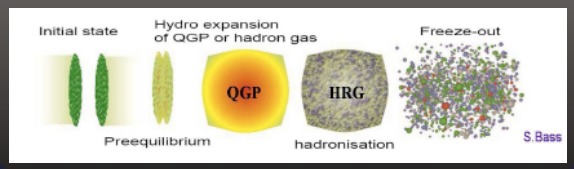
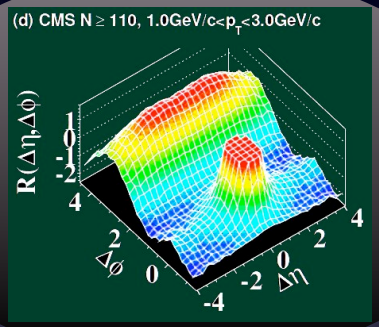
LECTURE X: Gauge/Gravity Correspondence Applications

HGS-HIRe Powerweek, Universität Frankfurt, July 16-21st 2012





by Matthias Kaminski (University of Washington)

Invitation

Properties of Gauge/Gravity

Negative	*Positive*
•only toy models	•strong coupling effects
	•models thermalization, etc
•no model of QCD or SM	•exact solutions exist
•no quantitative results (mass)	•qualitative results (scaling)
•QCD in this universality class?	•some universal results

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The Ridge Phenomenon

- strong coupling effects?
- pre-thermalization?
- needs qualitative explanation
- some “universal” results?

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Gauge/Gravity seems like an appropriate tool.

The Ridge Phenomenon

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- pre-thermalization?
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- some “universal” results?

Invitation

Gauge/Gravity Dictionary

Gauge Theory		*Gravity Theory*	
"Medium" after collision	$\longleftrightarrow$	Background geometry (metric, gauge fields, ...)	$g_{\mu\nu}(r)$
Temperature	$\longleftrightarrow$	Hawking T ~ horizon radius	$g_{\mu\nu}(r; r_{\text{Horizon}})$

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Pre-Equilibrium (difficult)	$\longleftrightarrow$	Shock-wave collision	

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Pre-Equilibrium (difficult)	$\longleftrightarrow$	Shock-wave collision	
Two-point correlations (relatively easy)	$\longleftrightarrow$	Fluctuations of gravity fields	$\delta g_{\mu\nu}(r, t, \vec{x})$

Outline

✓ Invitation

I. Review: Gauge/Gravity & Heavy-Ion-Collisions

- Gauge/Gravity
- Completed Hydrodynamics

II. Gauge/Gravity Models for the Ridge

- Shock-Wave Metric yields Pre-Equilibrium
- Fluctuations give Correlation Functions

III. Other Possibilities

IV. Conclusions

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→ Systematic scan for origin of ridge

→ Toy models for hydrodynamic flow vs. toy models of jets

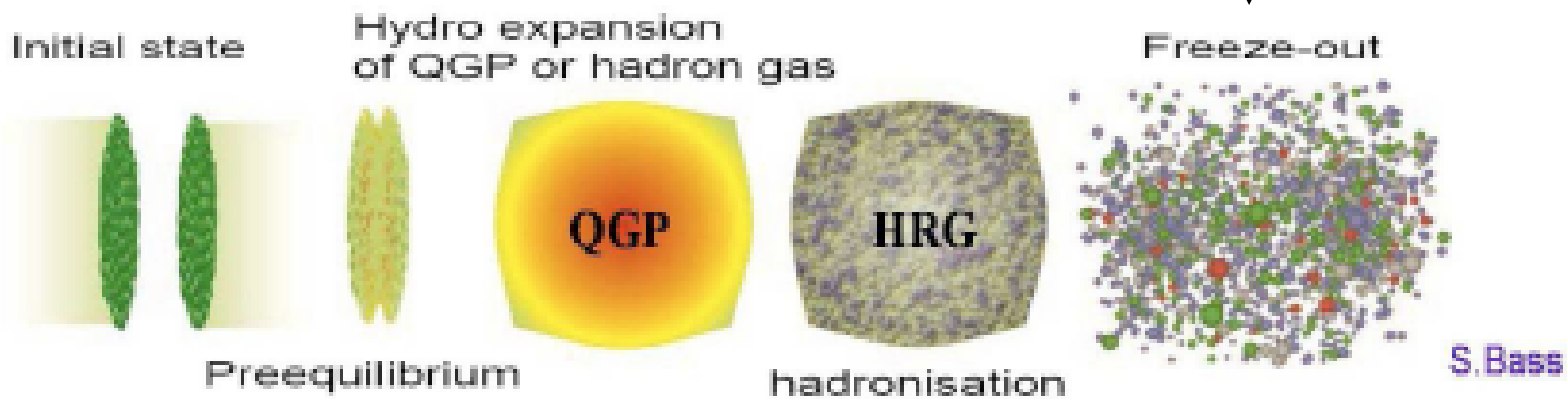
IV. Conclusions

I. Gauge/Gravity & Heavy-Ion-Collisions

What has been done to holographically model HIC?

A lot

Not much



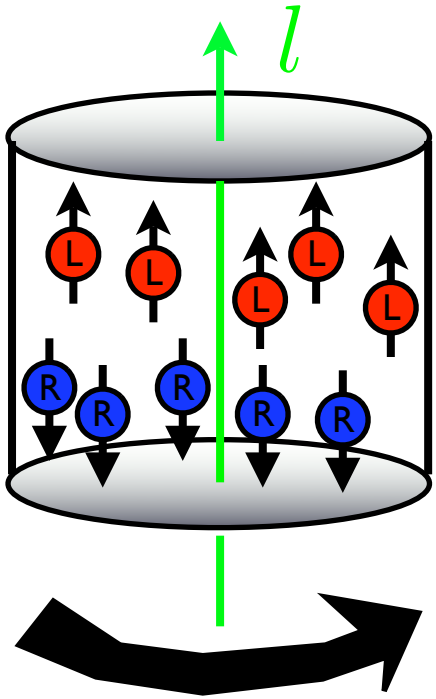
Quite a bit

Not much

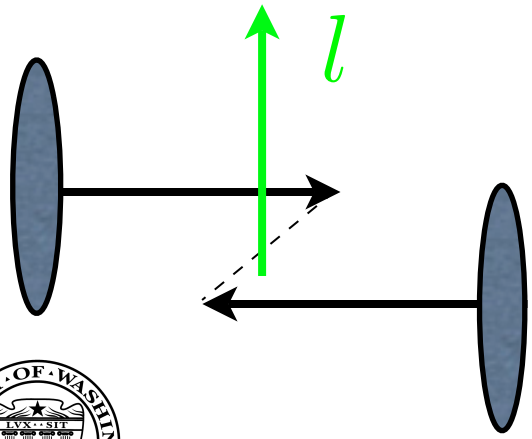
We are going to discuss only examples here.
This is not a full review. *Review: [Gubser, Karch 0901.0935]*

I. Gauge/Gravity & Heavy-Ion-Collisions

Chiral vortex effect



Heavy-ion-collision



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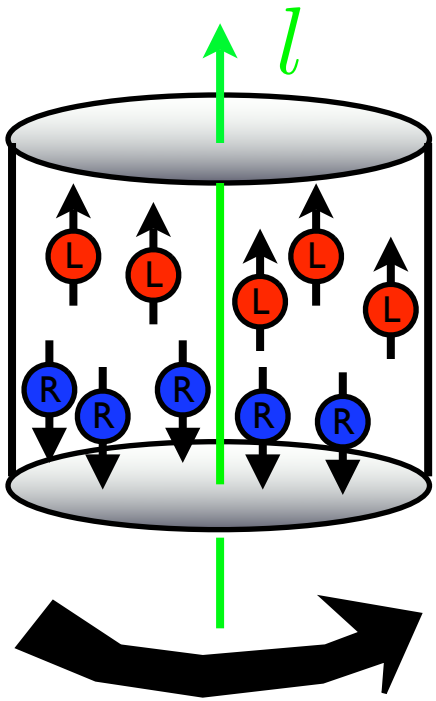

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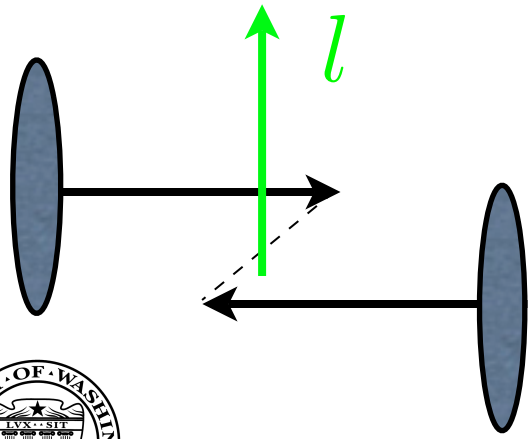

Fluid/Gravity [Baier et al. 2007] [Bhattacharyya et al. 0712.2456]

Einstein equations = hydrodyn. conservation equations + dynamical EOMs for gravity fields

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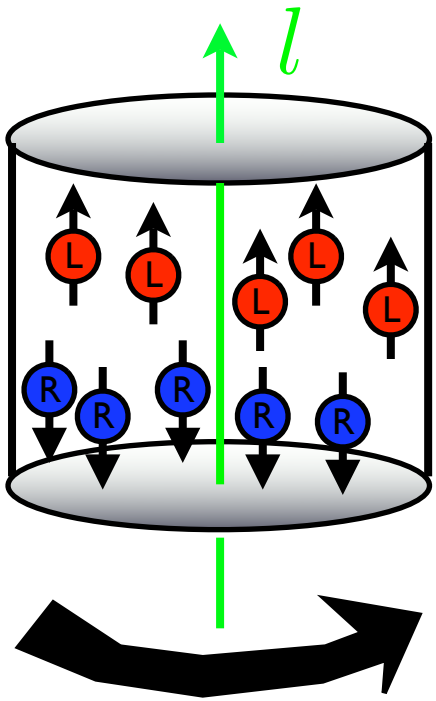

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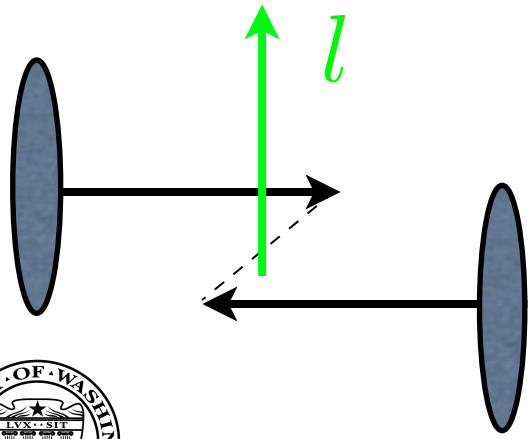

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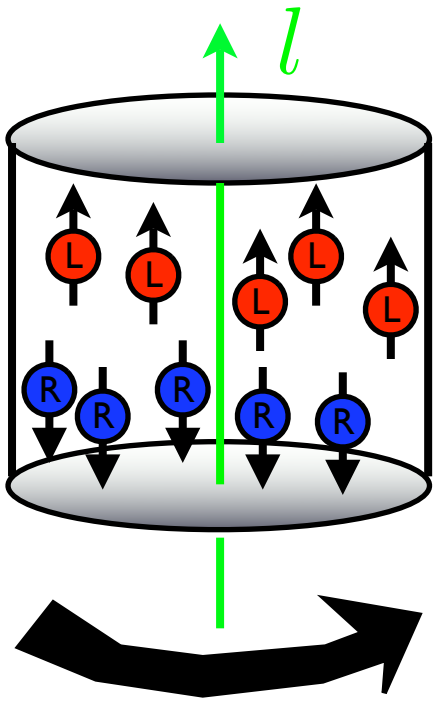

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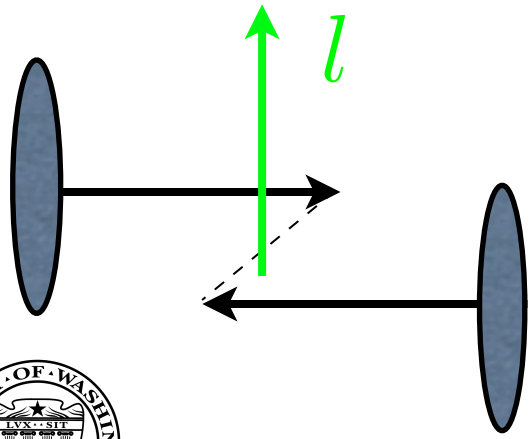

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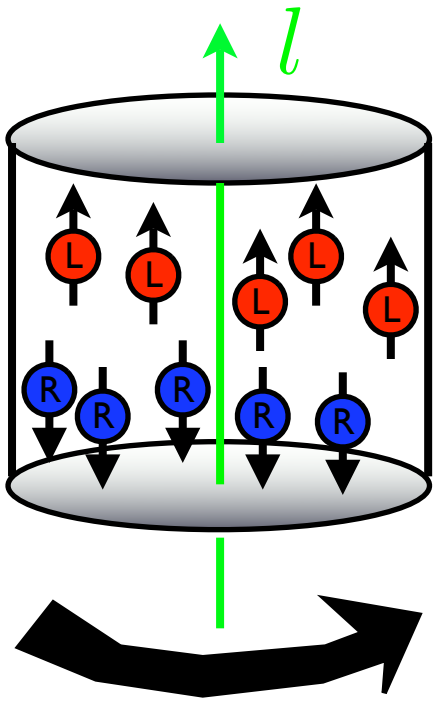

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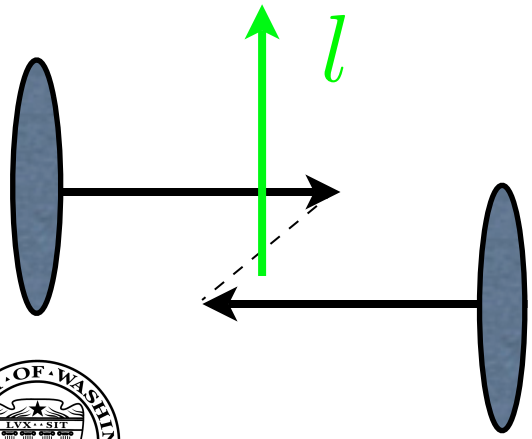

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Fluid/Gravity derivation of chiral vortex effect.

[Erdmenger, Haack, MK, Yarom 0809.2488]
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I. Gauge/Gravity & Heavy-Ion-Collisions

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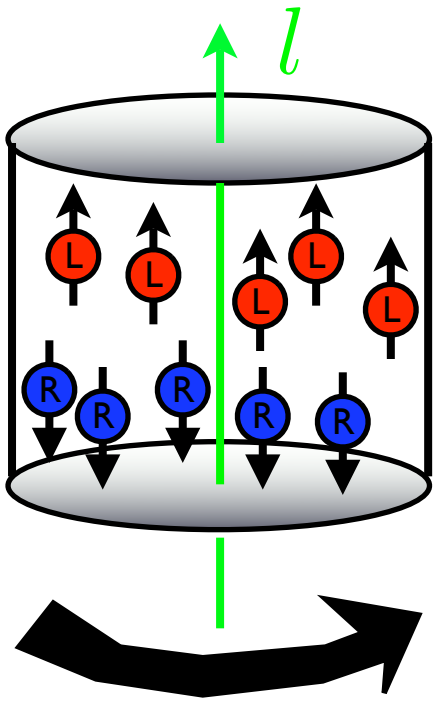

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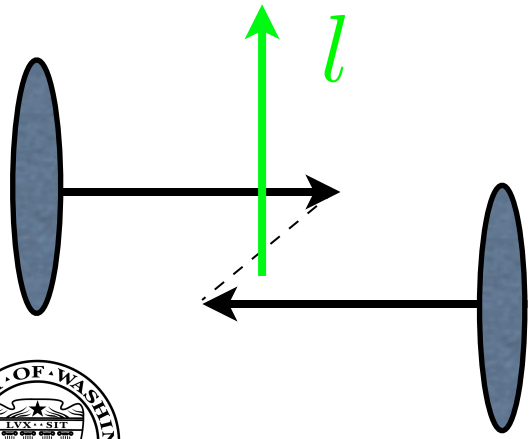

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Pure field theory derivation.

[Son,Surowka 0906.5044]

In parallel: chiral magnetic effect.

[Kharzeev et al., 2007]
[Fukushima et al., 2008]

I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamics

Hydrodynamics is an effective field theory, an expansion in gradients (equivalently: low frequencies and large momenta).

Example: Relativistic fluids with one conserved charge, with an anomaly (chiral)

Constitutive equations

[cf. talk by B. Müller]

$$T^{\mu\nu} = \frac{\epsilon}{3}(4u^\mu u^\nu + g^{\mu\nu}) + \tau^{\mu\nu}$$

$$j^\mu = nu^\mu - \sigma T(\underbrace{g^{\mu\nu} + u^\mu u^\nu}_{=:\Delta^{\mu\nu}})\partial_\nu\left(\frac{\mu}{T}\right) + \xi\omega^\mu$$

Vorticity $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_\nu\partial_\lambda u_\rho$

NEW!

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NEW!

from writing down all possible terms (respecting symmetries) with one derivative, built from $\{u, \epsilon, T, n, \mu, \epsilon^{\mu\nu\rho\ldots}\}$.

Examples $\{\nabla^\nu\mu, \nabla^\nu T, nu^\nu,$
$u^\nu u^\kappa \nabla_\kappa n, u^\nu n \nabla_\kappa u^\kappa, \ldots\}$

I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamics: first order traditional procedure

1. Write down all first order (pseudo)vectors and (pseudo)tensors

2. Restricted by conservation equations

$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda \qquad \nabla_\mu j^\mu = C E^\mu B_\mu$$

Example: no external fields

$$\Longrightarrow \quad 0 = \nabla_\mu n u^\mu = n \nabla_\mu u^\mu + u^\mu \nabla_\mu n$$

Possibly restricted by conformal symmetry

Example: $\nabla^\nu \left(\frac{\mu}{T} \right)$ invariant under Weyl rescaling

3. Further restricted by positivity of entropy production

[Landau, Lifshitz]

$$\nabla_\mu J_s^\mu \geq 0$$

I. Gauge/Gravity & Heavy-Ion-Collisions

(non-conformal) hydrodynamics in 3+1

[Son,Surowka 0906.5044]

Complete constitutive equations in 3+1 (with external gauge field)

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} - \eta \Delta^{\mu\alpha}\Delta^{\nu\beta}(\partial_\alpha u_\beta + \partial_\beta u_\alpha) - (\zeta - \frac{2}{3}\eta)\Delta^{\mu\nu}\nabla_\gamma u^\gamma$$

$$j^\mu = n u^\mu + \sigma V^\mu + \xi \omega^\mu + \xi_B B^\mu$$

$$V^\mu = E^\mu - T\Delta^{\mu\nu}\nabla_\nu\left(\frac{\mu}{T}\right)$$

$$E^\mu = F^{\mu\nu}u_\nu$$

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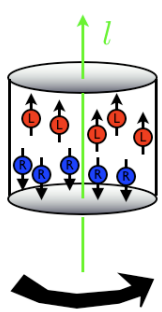
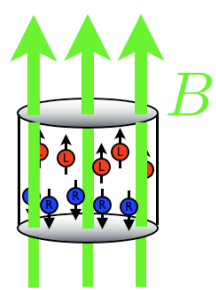
New transport coefficients restricted

$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\epsilon + P}\right), \quad \xi_B = C\left(\mu - \frac{1}{2}\frac{n\mu^2}{\epsilon + P}\right)$$

Chiral vortex effect



Chiral magnetic effect



Observable in heavy-ion collisions

Predicted values:
[Kharzeev, Son 1010.0038]

I. Gauge/Gravity & Heavy-Ion-Collisions

Un-biased predictive power

What we did not know:

Chiral magnetic effect predicted: [Kharzeev 2004]

Chiral vortical effect proposed: [Kharzeev, Zhitnitsky 2007]

Needs corrections: [Landau, Lifshitz]

Ignorance is bliss:

Complete first order constitutive equations in 3+1dim discovered in gravity without prejudice.

Gauge/Gravity method gives you everything there is inside a model.

Word of caution:

Gauge/Gravity is not entirely universal. Values of e.g. transport coefficients and features are generally model-dependent. But within the model you get “everything”.

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Gauge/Gravity method gives you everything there is inside a model.

→ Take a model, check for ridge, change model

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I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamic two-point-functions

Simplified example in 2+1 dim:

$$J^\mu = \rho_0 u^\mu + \sigma E^\mu$$

External sources $A_t,\, A_x \propto e^{-i\omega t + ikx}$

$$u^\mu = (1, 0, 0)$$

Allow response $\rho_0 = \delta\rho$ (fix T and u)

I. Gauge/Gravity & Heavy-Ion-Collisions

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One-point-functions from solving $\nabla_\mu J^\mu = 0$

$$\langle J^t \rangle = \delta\rho = -\frac{i\sigma k}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle J^x \rangle = \delta\rho = -\frac{i\sigma \omega}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle J^y \rangle = 0$$

Einstein relation for diffusion: $D = \dfrac{\sigma}{\chi}$

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$\Longrightarrow$ Two-point-functions $\langle J^t J^x \rangle = \dfrac{\delta \langle J^t \rangle}{\delta A_x} = -\dfrac{i\sigma\omega k}{\omega + iDk^2}$

$\Longrightarrow$ Kubo formulae for transport coefficients

I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamic two-point-functions

Simplified example in 2+1 dim:

$$J^\mu = \rho_0 u^\mu + \sigma E^\mu$$

External sources $A_t,\ A_x \propto e^{-i\omega t + ikx}$

possible: more sources

$$u^\mu = (1,0,0)$$

Allow response $\rho_0 = \delta\rho$ (fix T and u)

generally: T and u respond as well

[cf. talk by B. Müller]

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I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamic Frames

Decomposition (Lorentz-invariance implied)

$$T_{\mu\nu} = \mathcal{E} u_\mu u_\nu + \mathcal{P} \Delta_{\mu\nu} + (q_\mu u_\nu + q_\nu u_\mu) + t_{\mu\nu}$$

$$J_\mu = \mathcal{N} u_\mu + j_\mu$$

$$u_\mu q^\mu = 0, \quad u_\mu t^{\mu\nu} = 0, \quad u_\mu j^\mu = 0$$

Example: Temperature gradient $\quad j_\mu = \cdots + \chi_T \Delta_\mu{}^\nu \nabla_\nu T + \ldots$

Field redefinition ambiguity out-of-equilibrium

$$\boxed{\begin{aligned} u_\nu(x) &\to \hat{u}_\nu(x) \\ T(x) &\to \hat{T}(x) \\ \mu(x) &\to \hat{\mu}(x) \end{aligned}}$$

Fix by choice of a particular hydrodynamic frame

Example: Landau frame

$$q_\mu = 0 \qquad \mathcal{E} = \epsilon_0 \qquad \mathcal{N} = \rho_0$$

I. Gauge/Gravity & Heavy-Ion-Collisions

Hydro without entropy current

Two-point functions together with "equilibrium correlators" replace the entropy argument.

Proven for 2+1 dimensions:
[Jensen, MK, Kovtun, Meyer, Ritz, Yarom 1112.4498]

Proven for "equality type" conditions in d dimensions:
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Inequality type: $\sigma \geq 0$ $\eta \geq 0$ (from two-point functions)

Generating functional

$$W_m = \int d^d x \, \mathcal{L}[\text{sources}(x)]$$

$\Longrightarrow$ Example: Equality type $\chi_T = 0$

$\Longrightarrow$ Generally: m-point functions, simplifies higher order hydro (zero frequency)

Example: Ideal superfluid

$$W_0 = \int d^d x \sqrt{-g} P(T, \mu, \xi^2)$$

I. Gauge/Gravity & Heavy-Ion-Collisions

Summary of part I

- Relativistic hydrodynamics was completed at first and second order (Careful with "Causal Viscous Hydro").

  [Baier et al, Minwalla et al 2007]
  [Erdmenger, Haack, MK, Yarom 0809.2488]
  [Banerjee et al. 0809.2596]

- Chiral transport effects measured in heavy-ion-collisions?

  [Kharzeev, Son]

- New methods for hydrodynamic correlation functions
- New method restricting transport coefficients

- Gauge/Gravity provides playground without prejudice

- Various models of particle collisions exist

Outline

✓ Invitation

✓ Review: Gauge/Gravity & Heavy-Ion-Collisions

- Gauge/Gravity
- Completed Hydrodynamics

II. Gauge/Gravity Models for the Ridge

- Shock-Wave Metric yields Pre-Equilibrium
- Fluctuations give Correlation Functions

→ A "first guess"

→ Toy models of full collision

III. Other Possibilities

IV. Conclusions

II. Gauge/Gravity Models for the Ridge

Pre-Equilibrium Model I

[Janik, Peschanski, 2005]

Single gravitational shock-wave metric

$$ds^2 = \frac{L^2}{z^2}\left\{-2\,dx^+\,dx^- + t_1(x^-)\,z^4\,dx^{-\,2} + dx_\perp^2 + dz^2\right\}$$

$$t_1(x^-) \equiv \frac{2\,\pi^2}{N_c^2}\,\langle T_{1--}(x^-)\rangle$$

Energy-momentum tensor component

z is the radial AdS-direction

L is the AdS-radius

Solves Einstein's equations in AdS5

$$R_{\mu\nu} + \frac{4}{L^2}\,g_{\mu\nu} = 0$$

II. Gauge/Gravity Models for the Ridge

Pre-Equilibrium Model I

[Janik, Peschanski, 2005]

Single gravitational shock-wave metric

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Energy-momentum tensor component

z is the radial AdS-direction

L is the AdS-radius

Solves Einstein's equations in AdS5 $\quad R_{\mu\nu} + \frac{4}{L^2}\,g_{\mu\nu} = 0$

Collide two shock waves with $\quad t_1(x^-) = \mu_1\,\delta(x^-), \quad t_2(x^+) = \mu_2\,\delta(x^+)$

This gives

$$ds^2 = \frac{L^2}{z^2}\Big\{-\left[2 + G(x^+,x^-,z)\right]\,dx^+\,dx^- + \left[t_1(x^-)\,z^4 + F(x^+,x^-,z)\right]\,dx^{-\,2}$$
$$+\left[t_2(x^+)\,z^4 + \tilde{F}(x^+,x^-,z)\right]\,dx^{+\,2} + \left[1 + H(x^+,x^-,z)\right]\,dx_\perp^2 + dz^2\Big\}.$$

which is analytically known (perturbatively)

II. Gauge/Gravity Models for the Ridge

Pre-Equilibrium Model II

"Holography and colliding gravitational shock waves in asymptotically AdS_5 spacetime" [Chesler, Yaffe 1011.3562]

Evolution of two colliding initial states with finite energy density, finite thickness, Gaussian profile, in N=4 Super-Yang-Mills theory at strong coupling.

Full planar shock-wave, non-singular, time-dependent, numerical solution to Einstein's equations.

Contains strong coupling and "medium" effects.

Ansatz: $$ds^2 = -A\,dv^2 + \Sigma^2\left[e^B d\boldsymbol{x}_\perp^2 + e^{-2B} dz^2\right] + 2dv\,(dr + F\,dz)$$

II. Gauge/Gravity Models for the Ridge

Pre-Equilibrium Model II

"Holography and colliding gravitational shock waves in asymptotically AdS_5 spacetime"

[Chesler, Yaffe 1011.3562]

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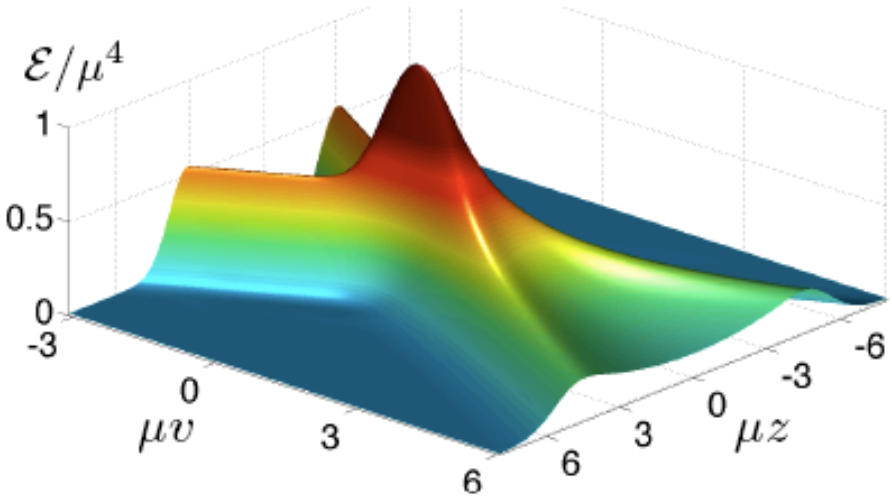


FIG. 1: Energy density $\mathcal{E}/\mu^4$ as a function of time v and longitudinal coordinate z.

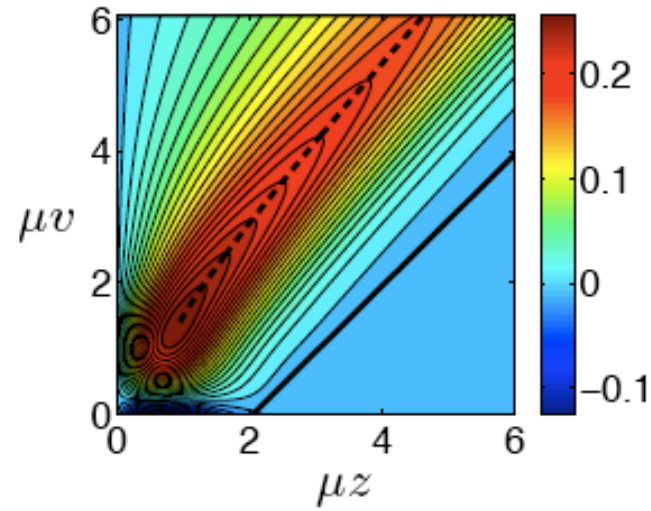


FIG. 2: Energy flux $\mathcal{S}/\mu^4$ as a function of time v and longitudinal coordinate z.

Initial data: $$ds^2 = r^2[-dx_+dx_- + d\boldsymbol{x}_\perp^2] + \frac{1}{r^2}\left[dr^2 + h(x_\pm)\, dx_\pm^2\right]$$

Pick Gaussian (arbitrary) $$h(x_\pm) \equiv \mu^3\,(2\pi w^2)^{-1/2}\, e^{-\frac{1}{2}x_\pm^2/w^2}$$

II. Gauge/Gravity Models for the Ridge

"Long-Range Rapidity Correlations in Heavy Ion Collisions at Strong Coupling from AdS/CFT" [Grigoryan, Kovchegov 1012.5431]

Basic idea:

Gauge		*Gravity*
Collision of nuclei	↔	Metric of Model I
Correlations at early times	↔	Fluctuations around this
Correlations at late times	↔	Fluctuations around dual to ideal Bjorken

II. Gauge/Gravity Models for the Ridge

Recipe: Two-point correlator from fluctuations [Son, Starinets 2002]

Action for gravity scalar field fluctuation (dual to glueball)

$$S^\phi = -\frac{N_c^2}{16\,\pi^2\,L^3}\int d^4x\,dz\,\sqrt{-g}\,g^{MN}\,\partial_M\phi(x,z)\,\partial_N\phi(x,z)$$

Solve equation of motion for that scalar

$$\frac{1}{\sqrt{-g}}\,\partial_M\left[\sqrt{-g}\;g^{MN}\partial_N\phi(x,z)\right] = 0$$

On-shell action

$$S_{cl}^\phi = \frac{N_c^2}{16\,\pi^2\,L^3}\int d^4x\,\left[\sqrt{-g}\,g^{zz}\,\phi(x,z)\,\partial_z\phi(x,z)\right]\Big|_{z=0} = \frac{N_c^2}{16\,\pi^2}\int d^4x\,\phi_B(x)\,\left[\frac{1}{z^3}\,\partial_z\phi(x,z)\right]\Big|_{z=0}$$

Real-time retarded Green's function

$$\boxed{G_R(x_1,x_2) = \frac{\delta^2[S_{cl}^\phi - S_0]}{\delta\phi_B(x_1)\,\delta\phi_B(x_2)}}$$

II. Gauge/Gravity Models for the Ridge

Implications

Large-rapidity glueball correlations in simplest background look very different from ridge data. But there are large-rapidity correlations at early times.

[cf. talk by K. Dusling]

$$C(k_1, k_2)\Big|_{|\Delta y| \gg 1} \sim \cosh(4\,\Delta y)$$

Computation in background dual to ideal Bjorken hydrodynamics gives no large-rapidity correlations at late times.

Outline

✓ Invitation

✓ Review: Gauge/Gravity & Heavy-Ion-Collisions

- Gauge/Gravity
- Completed Hydrodynamics

✓ Gauge/Gravity Models for the Ridge

- Shock-Wave Metric yields Pre-Equilibrium
- Fluctuations give Correlation Functions

III. Other Possibilities

→ Systematic scan for origin of ridge

→ Toy models for hydrodynamic flow vs. toy models of jets

IV. Conclusions

III. Other Possibilities

Correlations after collision of two nulei in a medium

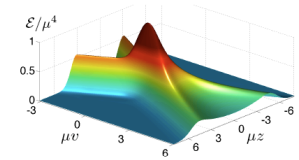


FIG. 1: Energy density $\mathcal{E}/\mu^4$ as a function of time v and longitudinal coordinate z.

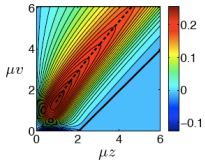


FIG. 2: Energy flux $\mathcal{S}/\mu^4$ as a function of time v and longitudinal coordinate z.

PROPOSAL

Compute fluctuations around the full numerical background metric of model II at different times to scan the full time evolution of correlations.

Step in this direction:

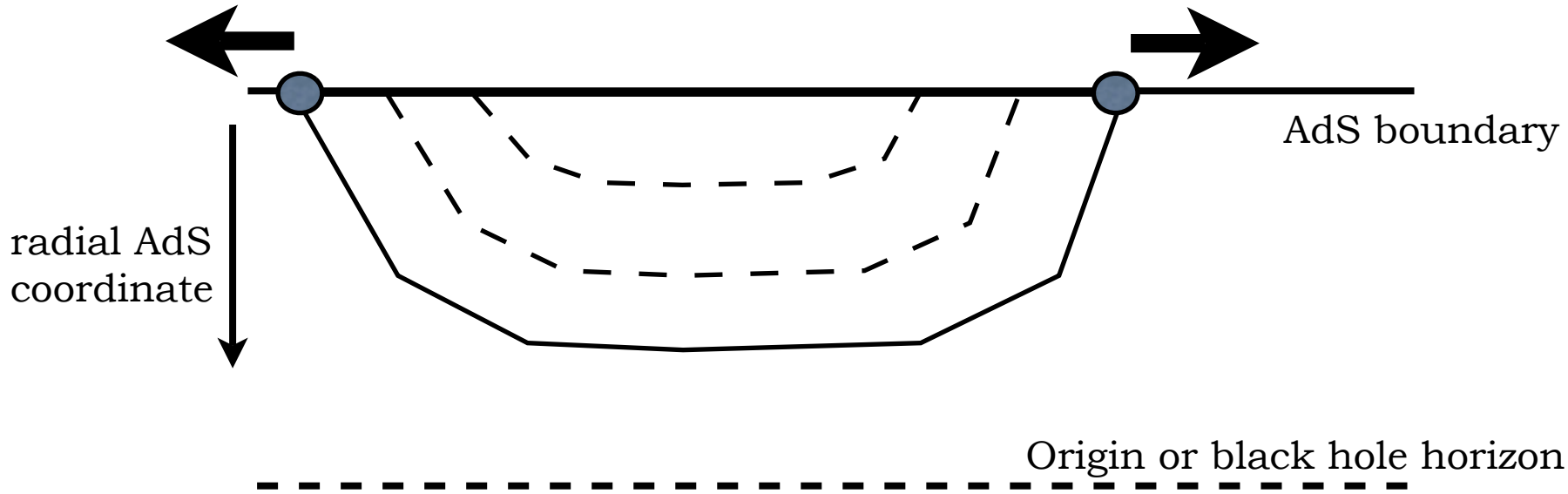
[Chesler, Teaney 2011]

Compute fluctuations around simplified version of model II (dual to two-point correlation functions). Check fluctuation dissipation theorem and equilibration.

III. Other Possibilities

Model of a jet

Take a string falling/being torn apart (backreacted)



Compute fluctuations around this background
(dual to two-point correlation functions)

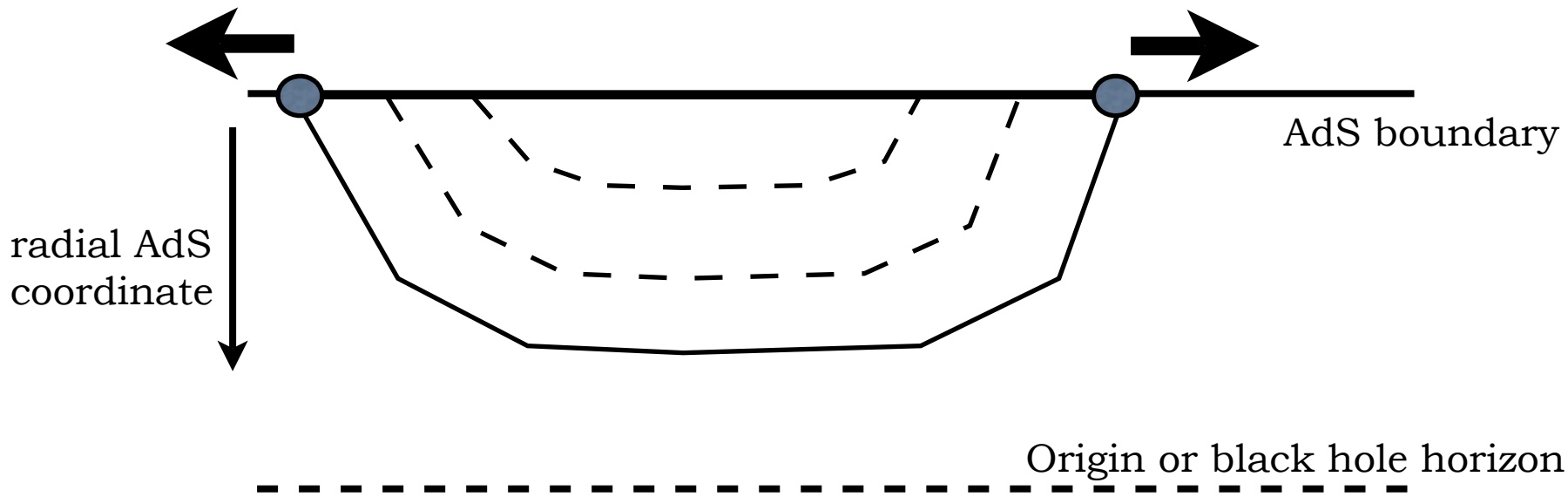
Initial conditions?

see also [Hofman, Maldacena 2008]

III. Other Possibilities

Model of a jet

Take a string falling/being torn apart (backreacted)



Compute fluctuations around this background
(dual to two-point correlation functions)

Initial conditions? → Toy model for jets?

see also [Hofman, Maldacena 2008]

IV. Conclusions

✓ complete first and second order hydro

✓ new method for restricting transport coeffs

✓ new method for zero-frequency m-point correlators

✓ candidate model for collision (ridge)

➡ fluctuations at different times, unique features?

➡ use “more of hydro”: fluctuations, 2nd O(), methods...

➡ measure chiral transport effects

APPENDIX

-Entropy production

Structure of divergence

$$\nabla_\alpha J_s^\alpha = \text{[redacted]} + \begin{pmatrix}\text{products of}\\ \text{first order data}\end{pmatrix} ,$$

$$\Longrightarrow \tilde{\nu}_2 = \nu_0 = \nu_1 = \nu_2 = \nu_3 = 0$$

APPENDIX

-Entropy production

Structure of divergence

$$\nabla_\alpha J_s^\alpha = + \left(\nu_2 - \frac{\nu_3}{T}\right) \nabla_\mu E^\mu + \nu_3 \Delta^{\mu\nu} \nabla_\mu \partial_\nu \frac{\mu}{T}$$
$$+ (\nu_0 + \nu_1)\, u^\alpha \nabla_\alpha \nabla_\mu u^\mu - \nu_1 u^\alpha u^\mu R_{\alpha\mu}$$
$$- \tilde{\nu}_2 u^\alpha \nabla_\alpha B + \left(\begin{smallmatrix}\text{products of}\\ \text{first order data}\end{smallmatrix}\right) ,$$
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APPENDIX

-Entropy production

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Products of first order data

$$\partial_\alpha J_s^\alpha = + \partial_\alpha J_{s\,\text{canon}}^\alpha$$
$$- \Omega(\partial\cdot u)$$
$$- B(\partial\cdot u)$$
$$+ U_2 \cdot \tilde{U}_3$$
$$+ U_1 \cdot \tilde{U}_3$$
$$+ U_1 \cdot \tilde{U}_2$$

APPENDIX

-Entropy production

Structure of divergence

$$\nabla_\alpha J_s^\alpha = + \left(\nu_2 - \frac{\nu_3}{T}\right) \nabla_\mu E^\mu + \nu_3 \Delta^{\mu\nu} \nabla_\mu \partial_\nu \frac{\mu}{T}$$
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Products of first order data

$$\partial_\alpha J_s^\alpha = + \partial_\alpha J_{s\,\text{canon}}^\alpha$$
$$- \Omega(\partial\cdot u) \left[T \left(\frac{\partial P_0}{\partial \epsilon_0}\right)_{\rho_0} (\partial_T \tilde{\nu}_5 + \tilde{\nu}_1) + \frac{1}{T} \left(\frac{\partial P_0}{\partial \rho_0}\right)_{\epsilon_0} (\partial_{\bar{\mu}} \tilde{\nu}_5 + \tilde{\nu}_3) \right]$$
$$- B(\partial\cdot u) \left[T \left(\frac{\partial P_0}{\partial \epsilon_0}\right)_{\rho_0} \partial_T \tilde{\nu}_4 + \frac{1}{T} \left(\frac{\partial P_0}{\partial \rho_0}\right)_{\epsilon_0} \partial_{\bar{\mu}} \tilde{\nu}_4 \right]$$
$$+ U_2 \cdot \tilde{U}_3 \left[R_0 T (\partial_T \tilde{\nu}_3 - \partial_{\bar{\mu}} \tilde{\nu}_1) - \partial_{\bar{\mu}} \tilde{\nu}_4 + R_0 T^2 \partial_T \tilde{\nu}_4 \right]$$
$$+ U_1 \cdot \tilde{U}_3 \left[-R_0 T^2 (\partial_T \tilde{\nu}_5 + \tilde{\nu}_1) + (\partial_{\bar{\mu}} \tilde{\nu}_5 + \tilde{\nu}_3) + T(\partial_{\bar{\mu}} \tilde{\nu}_1 - \partial_T \tilde{\nu}_3) \right]$$
$$+ U_1 \cdot \tilde{U}_2 \left[\frac{\partial_{\bar{\mu}} \tilde{\nu}_5 + \tilde{\nu}_3}{T} + \partial_{\bar{\mu}} \tilde{\nu}_1 - \partial_T \tilde{\nu}_3 - T \partial_T \tilde{\nu}_4 \right] ,$$

APPENDIX

-*Entropy production*

Canonical part

$$\partial_\alpha J^\alpha_{s\,\mathrm{canon}} = -\left(\frac{1}{2}\Delta_{\mu\nu}\tau^{\mu\nu} - \left(\frac{\partial P_0}{\partial \epsilon_0}\right)_{\rho_0} u_\mu u_\nu \tau^{\mu\nu} + \left(\frac{\partial P_0}{\partial \rho_0}\right)_{\epsilon_0} u_\mu \Upsilon^\mu\right)\frac{\partial\cdot u}{T}$$
$$- \left(R_0 u_\mu \tau^{\mu\nu} + \Upsilon^\nu\right)\Delta_{\nu\alpha} U_3^\alpha$$
$$- \frac{\tau^{\mu\nu}\sigma_{\mu\nu}}{2T}\,.$$

Transform back to Landau frame

Thermodynamic response parameters

$$\tilde{\chi}_B = \frac{\partial P_0}{\partial \epsilon_0}\left(T\frac{\partial \mathcal{M}_B}{\partial T} + \mu\frac{\partial \mathcal{M}_B}{\partial \mu} - \mathcal{M}_B\right) + \frac{\partial P_0}{\partial \rho_0}\frac{\partial \mathcal{M}_B}{\partial \mu}\,,$$
$$\tilde{\chi}_\Omega = \frac{\partial P_0}{\partial \epsilon_0}\left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu\frac{\partial \mathcal{M}_\Omega}{\partial \mu} + f_\Omega(T) - 2\mathcal{M}_\Omega\right) + \frac{\partial P_0}{\partial \rho_0}\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right)\,,$$
$$\tilde{\chi}_E = \frac{\partial \mathcal{M}_B}{\partial \mu} - R_0\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right),$$
$$T\tilde{\chi}_T = \left(T\frac{\partial \mathcal{M}_B}{\partial T} + \mu\frac{\partial \mathcal{M}_B}{\partial \mu} - \mathcal{M}_B\right) - R_0\left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu\frac{\partial \mathcal{M}_\Omega}{\partial \mu} + f_\Omega(T) - 2\mathcal{M}_\Omega\right)\,,$$

Matching to two-point functions later gives: $\mathcal{M}_B = \dfrac{\partial P}{\partial B}\,, \qquad \mathcal{M}_\Omega = \dfrac{\partial P}{\partial \Omega}$

APPENDIX

-Two-point-functions

Most general parity-violating case is more complicated

$$\begin{pmatrix} k^2\sigma - i\omega\frac{\partial\rho_0}{\partial\mu} & -k^2\left(\frac{\mu}{T}\sigma+\chi_T\right) - i\omega\frac{\partial\rho_0}{\partial T} & ik\rho_0 & 0 \\ -i\omega\frac{\partial\epsilon_0}{\partial\mu} & -i\omega\frac{\partial\epsilon_0}{\partial T} & ik(\epsilon_0+P_0) & 0 \\ ik\rho_0 & iks_0 & k^2(\eta+\zeta) - i\omega(\epsilon_0+P_0) & k^2\left(\tilde{\chi}_\Omega+\tilde{\eta}\right) \\ 0 & 0 & -k^2\tilde{\eta} & k^2\eta - i\omega(\epsilon_0+P_0) \end{pmatrix} \begin{pmatrix} \delta\mu \\ \delta T \\ \delta u^x \\ \delta u^y \end{pmatrix}$$

= vector containing external sources $h_{\mu\nu},\ A_\mu$

For example, we get a Kubo formula for

$$\lim_{k\to 0}\frac{1}{ik}\langle \mathcal{C}^0\mathcal{T}^{02}\rangle_R(0,k) = \tilde{\chi}_\Omega$$

APPENDIX

-Two-point-functions

Most general parity-violating case is more complicated

$$\begin{pmatrix} k^2\sigma - i\omega\frac{\partial\rho_0}{\partial\mu} & -k^2\left(\frac{\mu}{T}\sigma+\chi_T\right) - i\omega\frac{\partial\rho_0}{\partial T} & ik\rho_0 & 0 \\ -i\omega\frac{\partial\epsilon_0}{\partial\mu} & -i\omega\frac{\partial\epsilon_0}{\partial T} & ik(\epsilon_0+P_0) & 0 \\ ik\rho_0 & iks_0 & k^2(\eta+\zeta) - i\omega(\epsilon_0+P_0) & k^2\left(\tilde{\chi}_\Omega+\tilde{\eta}\right) \\ 0 & 0 & -k^2\tilde{\eta} & k^2\eta - i\omega(\epsilon_0+P_0) \end{pmatrix} \begin{pmatrix} \delta\mu \\ \delta T \\ \delta u^x \\ \delta u^y \end{pmatrix}$$

= vector containing external sources $h_{\mu\nu},\ A_\mu$

For example, we get a Kubo formula for

$$\lim_{k\to 0}\frac{1}{ik}\langle \mathcal{C}^0\mathcal{T}^{02}\rangle_R(0,k) = \tilde{\chi}\Omega$$

APPENDIX

-Two-point-functions

Restrictions from Onsager relations

$$G_R^{ij}(\omega, \mathbf{k}; b_a) = n_i n_j G_R^{ji}(\omega, -\mathbf{k}; -b_a)$$

where under time-reversal $\Theta \mathcal{O}_i \Theta^{-1} = n_i \mathcal{O}_i$

APPENDIX

-Two-point-functions

Restrictions from Onsager relations

$$G_R^{ij}(\omega, \mathbf{k}; b_a) = n_i n_j G_R^{ji}(\omega, -\mathbf{k}; -b_a)$$

where under time-reversal $\Theta \mathcal{O}_i \Theta^{-1} = n_i \mathcal{O}_i$

From time-reversal covariance plus translation invariance

$$G_R^{ij}(x) \equiv i\theta(t)\,\mathrm{Tr}\left(\varrho[\mathcal{O}_i(t,\mathbf{x}), \mathcal{O}_j(0)]\right) = i\theta(t)\, n_i n_j \mathrm{Tr}\left(\varrho'[\mathcal{O}_j(t,-\mathbf{x}),\, \mathcal{O}_i(0)]\right)$$

Parameters b_a break time-reversal invariance,
i.e. time-reversal and $b_a \to -b_a$
together are a symmetry

APPENDIX

-Two-point-functions

Restrictions from susceptibility constraints

Examples $\lim_{\mathbf{k}\to 0}\langle J^0 J^0\rangle(\omega=0,\mathbf{k}) = \left(\frac{\partial\rho_0}{\partial\mu}\right)_T$

Partition function in grand canonical ensemble

$$Z[T,\,\mu] = \mathrm{Tr}\left[\exp\left(-\frac{H}{T}+\frac{\mu Q}{T}\right)\right]$$

Constant external sources $A_0,\, h_{00},\, h_{0i}$
can be eliminated by shifting thermodynamic variables

$$Z[T,\,\mu;\,A_0,\,h_{00},\,h_{0i}] = Z\left[T\left(1+\frac{h_{00}}{2}\right),\,\mu\left(1+\frac{h_{00}}{2}\right)+A_0;\,0,\,0,\,0\right]$$

Thus we get relations for zero-momentum limits of zero-frequency correlators.

APPENDIX

-*Magnetovortical frame*

Thermodynamics depending on vorticity and magnetic field

$$P = P(T, \mu, B, \Omega) \qquad dP = s\, dT + \rho\, d\mu + \frac{\partial P}{\partial B} B + \frac{\partial P}{\partial \Omega} \Omega\,,$$

$$\epsilon + P = sT + \mu\rho\,.$$

Constitutive relations

$$\begin{aligned} T^{\mu\nu} &= (\epsilon - \mathcal{M}_\Omega \Omega + f_\Omega \Omega)\, u^\mu u^\nu \\ &\quad + (P - \zeta \nabla_\alpha u^\alpha - \tilde{x}_B B - \tilde{x}_\Omega\, \Omega)\, \Delta^{\mu\nu} - \eta \sigma^{\mu\nu} - \tilde{\eta} \tilde{\sigma}^{\mu\nu}\,, \\ J^\mu &= (\rho - \mathcal{M}_B \Omega)\, u^\mu + \sigma V^\mu + \tilde{\sigma} \tilde{V}^\mu + \tilde{\chi}_E \tilde{E}^\mu + \tilde{\chi}_T \epsilon^{\mu\nu\rho} u_\nu \nabla_\rho T\,, \end{aligned}$$

where $\mathcal{M}_B = \dfrac{\partial P}{\partial B}\,, \qquad \mathcal{M}_\Omega = \dfrac{\partial P}{\partial \Omega}$

Matching

$$\tilde{x}_B = \frac{\partial P}{\partial B}\,, \qquad \tilde{x}_\Omega = \frac{\partial P}{\partial \Omega}\,,$$

$$T\tilde{\chi}_T = \frac{\partial \epsilon}{\partial B} + R_0 \left(\frac{\partial P}{\partial \Omega} - \frac{\partial \epsilon}{\partial \Omega} - f_\Omega \right)\,, \qquad \tilde{\chi}_E = \frac{\partial \rho}{\partial B} + R_0 \left(\frac{\partial P}{\partial B} - \frac{\partial \rho}{\partial \Omega} \right)\,.$$

APPENDIX

-2+1 dimensional results

[Jensen,MK,Kovtun,Meyer, Ritz,Yarom 1112.4498]

Conservation equations

$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \qquad \nabla_\mu J^\mu = 0$$

APPENDIX

-2+1 dimensional results

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Conservation equations

$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \qquad \nabla_\mu J^\mu = 0$$

Constitutive equations

$$T^{\mu\nu} = \epsilon_0 u^\mu u^\nu + \left(P_0 - \zeta \nabla_\alpha u^\alpha - \tilde{\chi}_B B - \tilde{\chi}_\Omega \Omega\right) \Delta^{\mu\nu} - \eta \sigma^{\mu\nu} - \tilde{\eta} \tilde{\sigma}^{\mu\nu}$$

$$J^\mu = \rho_0 u^\mu + \sigma V^\mu + \tilde{\sigma} \tilde{V}^\mu + \tilde{\chi}_E \tilde{E}^\mu + \tilde{\chi}_T \epsilon^{\mu\nu\rho} u_\nu \nabla_\rho T$$

"New" transport terms arise!

APPENDIX

-2+1 dimensional results

[Jensen,MK,Kovtun,Meyer, Ritz,Yarom 1112.4498]

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$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \qquad \nabla_\mu J^\mu = 0$$

Constitutive equations

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$$J^\mu = \rho_0 u^\mu + \sigma V^\mu + \tilde{\sigma} \tilde{V}^\mu + \tilde{\chi}_E \tilde{E}^\mu + \tilde{\chi}_T \epsilon^{\mu\nu\rho} u_\nu \nabla_\rho T$$

"New" transport terms arise!

thermodynamic interpretation of $\tilde{\chi}_E$, $\tilde{\chi}_\Omega$, $\tilde{\chi}_B$

$\tilde{\eta}$ Hall viscosity

$\tilde{\chi}_E, \tilde{\sigma}$ off-diagonal conductivity (anomalous Hall conductivty)

$\tilde{\chi}_T$ "thermal Hall conductivity"