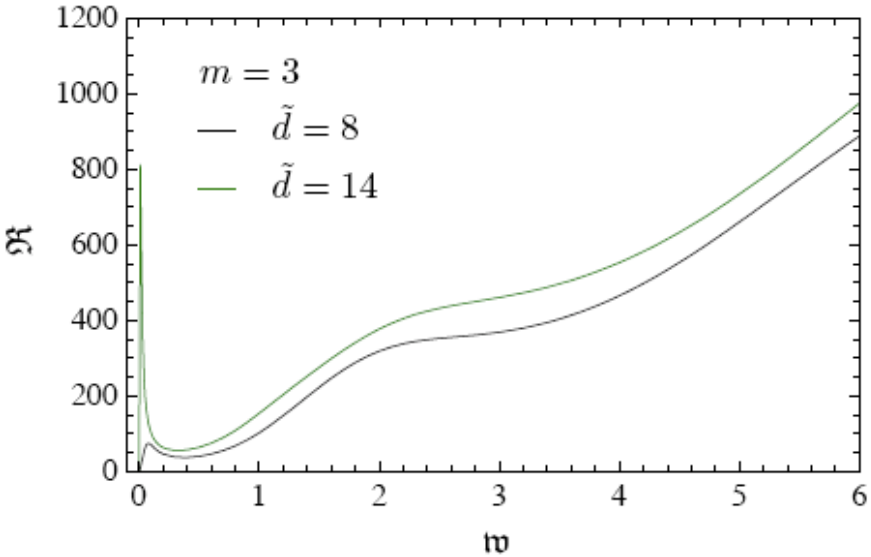
$$M_{\rm v} = \frac{2L}{R^2} \sqrt{(n + l + 1)(n + l + 2)}$$

n counts nodes of solution in radial AdS direction

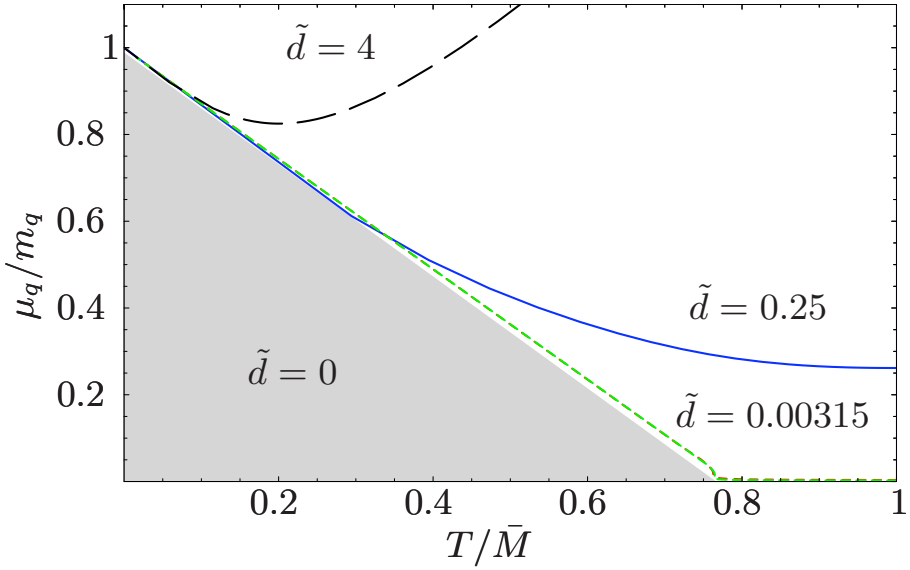
High isospin densities: instabilities

[Erdmenger, M.K., Kerner, Rust 0807.2663]

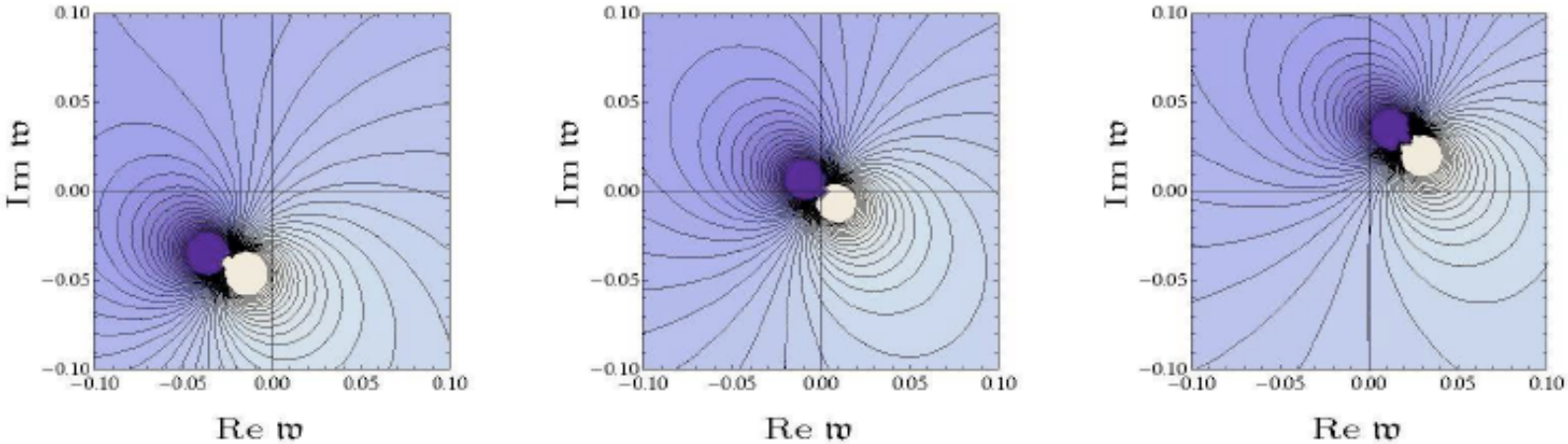
New (lowest) mesonic excitation



Baryonic phases (reminder)



Stability



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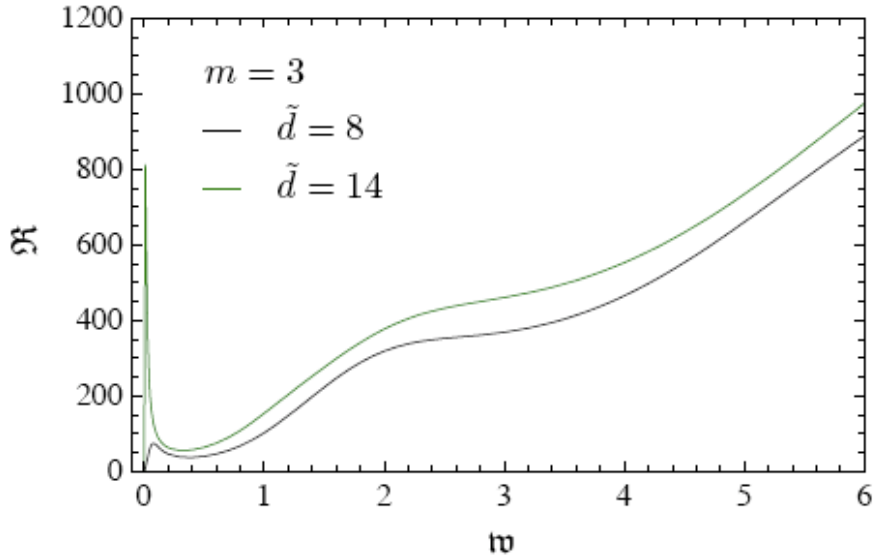
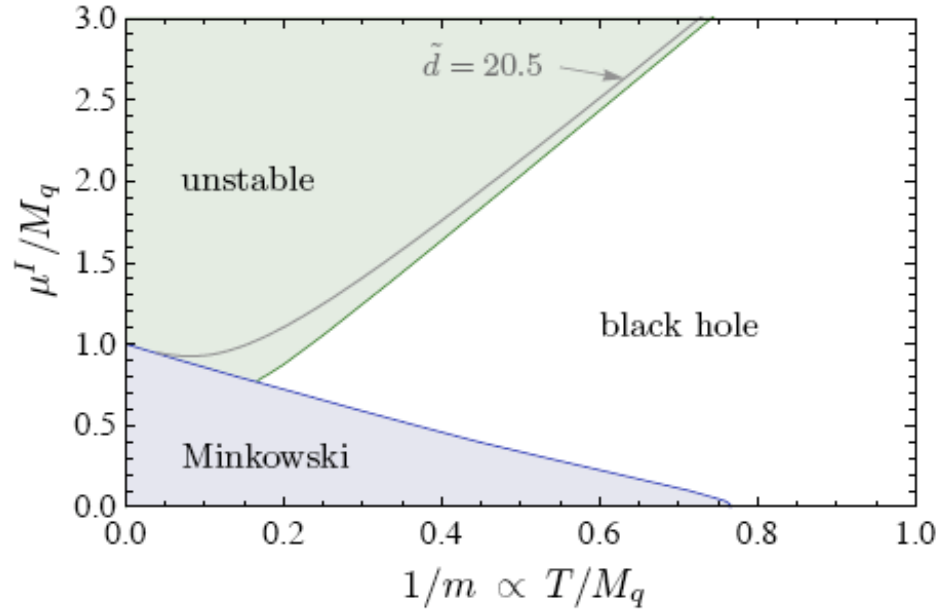


New phase



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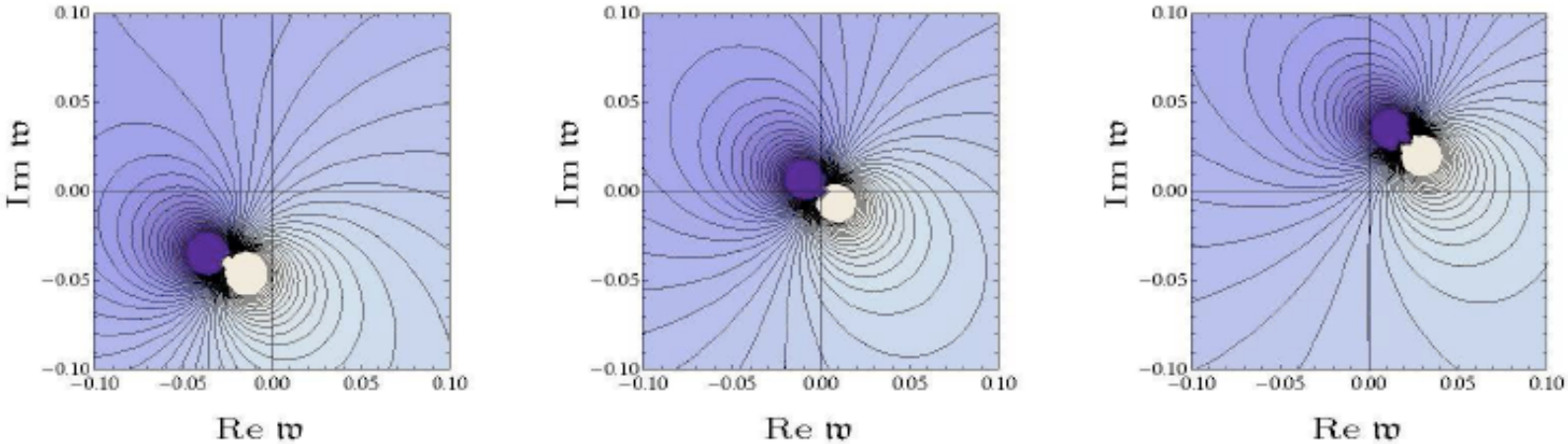


Building a holographic superfluid

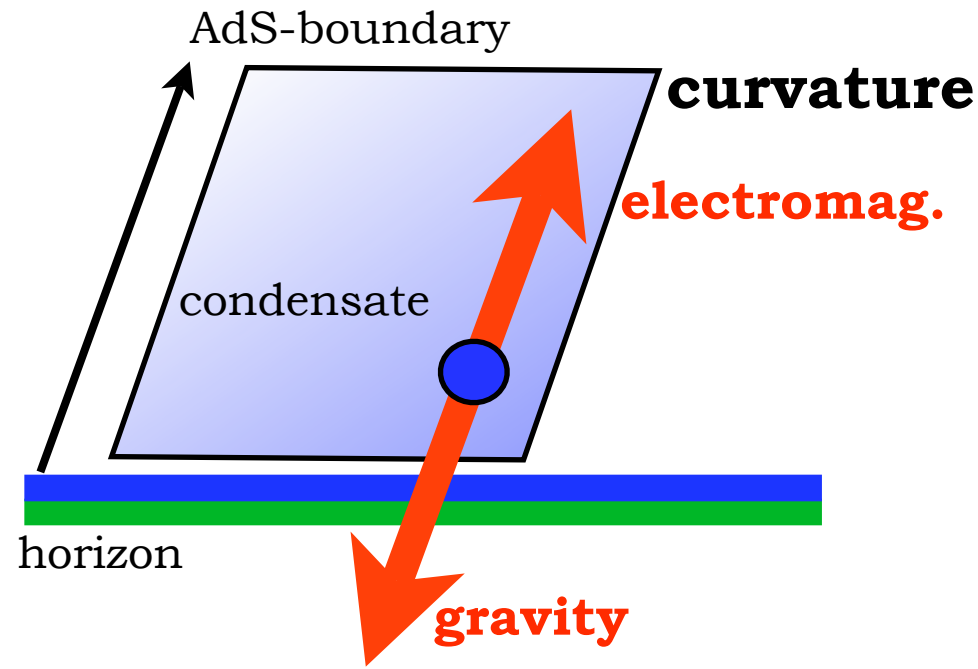
Field Theory

What do we need?

- charged condensate (vev)
- no source
- condensate of charge carriers
- finite temperature, chem. pot.

Gravity

[Gubser 0801.2977]
[Gubser, Pufu 0805.2960]



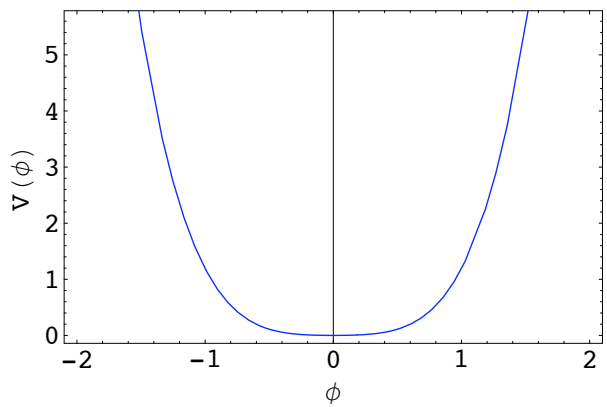
- introduce normalizble mode
- no non-normalizable mode
- condensate hovers over horizon
- black hole, charge

Is this stable?

Get some intuition

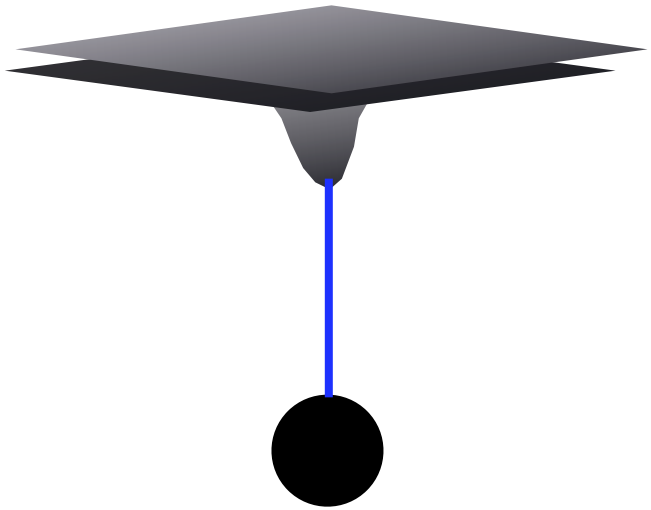
Field Theory

$$\mathcal{L} \sim D_\nu \phi D^\nu \phi \sim ({M_q}^2 - {\mu_{\text{isospin}}}^2)\phi^2$$



- charged particles condense at large enough chemical potential $\mu_{\text{isopin}} \sim M_q$

Gravity



- strings (D3-D5) give FT charges
- cannot put infinitely many
- second brane is important (probe limit)

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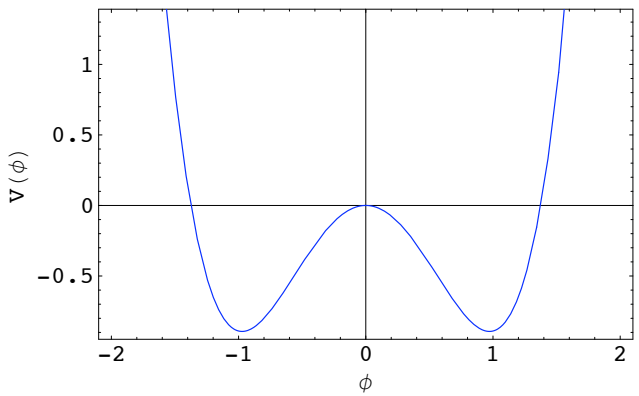

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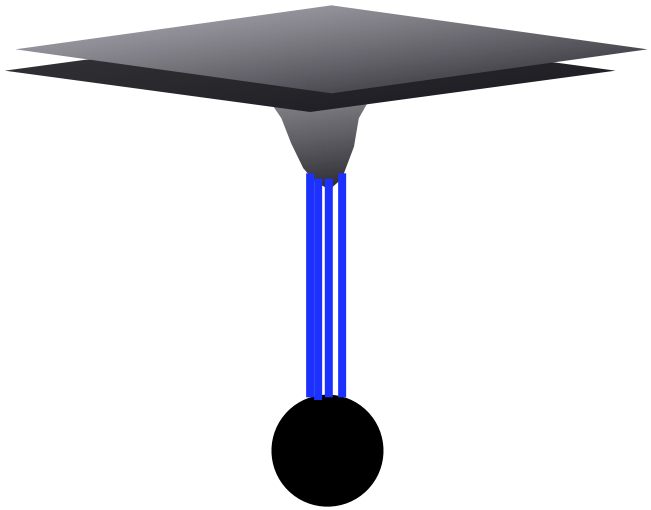

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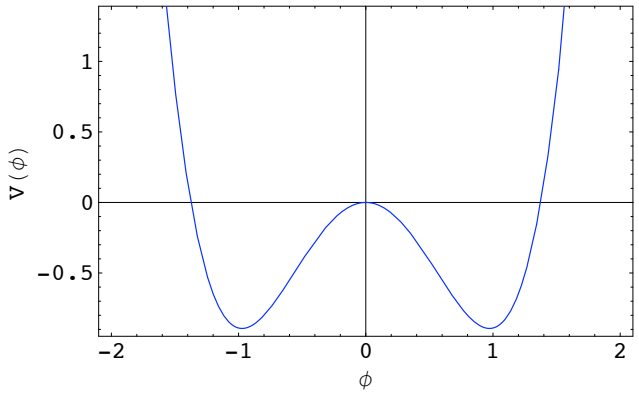

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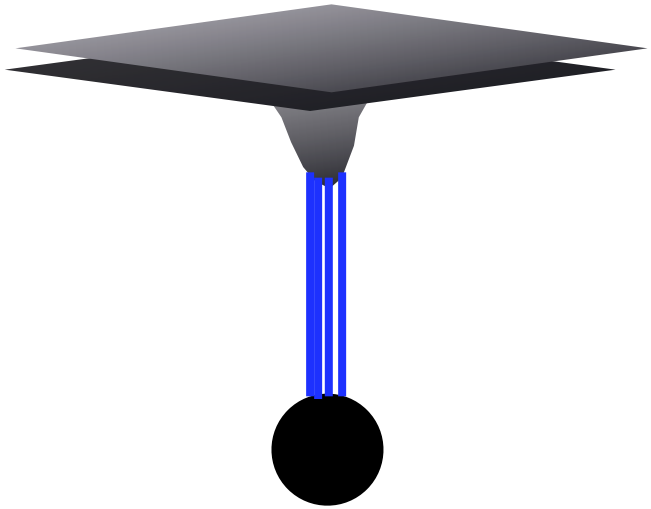

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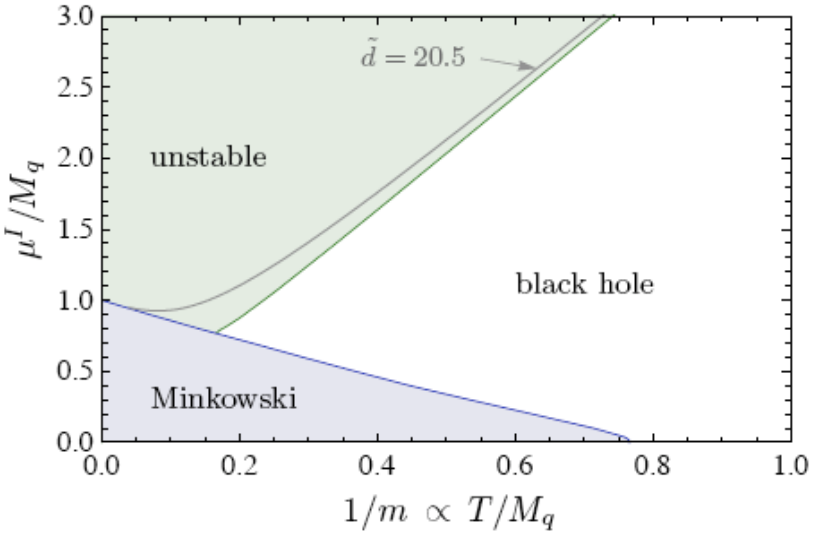
We need a non-Abelian structure!

Vector meson superfluid

General idea



[Erdmenger, M.K., Kerner, Rust 0807.2663]

$$A_0^3 = \mu + \frac{d}{\rho^2} + \ldots$$

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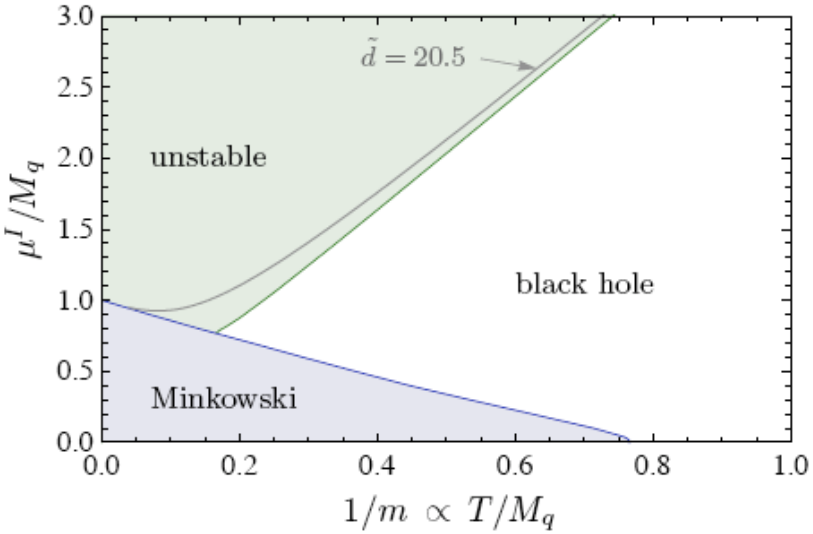

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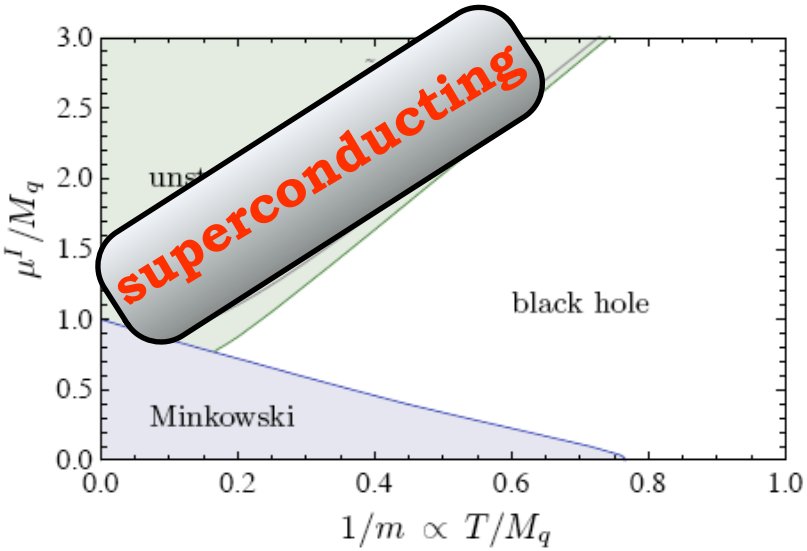

[Ammon, Erdmenger, M.K., Kerner 0810.2316]

$$A_0^3 = \mu + \frac{d_0^3}{\rho^2} + \ldots$$

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[Gubser, Pufu 0805.2960]

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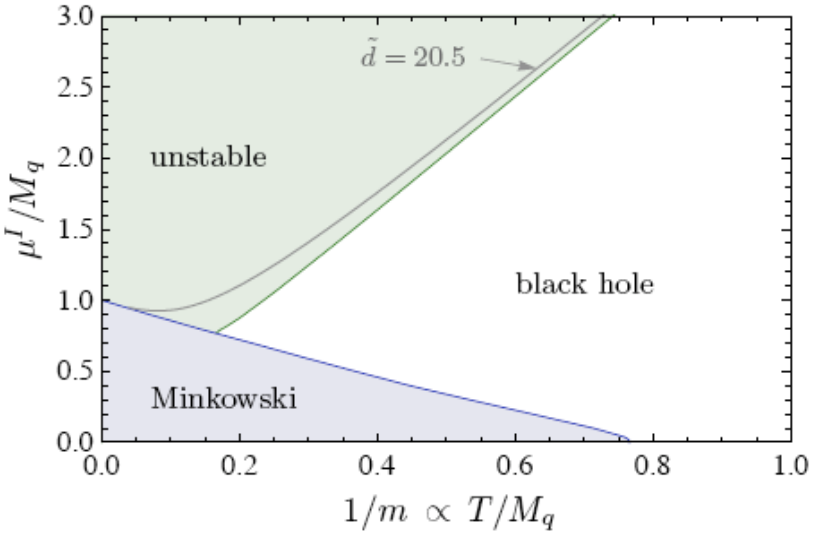

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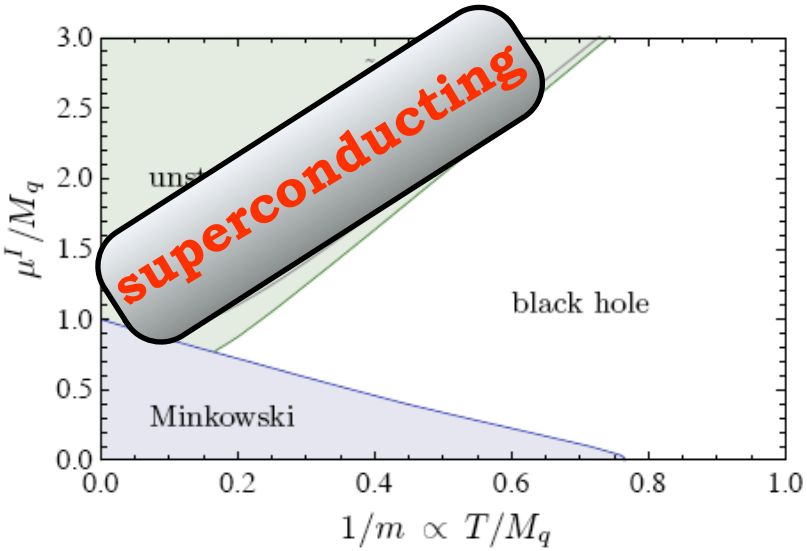

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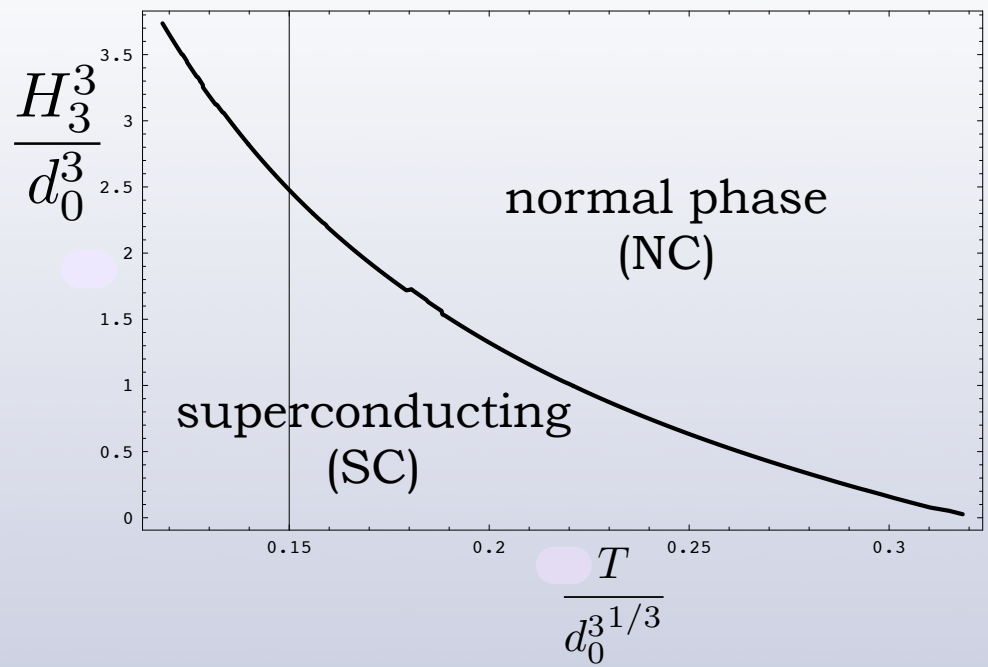
← spont. breaks U(1)

[Gubser, Pufu 0805.2960]

Phenomenology of a superfluid/ superconductor

- second order phase transition
- mean-field theory critical exponents
- energy gap in the conductivity ("Cooper pair" binding)
- Meissner-Ochsenfeld effect, condensate destroyed at large B

Finite magnetic field:



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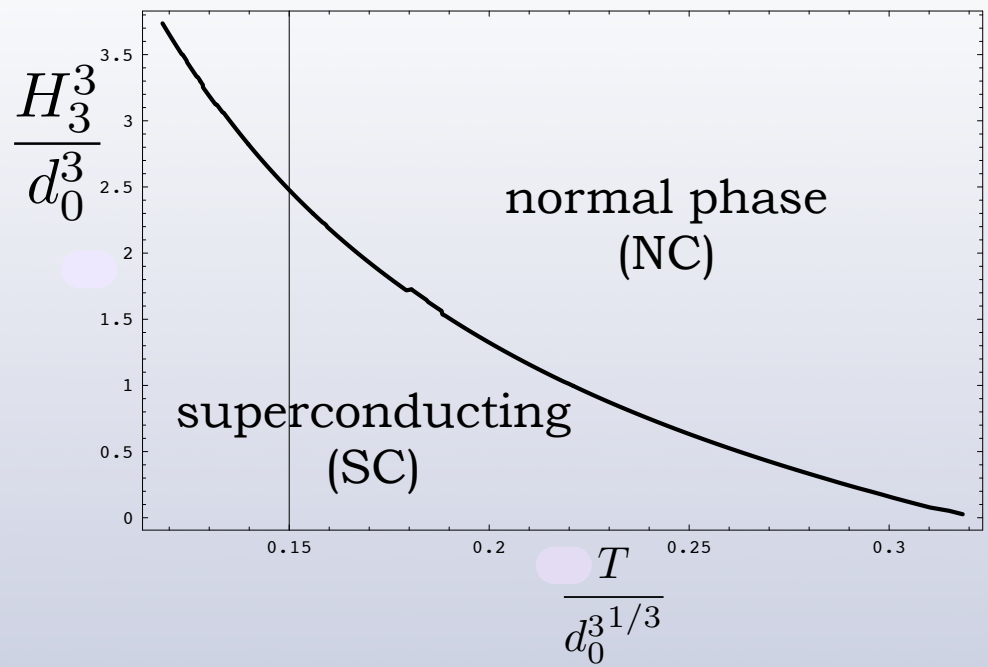
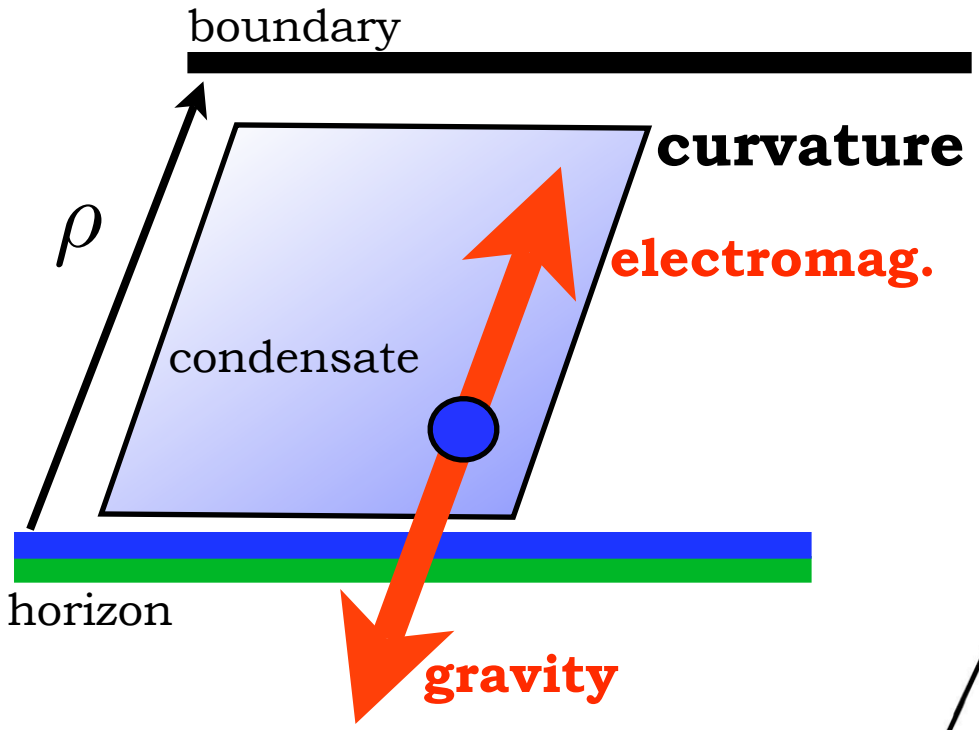
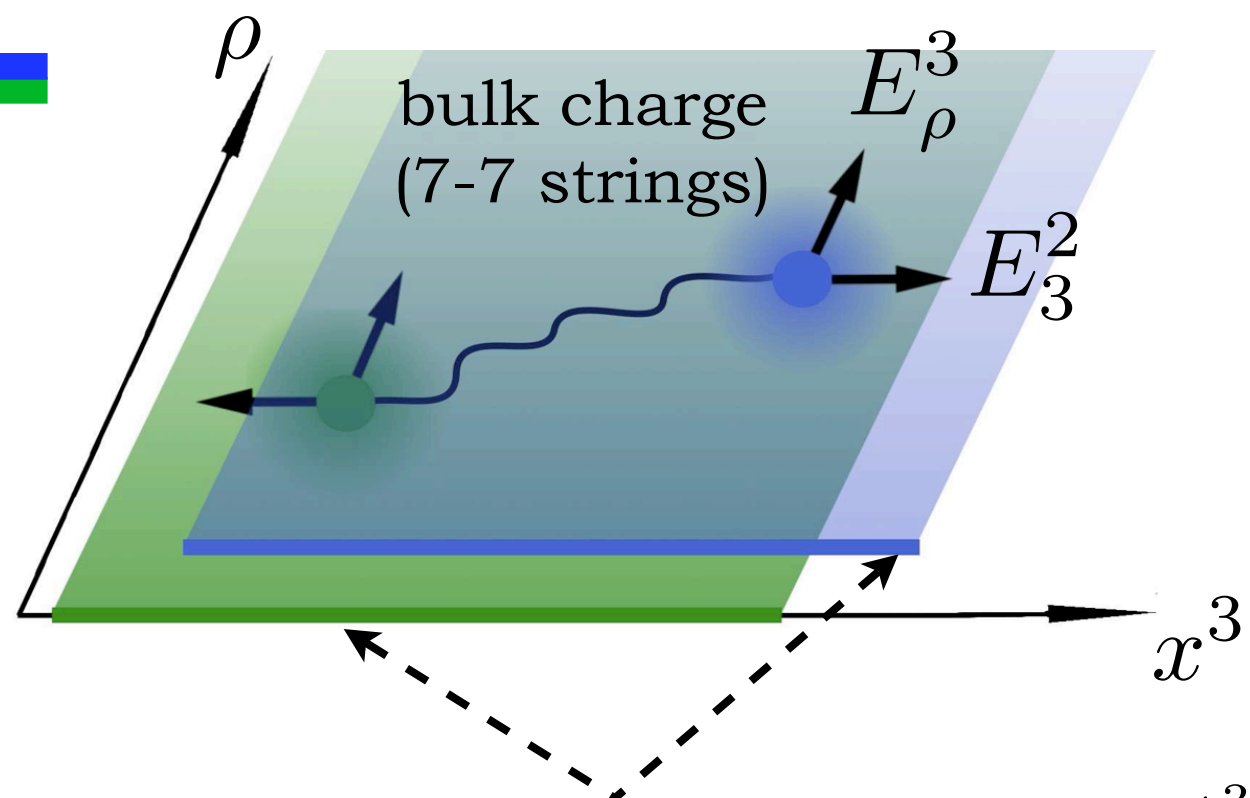


This looks like a superfluid/conductor!

Stringy pairing picture



$$E^3_\rho = F^3_{\rho 0} = \partial_\rho A^3_0$$
$$B^1_{\rho 3} = F^1_{\rho 3} = \partial_\rho A^1_3$$
$$E^2_3 = F^2_{30} = A^1_3 A^3_0$$



7-7 strings generate A^1_3 i.e. they break the U(1) and are thus dual to Cooper pairs.

charged horizon (3-7 strings generate A^3_0)