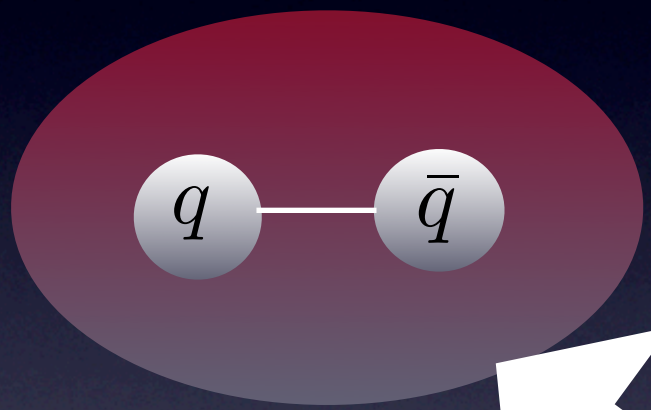
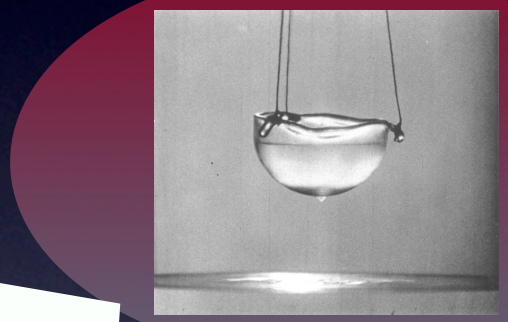
Fermions in a holographic p-wave superfluid

London Imperial College, HET Seminar, June 23rd 2010





AdS/CFT

[arXiv: 0810.2316]
[arXiv: 0903.1864]
[arXiv: 1003.1134]

by Matthias Kaminski (Princeton University)

in collaboration with M.Ammon, J.Erdmenger, A.O'Bannon (MPI Munich)

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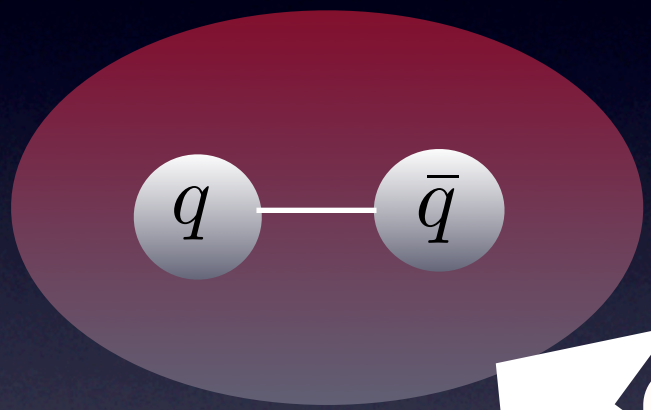

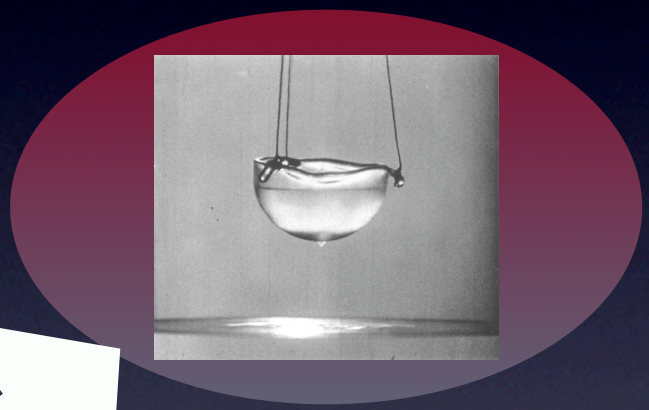

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Outline

I. Invitation: Fermions in Experiments & String Theory

II. Holographic Superfluids

III. Flavored Fermions from String Theory

IV. Results (spec. func., scaling, reduced Fermi surface)

V. Discussion

I. Invitation

Why can string theory be useful for condensed matter physics?

I. Invitation: Superfluids & Fermions

Superfluid: global symmetry spontaneously broken

Superconductor: charged superfluid, gauge symmetry

→ weakly gauge AdS-boundary theory

Weak coupling concepts

- (charged) condensate of Cooper-pairs (BCS)
- (gauge) symmetry spontaneously broken
- strongly correlated electron systems → ?

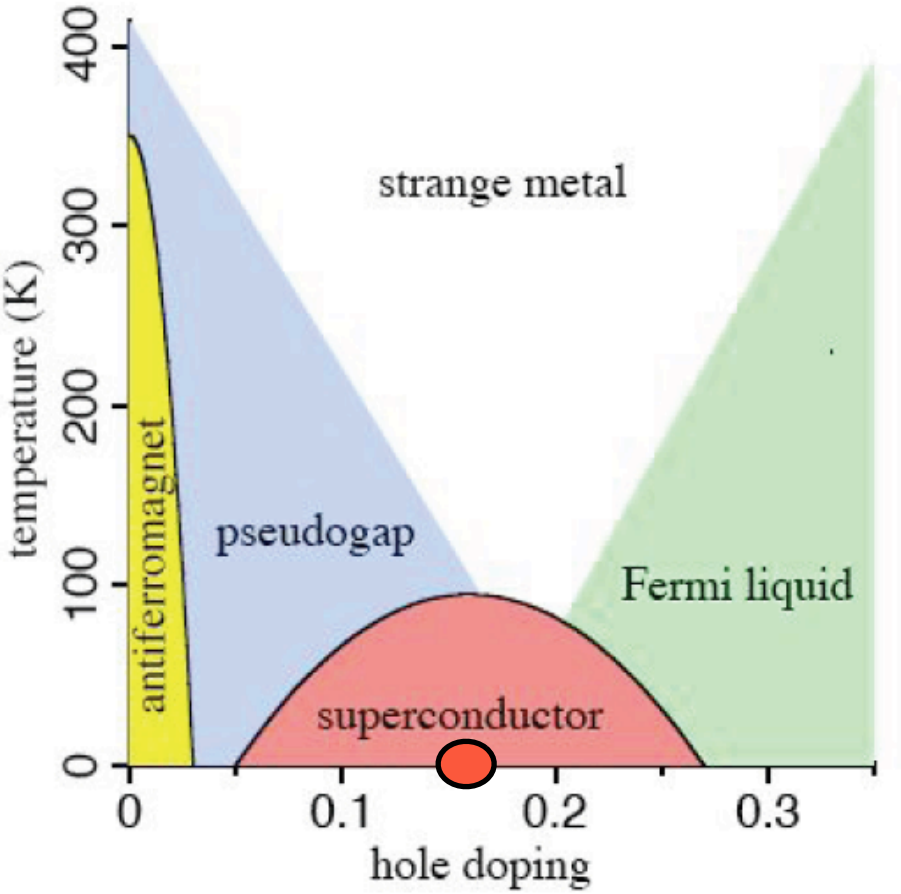
Examples

- Color superconducting phase at high densities
  [Alford, Rajagopal, Wilczek '97]
- superfluid He3, (high Tc) superconductors

What happens to Fermions?

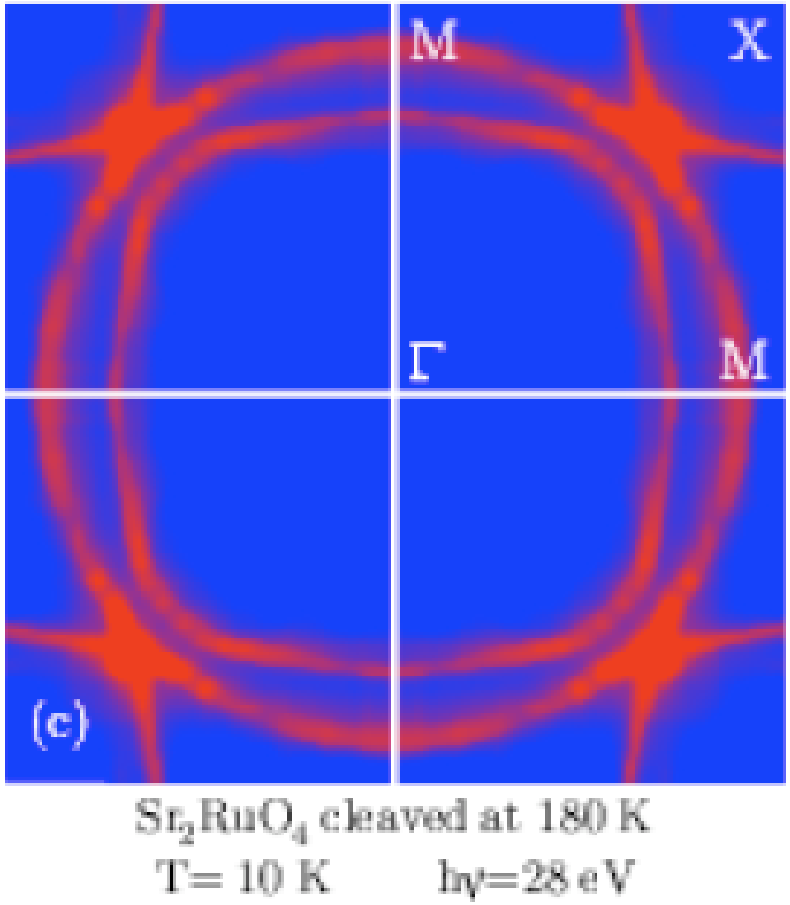
I. Invitation: Fermions in Experiments

Quantum Critical Point



high Tc cuprate (d-wave)

Fermi Surface from ARPES



unconventional low Tc superconductor (p-wave)

I. Invitation: Holographic Fermions

Gravity model [Liu et al. 0903.2477]

charged AdS4 black hole:

$$ds^2 \equiv g_{MN} dx^M dx^N = \frac{r^2}{R^2}(-f dt^2 + d\vec{x}^2) + \frac{R^2}{r^2}\frac{dr^2}{f}$$

$$f = 1 + \frac{Q^2}{r^{2d-2}} - \frac{M}{r^d}, \qquad A_t = \mu\left(1 - \frac{r_0^{d-2}}{r^{d-2}}\right)$$

Dirac action:

$$S = \int d^{d+1}x\sqrt{-g}\, i(\bar{\psi}\Gamma^M \mathcal{D}_M \psi - m\bar{\psi}\psi)$$

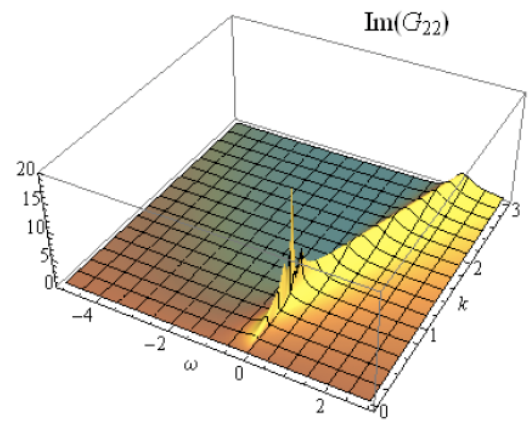
$$\mathcal{D}_M = \partial_M + \frac{1}{4}\omega_{abM}\Gamma^{ab} - iqA_M$$

Critical Scaling: non-Fermi liquid

$$\omega_* \sim (k - k_F)^z$$

$$\mathrm{Im}\, G_{22}(\omega_*, k) \sim (k - k_F)^{-\alpha}$$

fermionic quasiparticle peak:



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Fermi liquid:

$$\alpha = 1 = z$$

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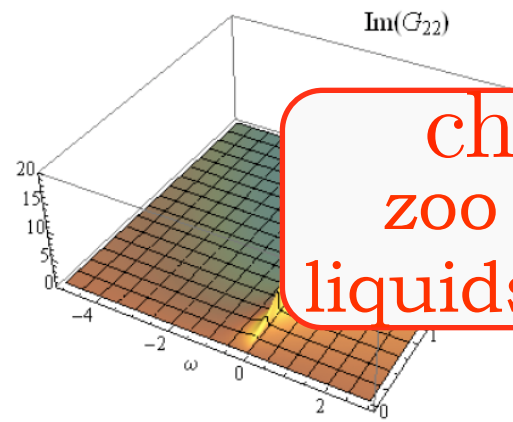

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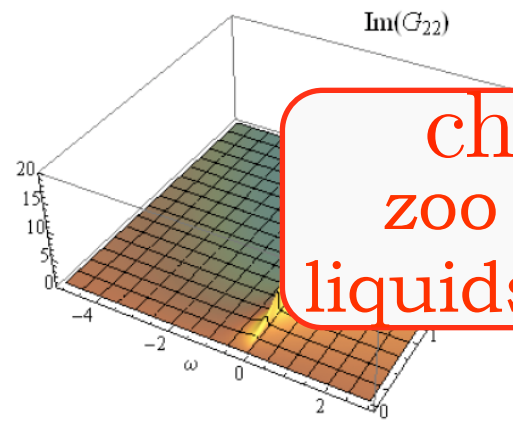

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Problems

- arbitrary parameters (fermion charge/mass,...)
- low T but condensation NOT considered

Solutions

- fix parameters by generic string construction (D3/D5)
- background with condensate (SC black hole)

Navigator

✓ Invitation: Fermions in Experiments & String Theory

II. Holographic Superfluids

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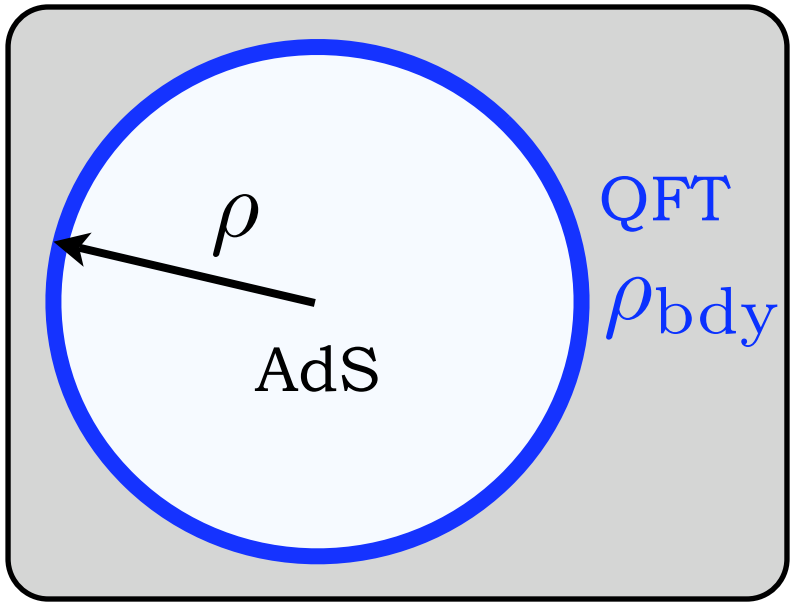
II. Holo SF: Gauge/Gravity Correspondence

gravity theory (weak)	$\longleftrightarrow$	gauge theory (strong)

Near AdS-boundary $(\rho \to \infty)$

$$A = \underbrace{A^{(0)}}_{\text{non-normalizable (source)}} + \frac{\overbrace{A^{(2)}}^{\text{normalizable (vev)}}}{\rho^2} + \cdots$$



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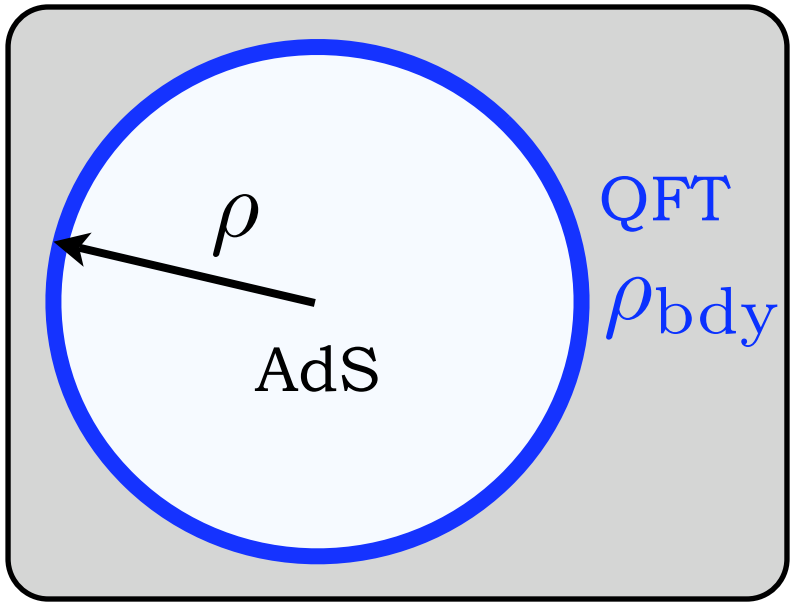


Dictionary

QFT FEATURE (energy scale) $\longleftrightarrow$ GEOMETRY (radial coord. ρ)

operator J_μ $\longleftrightarrow$ field A_μ (gauge)

vev $\longleftrightarrow$ $A^{(2)}$ (charge)

source $\longleftrightarrow$ $A^{(0)}$ (chem. pot.)

Building a Holographic Superfluid

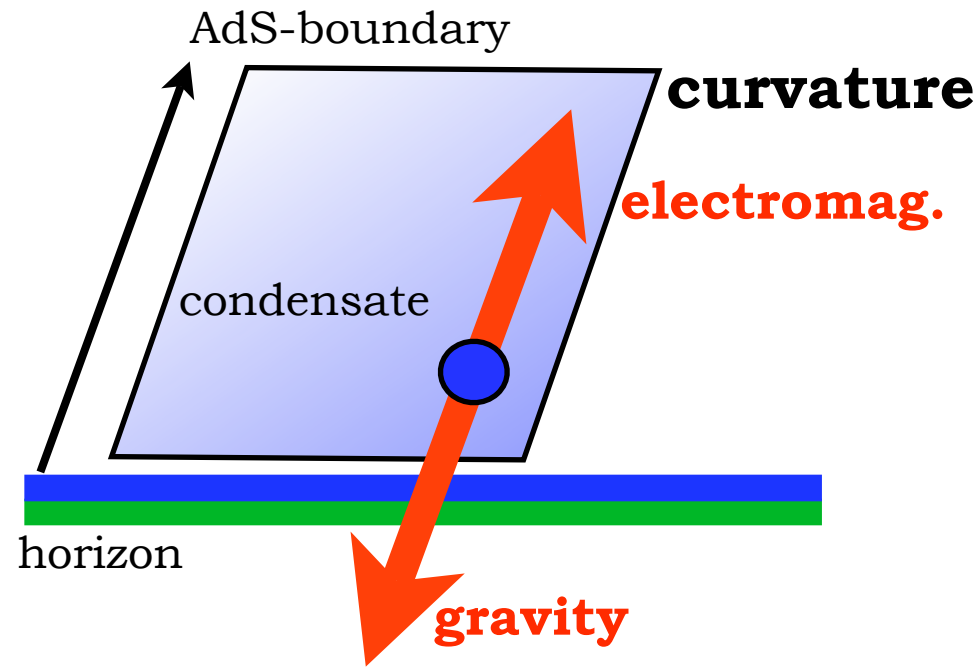
Field Theory

What do we need?

- charged condensate (vev)
- no source
- condensate of charge carriers
- finite temperature, chem. pot.

Gravity

[Gubser 0801.2977]
[Gubser, Pufu 0805.2960]



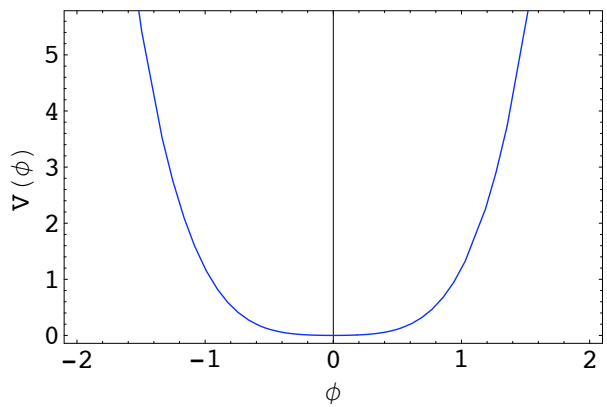
- introduce normalizble mode
- no non-normalizable mode
- condensate hovers over horizon
- black hole, charge

Is this stable?

Get some intuition

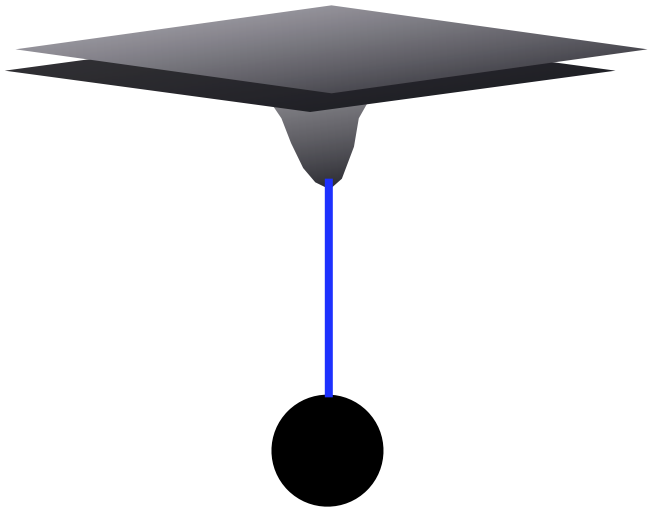
Field Theory

$$\mathcal{L} \sim D_\nu \phi D^\nu \phi \sim ({M_q}^2 - {\mu_{\text{isospin}}}^2)\phi^2$$



- charged particles condense at large enough chemical potential $\mu_{\text{isopin}} \sim M_q$

Gravity



- strings (D3-D5) give FT charges
- cannot put infinitely many
- second brane is important (probe limit)

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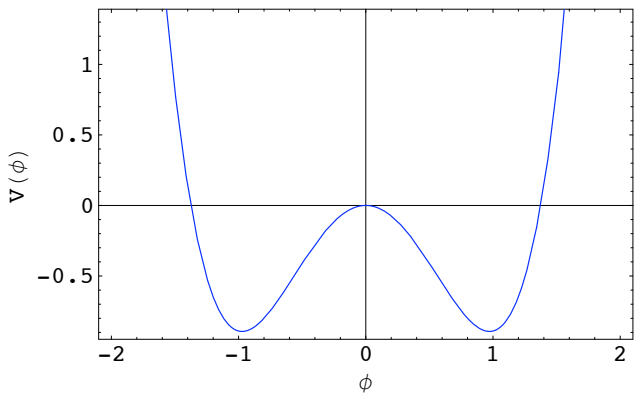

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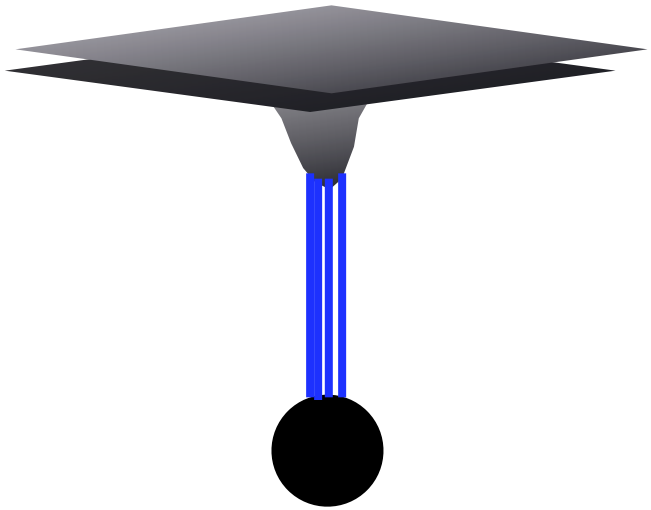

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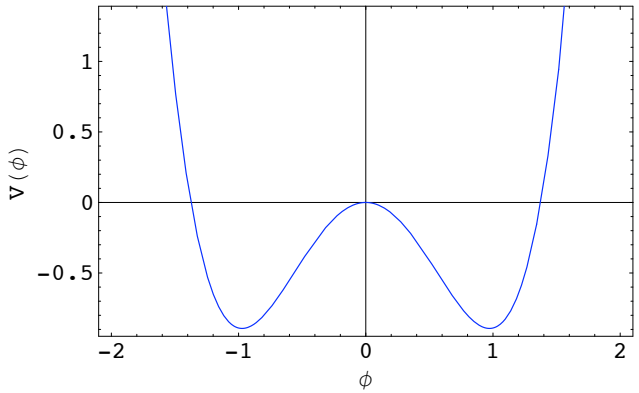

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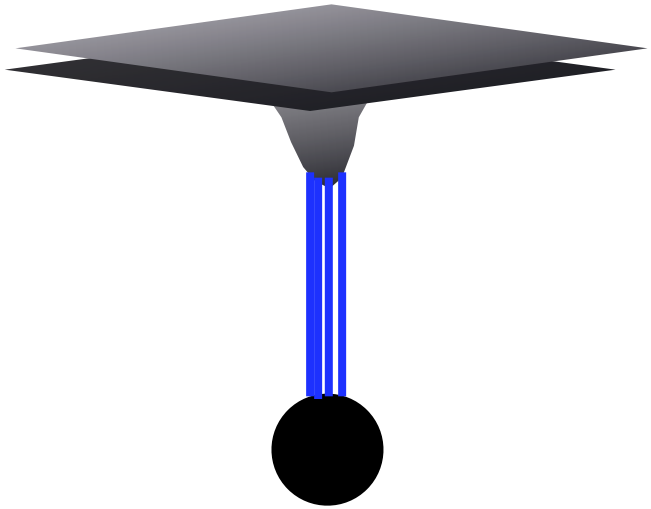

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We need a non-Abelian structure!

II.b) Holographic Superfluid: D3/D5-Branes

- N_c D3-branes
  dual to $\mathcal{N} = 4$ SYM with $SU(N_c)$

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N_f D5-branes

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[Karch,Katz hep-th/0205236]

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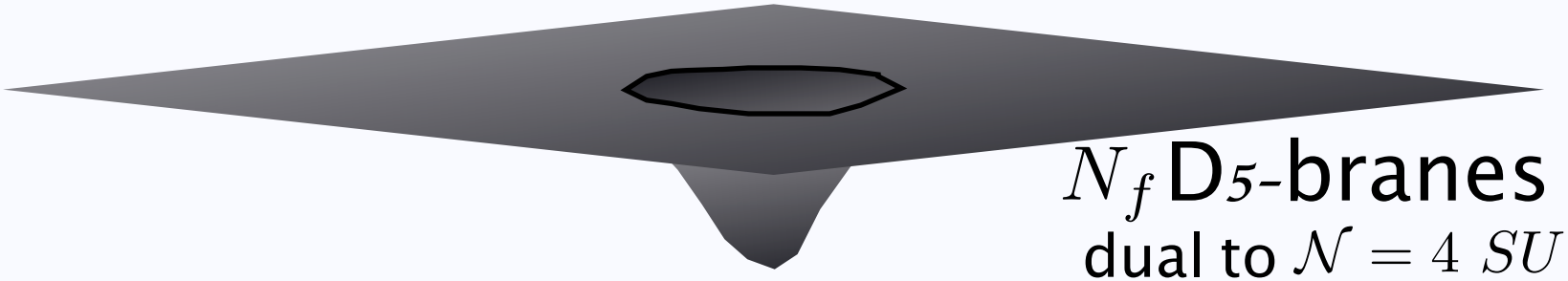

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N_c D3-branes (black)

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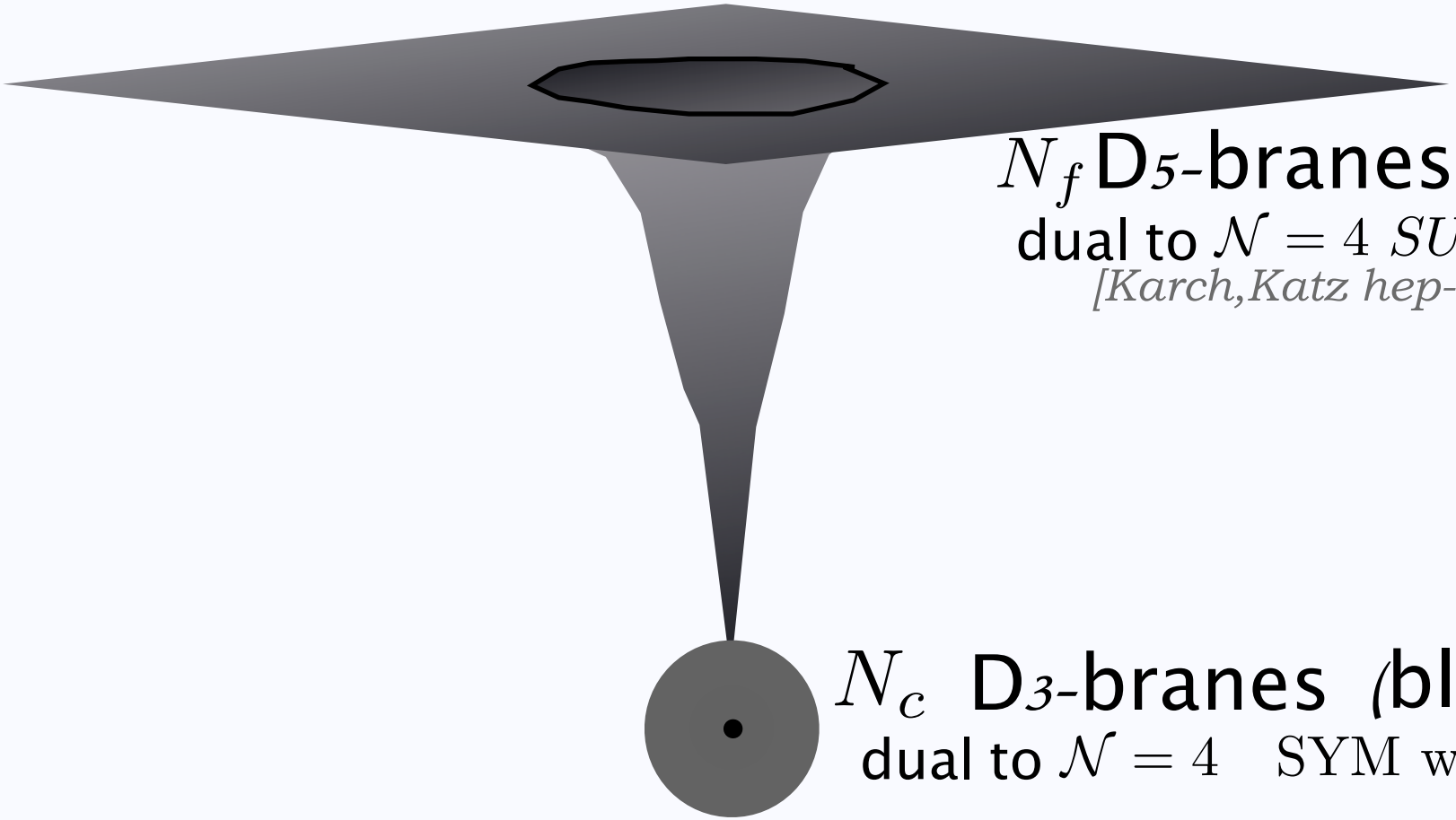

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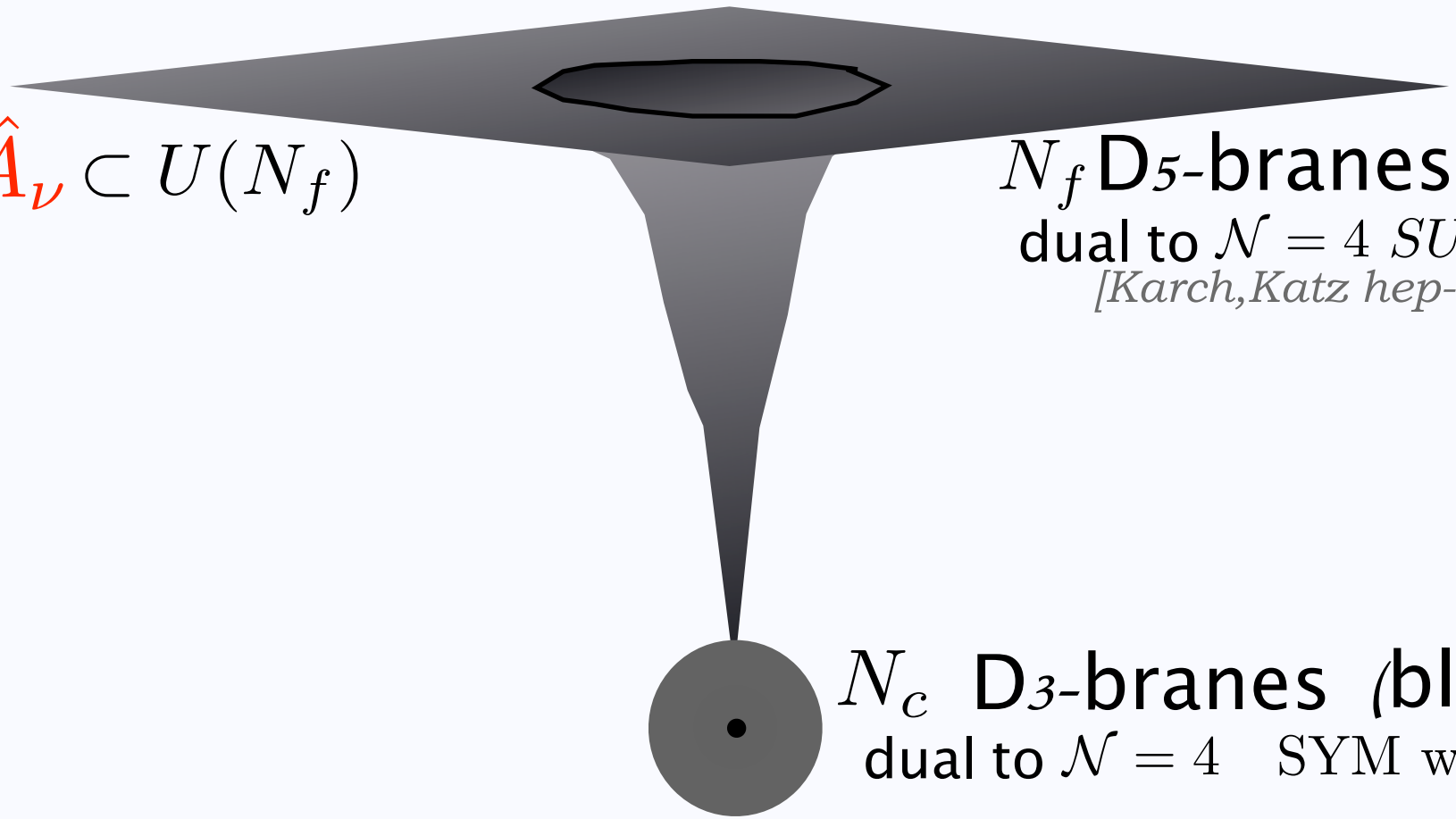

$\hat{A}_\nu \subset U(N_f)$

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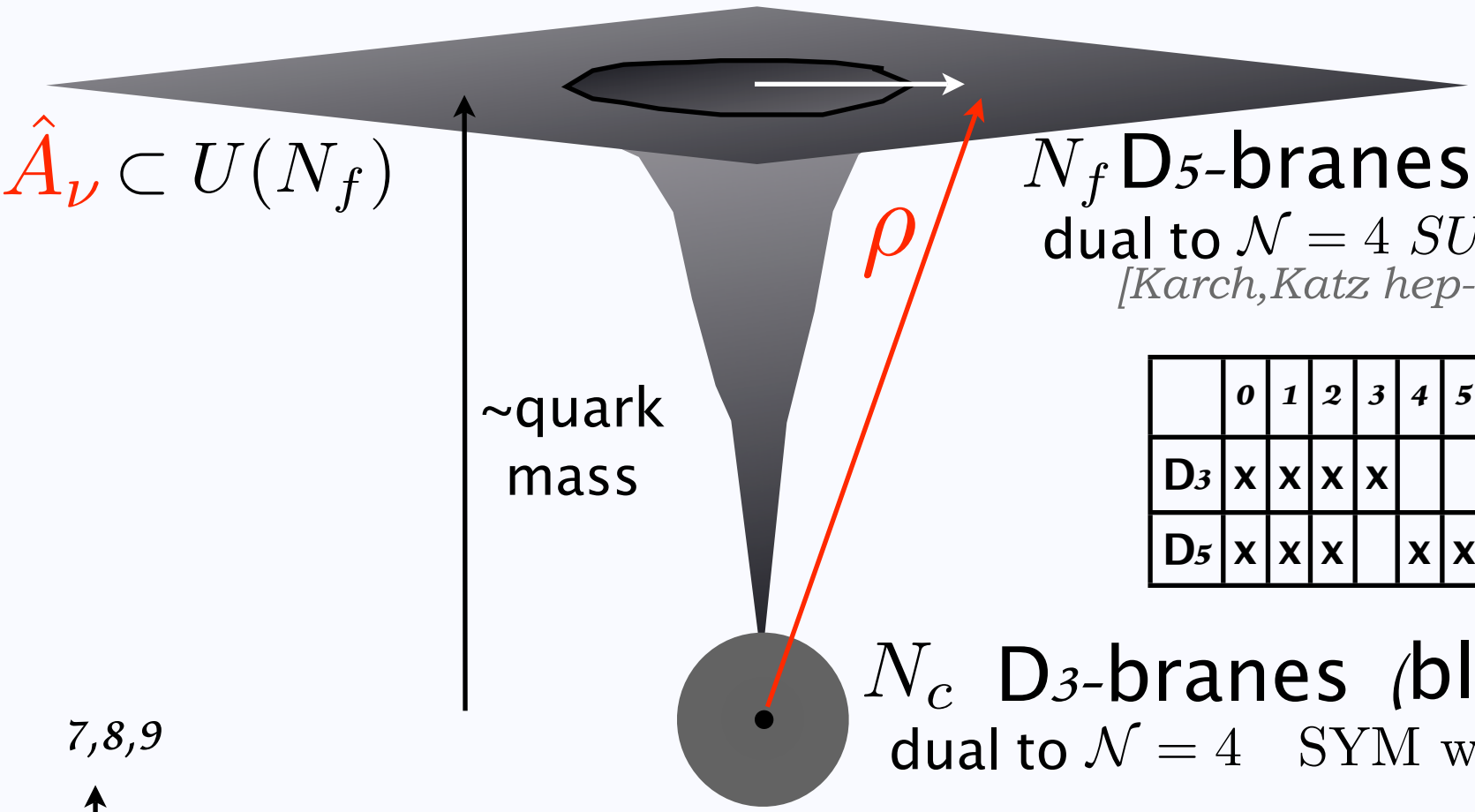

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[Karch,Katz hep-th/0205236]

	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x						
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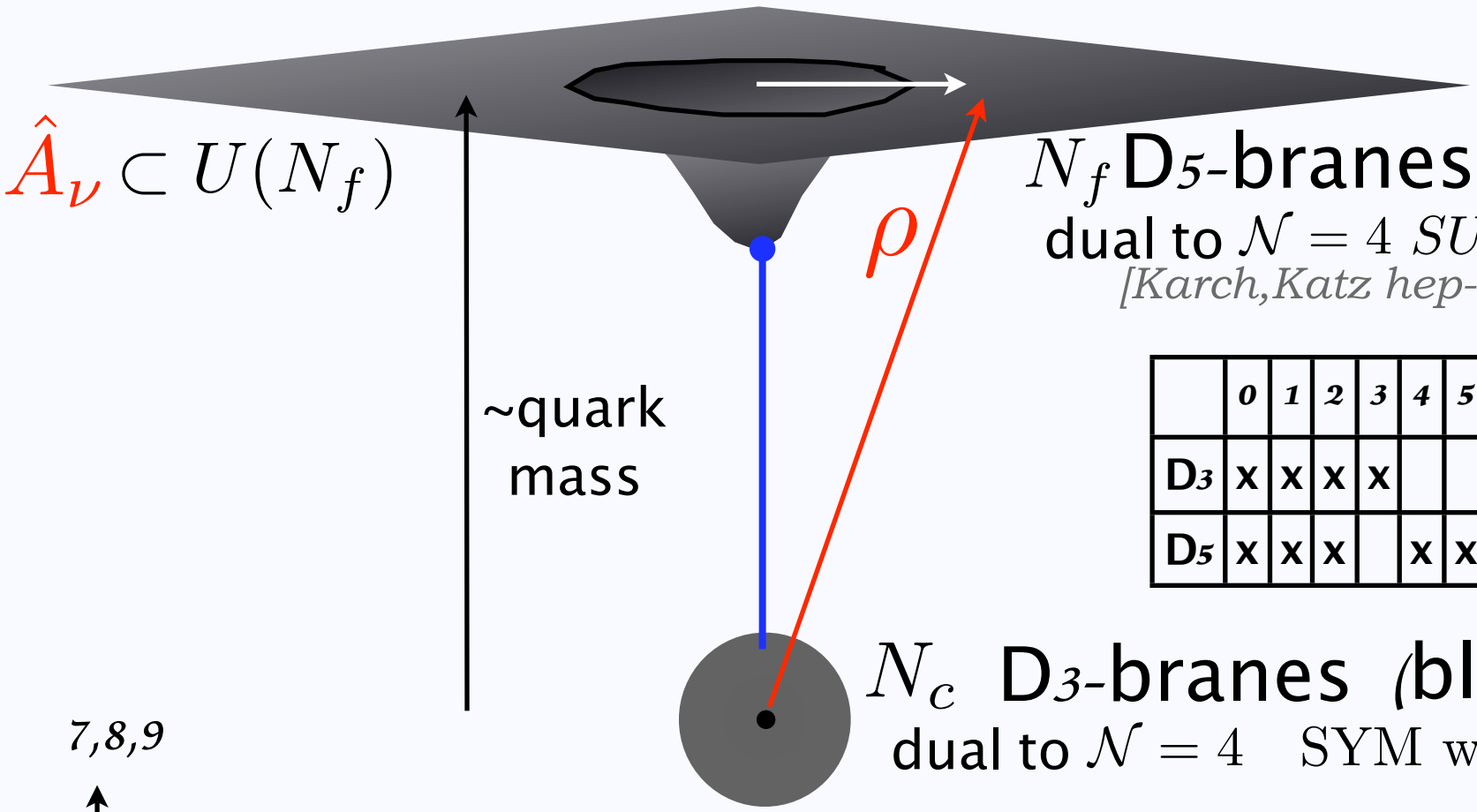

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Chemical potential: $\hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu$
(cf. therm. FT)

[Myers et al., hep-th/0611099]
[Mateos et al. 0709.1225]

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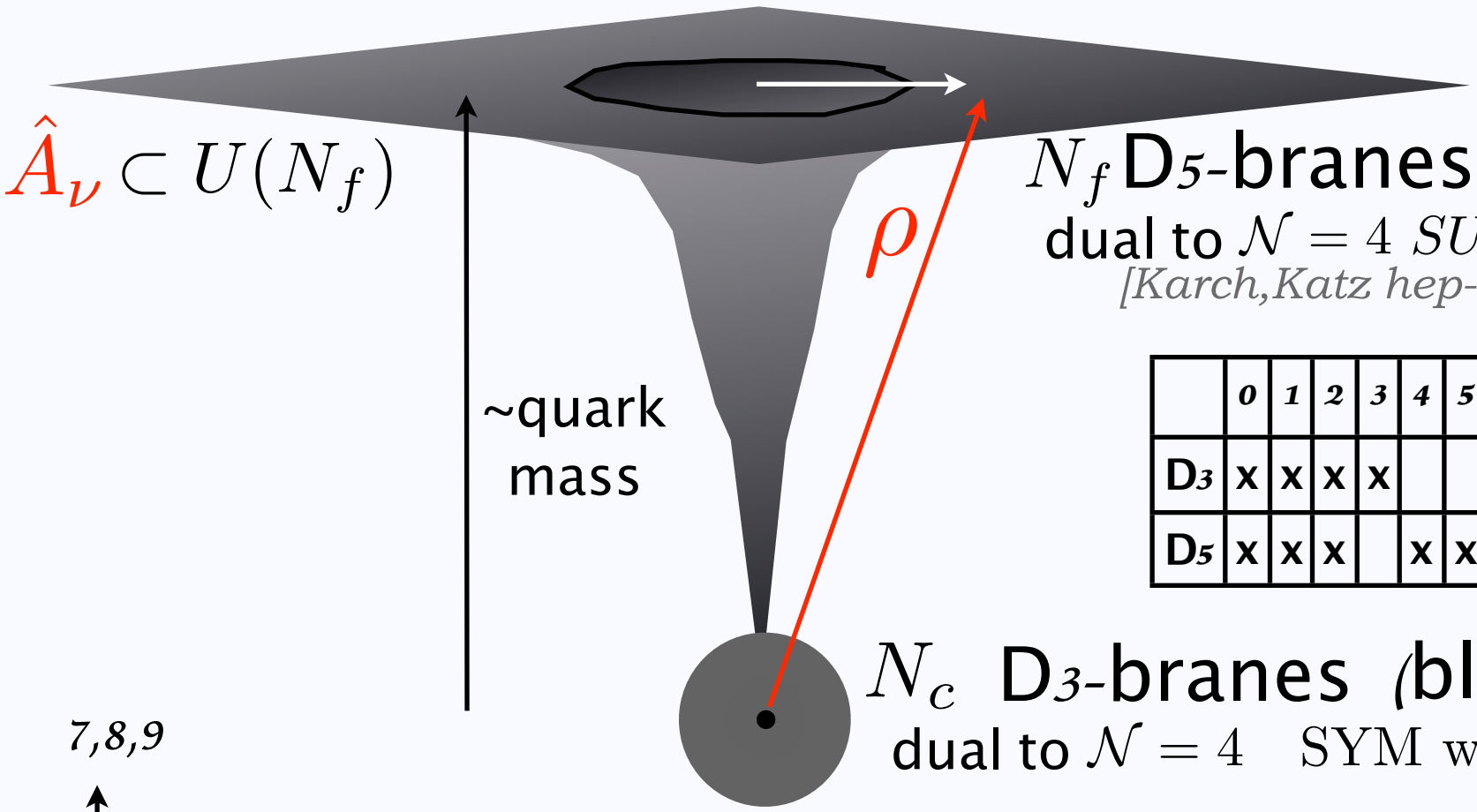

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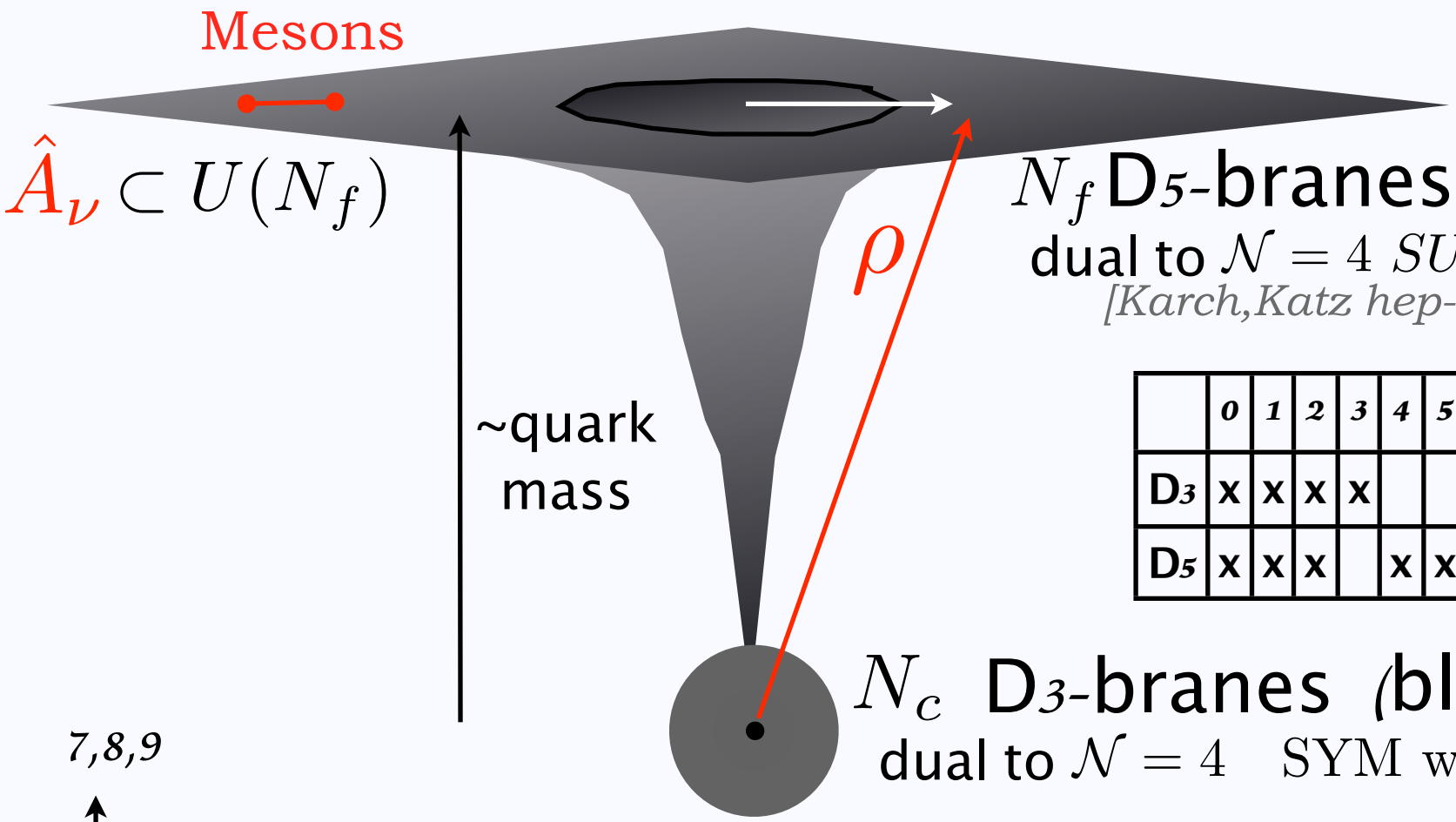

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Non-Abelian DBI with Chemical Potential

AdS black hole metric

↓ pullback

Induced metric on Dp-branes

$$ds^2_{Dp} = \frac{1}{u^2}\left(\frac{du^2}{f(u)} - f(u)dt^2 + d\vec{x}^2\right) + ds^2_{S^Q}$$

with $f(u) = 1 - \frac{u^4}{u_h^4}, \qquad u_h = \frac{1}{\pi T}$

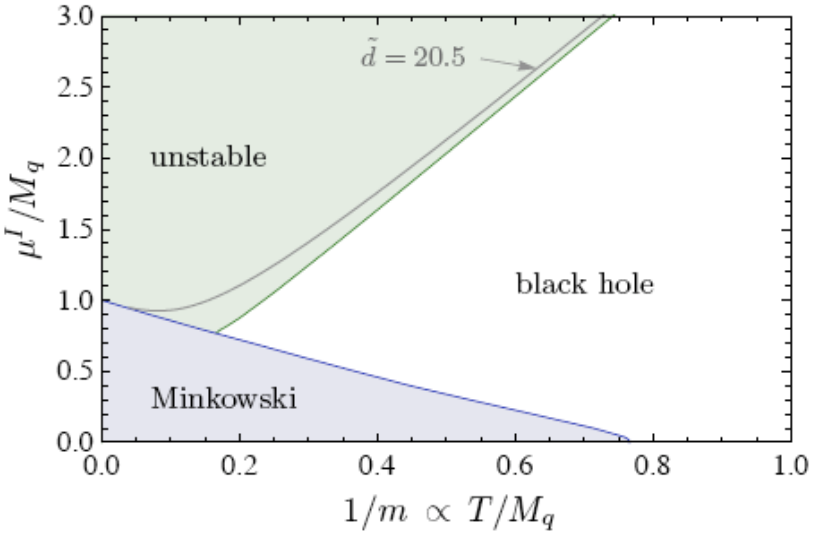
Bosonic DBI action (second order expansion)

$$S_{Dp} = -T_{Dp} N_f \int d^{p+1}\xi \sqrt{-g_{Dp}} \left[1 + (2\pi\alpha')^2 \frac{1}{2} Tr\left(F_{\mu\nu}F^{\mu\nu}\right)\right]$$

chemical potential
(cf. thermal QFT)

Flavor Superfluid Phase

General idea



[Erdmenger, M.K., Kerner, Rust 0807.2663]

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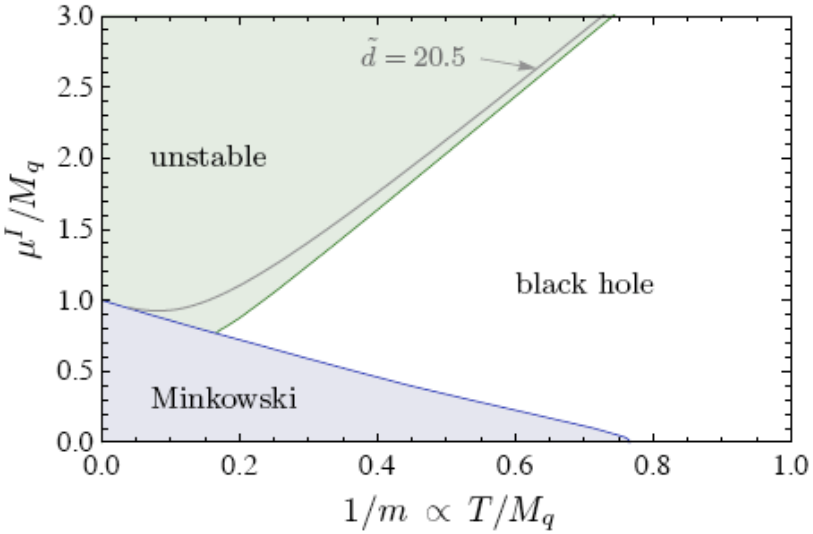

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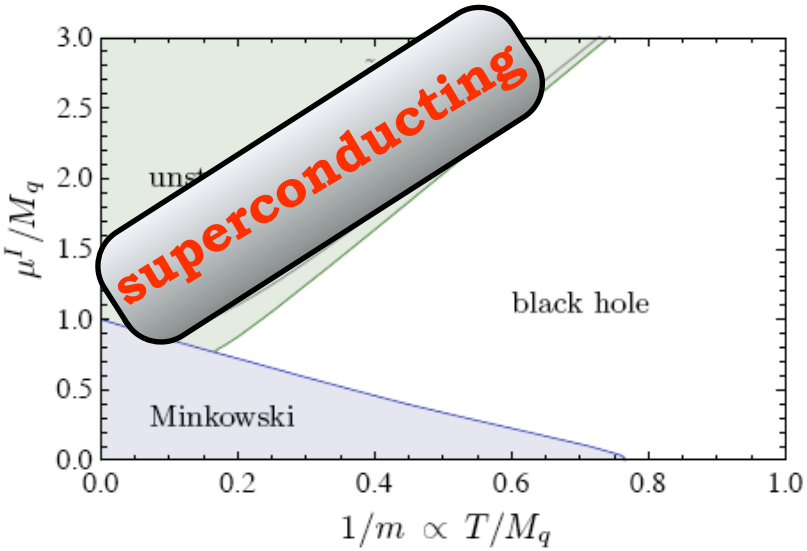

[Ammon, Erdmenger, M.K., Kerner 0810.2316]

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[Gubser, Pufu 0805.2960]

Flavor Superfluid Phase

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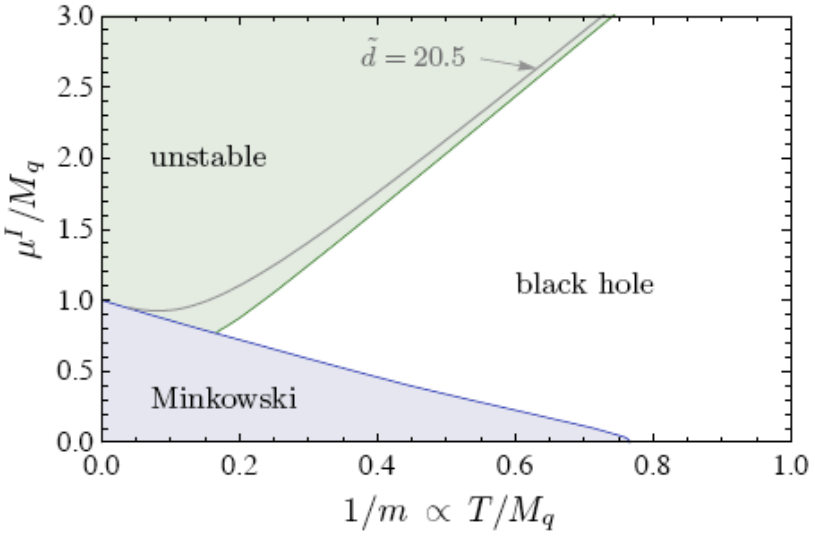

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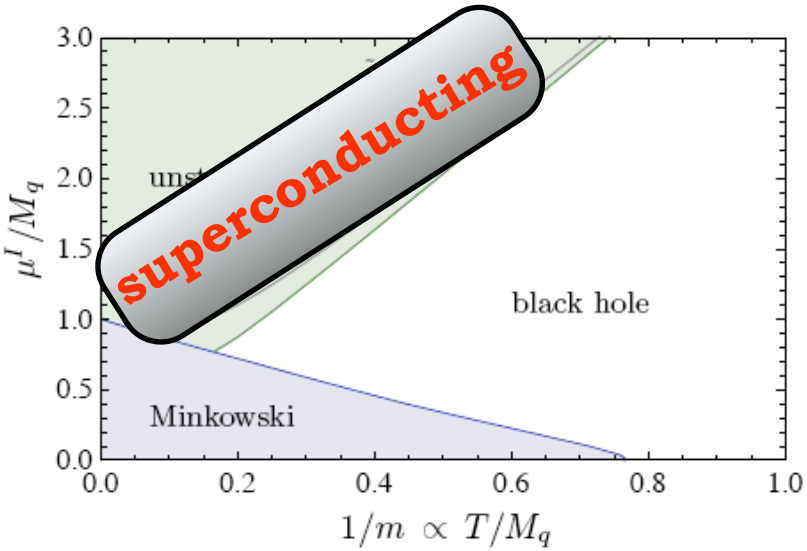

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← spont. breaks U(1)

[Gubser, Pufu 0805.2960]

Field Theory Picture

Gravity field $A_0^3 = \mu + \frac{d_0^3}{\rho^2} + \dots$

dual to current $J_0^3 \propto \bar{\psi}\tau^3\gamma_0\psi + \phi\tau^3\partial_0\phi = n_u - n_d$

explicitly breaks $U(2) \sim U(1)_B \times SU(2)_I \to U(1)_B \times U(1)_3$

New field $A_3^1 = \frac{d_3^1}{\rho^2} + \dots$

dual to current

$$J_3^1 \propto \bar{\psi}\tau^1\gamma_3\psi + \phi\tau^1\partial_3\phi$$
$$= \bar{\psi}_u\gamma_3\psi_d + \bar{\psi}_d\gamma_3\psi_u + \text{bosons}$$

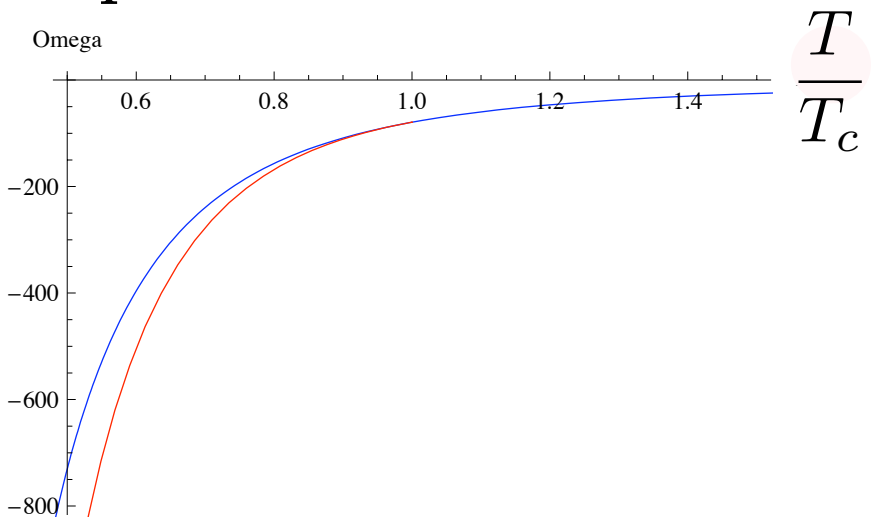
spontaneously breaks $U(1)_3 \leftrightarrow U(1)_{\text{em}}$ and $SO(2)$

Condensation of SU(2) adjoint vector mesons

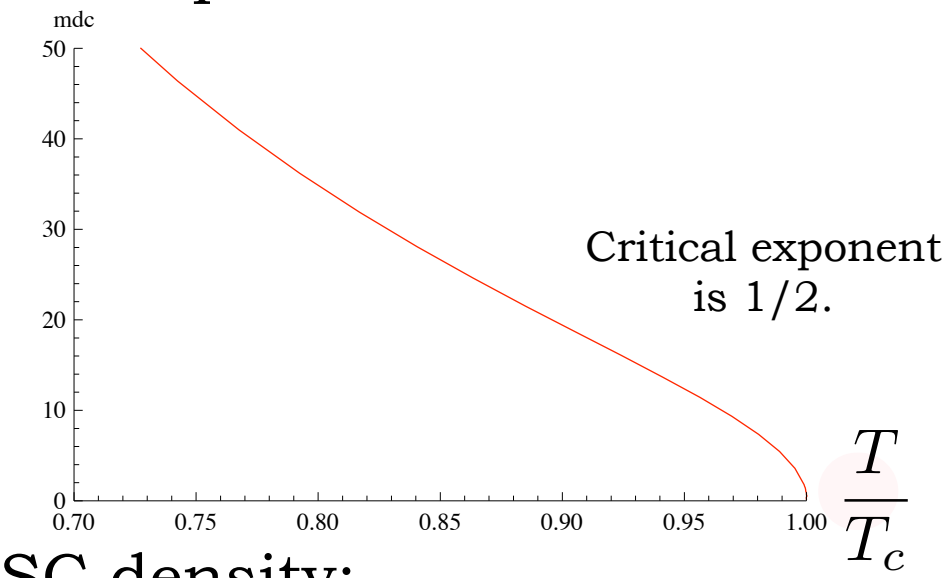
Thermodynamics results

$S_{\mathrm{DBI}} \propto$ grand potential

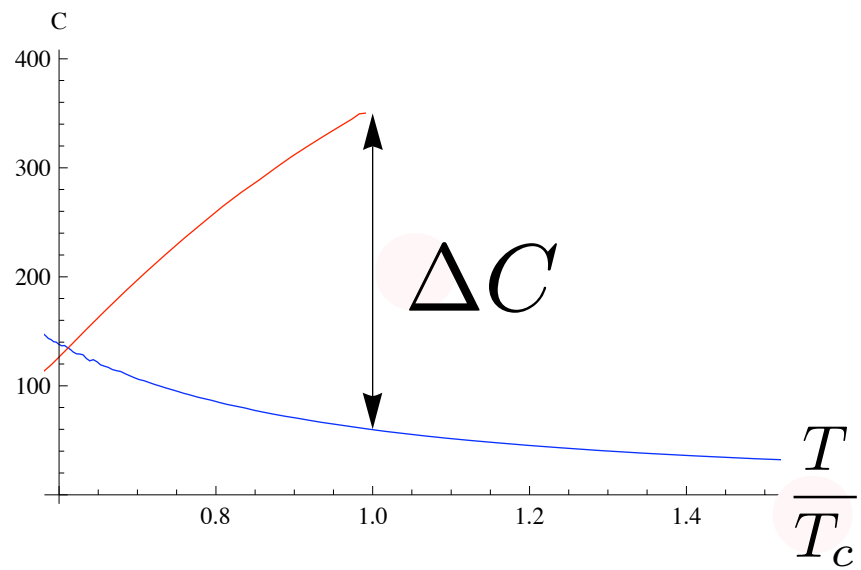
Grand potential:



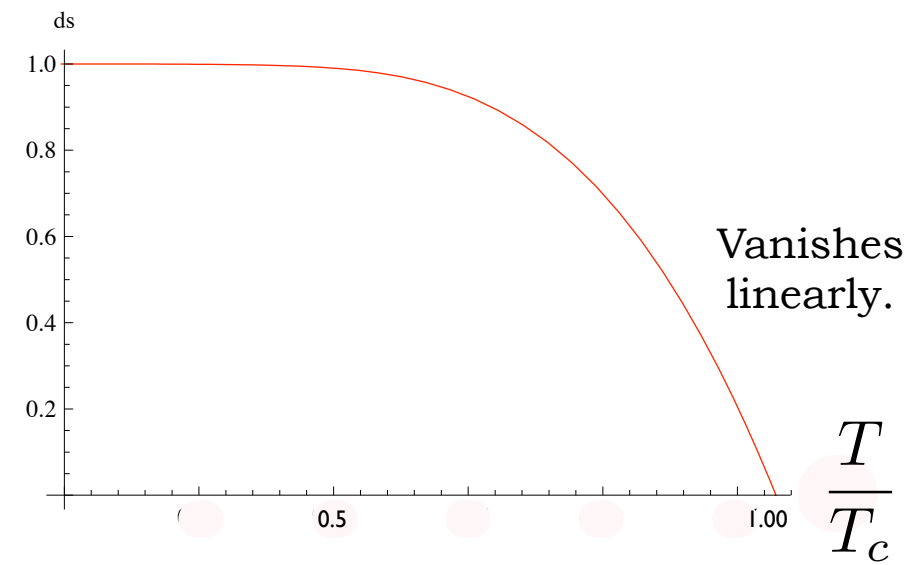
Order parameter:



Specific heat:



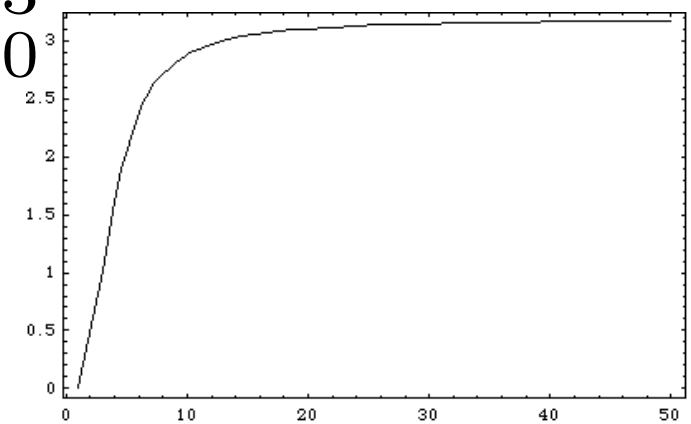
SC density:



Background field configuration

Gravity fields:

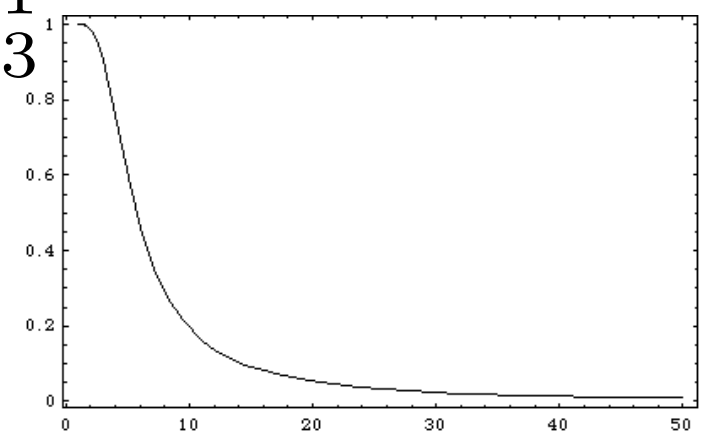
A_0^3



$\rightarrow \mu_0^3$

ρ

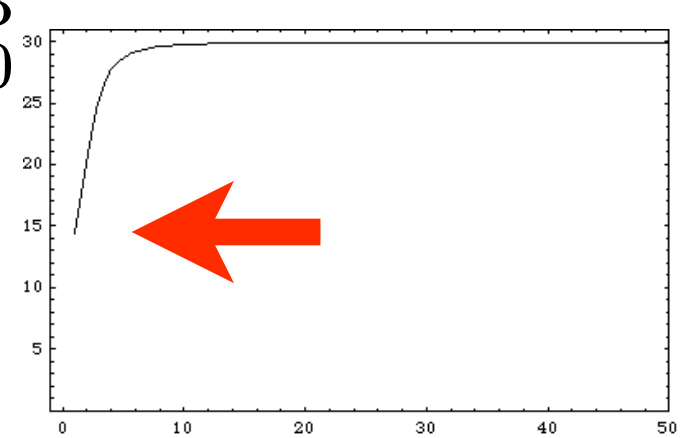
A_3^1



ρ

Conjugate momenta:

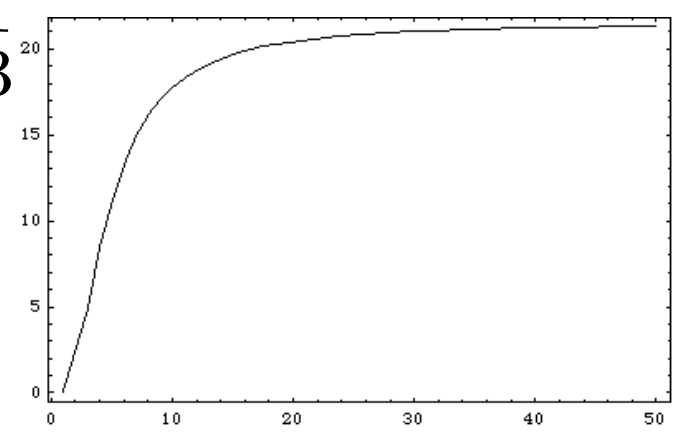
p_0^3



$\rightarrow d_0^3$

ρ

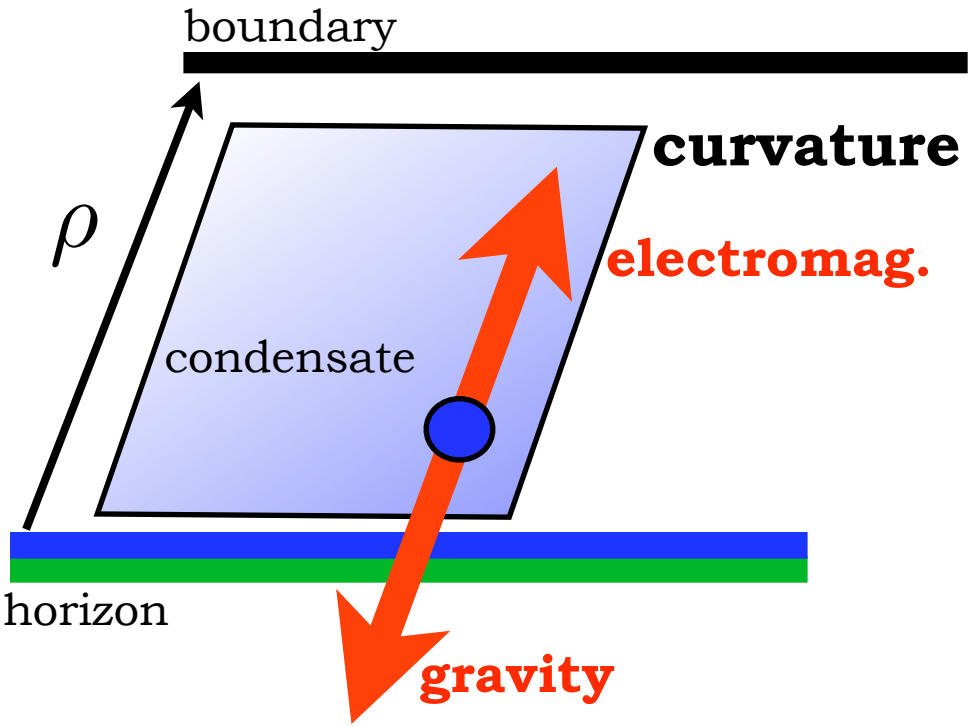
p_3^1



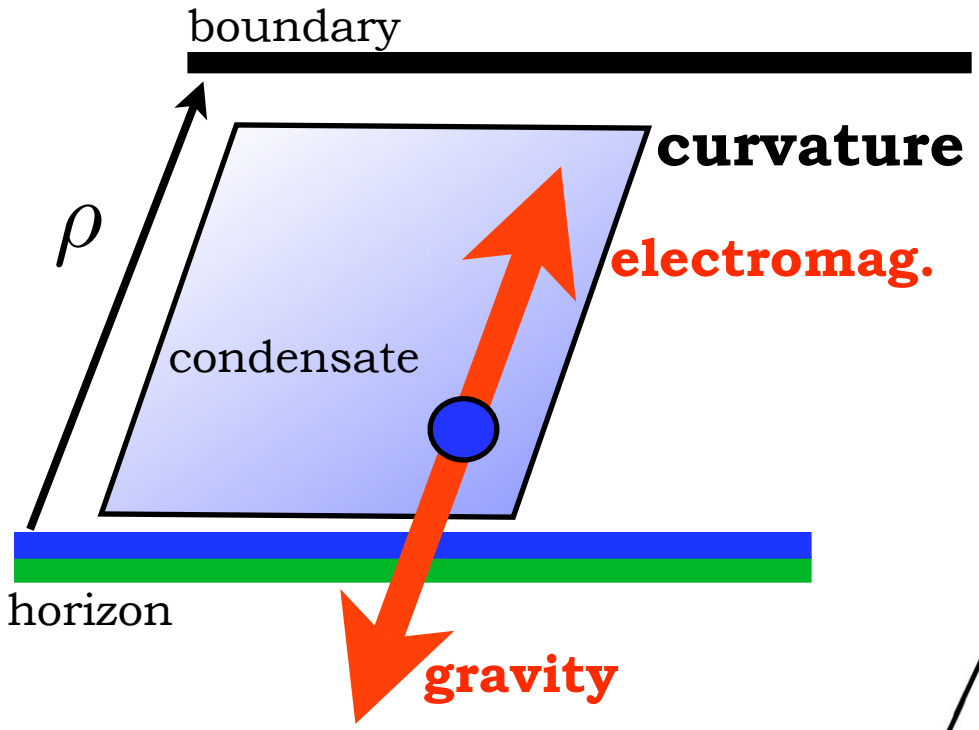
$\rightarrow d_3^1$

ρ

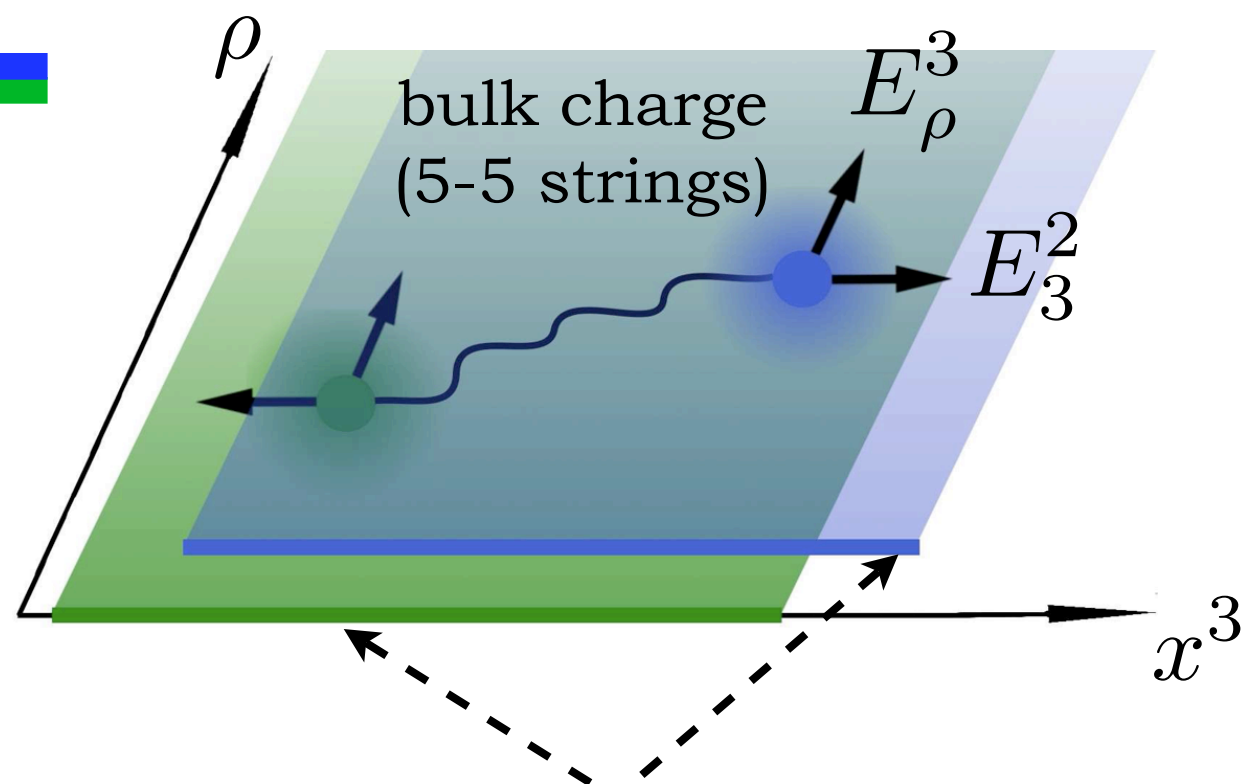
String Theory Picture



String Theory Picture



$$E^3_\rho = F^3_{\rho 0} = \partial_\rho A^3_0$$
$$B^1_{\rho 3} = F^1_{\rho 3} = \partial_\rho A^1_3$$
$$E^2_3 = F^2_{30} = A^1_3 A^3_0$$



5-5 strings generate A^1_3 i.e. they break the U(1) and are thus dual to Cooper pairs.

charged horizon (3-5 strings generate A^3_0)

Navigator

✓ Invitation: Fermions in Experiments & String Theory

✓ Holographic Superfluids

III. Flavored Fermions from String Theory

IV. Results (spec. func., scaling, reduced Fermi surface)

V. Discussion

III. Fermions from ST: D3/D5-Brane Setup

SUSY: meson superpartners

$\hat{A}_\nu \subset U(N_f)$

N_f D5-branes

ρ

$L(\rho)$

	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x						
D5	x	x	x		x	x	x			

N_c D3-branes (black)

7,8,9

4,5,6

Chemical potential: $\hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu$

(cf. therm. FT)

[Myers et al., hep-th/0611099]
[Mateos et al. 0709.1225]

Worldvolume Fermions

Quadratic action for fermionic fluctuations:

$$S_{Dp} = N_f T_{Dp} \int d^{p+1}\xi \sqrt{-g_{Dp}} \frac{1}{2} Tr \left[\hat{\bar{\Psi}} P_- \Gamma^{\hat{A}} \left(D_{\hat{A}} + \frac{1}{8} \frac{i}{2*5!} F_{\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}} \Gamma^{\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}} \Gamma_{\hat{A}} \right) \hat{\Psi} \right]$$

[Martucci et al. hep-th/0504041]

Dirac equation:

$$\left[\Gamma^{\hat{A}} D_{\hat{A}} + \frac{1}{8} \frac{i}{2*5!} \Gamma^{\hat{A}} F_{\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}} \Gamma^{\hat{N}\hat{P}\hat{Q}\hat{R}\hat{S}} \Gamma_{\hat{A}} \right] \hat{\Psi} = 0$$

reduces to $\left(\not{D}_{AdS_P} - m_l^{\pm} \right) \Psi_l^{\pm} = 0$

Spelling out the gauge and spacetime covariant derivative

$$\left[\left(\not{D}_{AdS_P} - m \right) \Psi \right]_a = \left(u\sqrt{f}\, \gamma^u \partial_u + \frac{u}{\sqrt{f}} \gamma^t \partial_t + u\, \gamma^i \partial_i + \left[-\frac{P-1}{2} \sqrt{f} + \frac{1}{4} u \frac{f'}{\sqrt{f}} \right] \gamma^u \right) \Psi_a$$
$$+ e^M_A\, i\, \gamma^A \left[A_M , \Psi \right]_a - m\, \Psi_a$$

adjoint fermions couple through commutator to gauge field

Fixing the parameters

Dirac equation masses in $AdS_P \times S^Q$

$$m_\ell^+ = \ell + \frac{P}{2}, \qquad m_\ell^- = -\left(\ell + Q - \frac{1}{2}P\right)$$

Corresponding operator dimensions $\Delta_\ell^\pm = \frac{P-1}{2} + |m_\ell^\pm|$

Choose massless fermions $P = 4,\, Q = 2$ $\quad m_\ell^+ = \ell + 2$

$$\boxed{m_\ell^- = -\ell}$$

Fermion charges under $U(1)_3$ $\quad q = \pm 1,\, 0$

Dual Field Theory: (2+1)dim SYM with $\mathcal{N} = 4$

$SO(4,2) \to SO(3,2)$ $\qquad$ $SO(6)_R \to SU(2)_H \times SU(2)_V$

contains quarks and squarks $\quad \psi\,,\ \tilde{\psi}$

[DeWolfe et al., hep-th/0111135]
[Erdmenger et al., hep-th/0203020]

Operator dual to fermionic D5-brane fluctuations Ψ_ℓ^-

$$\mathcal{F}_\ell = \bar{\psi}(X_H)^{(\dots)}\tilde{\psi} + \tilde{\psi}^\dagger(X_H)^{(\dots)}\psi$$

$$\Delta_\ell^- = \frac{3}{2} + \ell$$

is neutral under baryon, adjoint under isospin symmetry.

Consider massless $\quad \mathcal{F}_{\ell=0} = \bar{\psi}\tilde{\psi} + \tilde{\psi}^\dagger\psi$

with gauge indices a=1,2,3 corresponding to bulk charges $q = \pm 1,\ 0$

Equations of motion for gravity fermions

$$\Psi_a(u, x^\mu) = u^{3/2} f(u)^{-1/4} e^{ik\cdot x} \psi_a(u)$$

$$0 = \left(\sqrt{f}\gamma^u\partial_u - \frac{i\omega}{\sqrt{f}}\gamma^t + ik_i\gamma^i - \frac{1}{u}m\right)\psi_+ + i\frac{A_t^3(u)}{\sqrt{f}}\gamma^t\psi_+ + A_x^1(u)\gamma^x\psi_0$$

$$0 = \left(\sqrt{f}\gamma^u\partial_u - \frac{i\omega}{\sqrt{f}}\gamma^t + ik_i\gamma^i - \frac{1}{u}m\right)\psi_- - i\frac{A_t^3(u)}{\sqrt{f}}\gamma^t\psi_- + A_x^1(u)\gamma^x\psi_0$$

$$0 = \left(\sqrt{f}\gamma^u\partial_u - \frac{i\omega}{\sqrt{f}}\gamma^t + ik_i\gamma^i - \frac{1}{u}m\right)\psi_0 - \frac{1}{2}A_x^1(u)\gamma^x(\psi_+ + \psi_-).$$

$\boxed{A_x^1 = 0}$ diagonal correl.

[Liu et al. 0903.2477]

$$\psi_\alpha \equiv \Pi_\alpha\psi = a_\alpha\begin{pmatrix}0\\1\end{pmatrix} + b_\alpha\begin{pmatrix}1\\0\end{pmatrix} + \dots$$

$$G_\alpha^R(\omega, k_x, k_y) = b_\alpha/a_\alpha$$

$\boxed{A_x^1 \neq 0}$ superfluid phase, coupled bulk eoms, operator mixing

[Cubrovic et al. 0904.1993]
[Kaminski et al. 0911.3610]

Navigator

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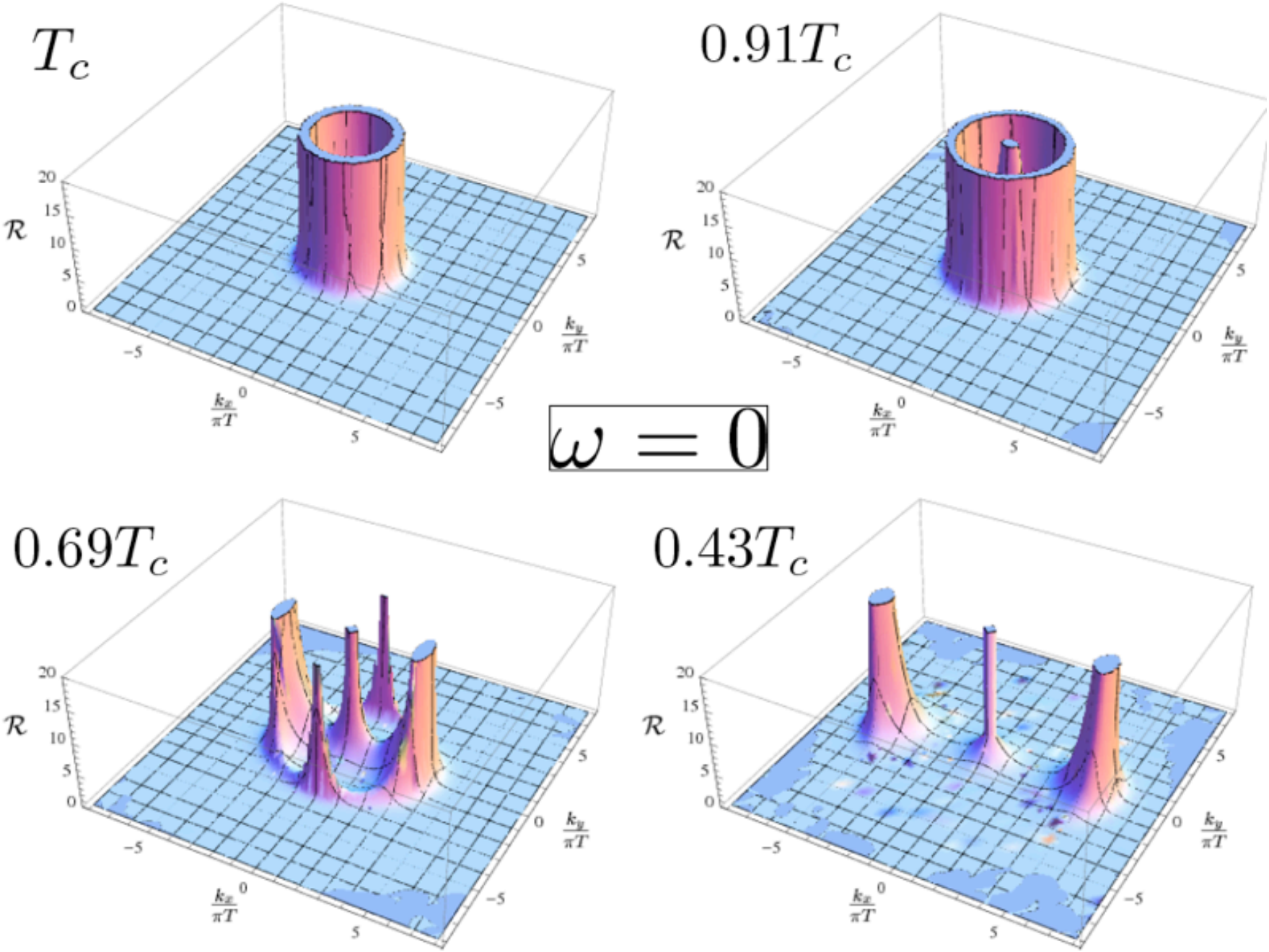
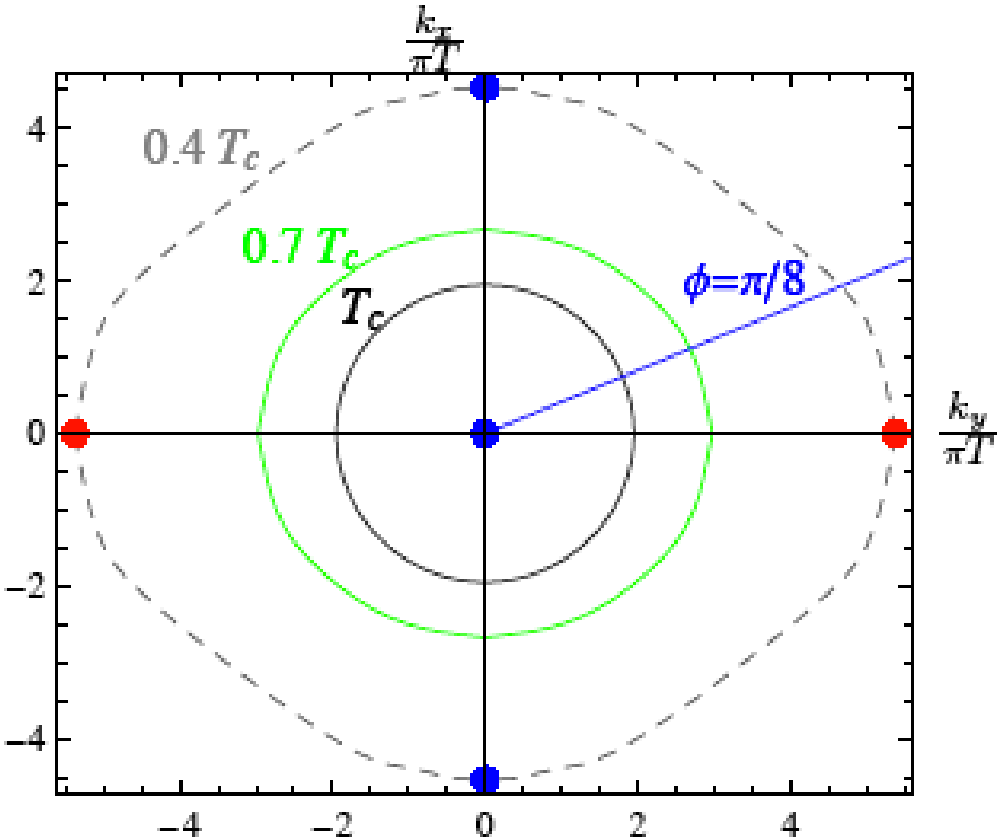
Main results in wrong phase

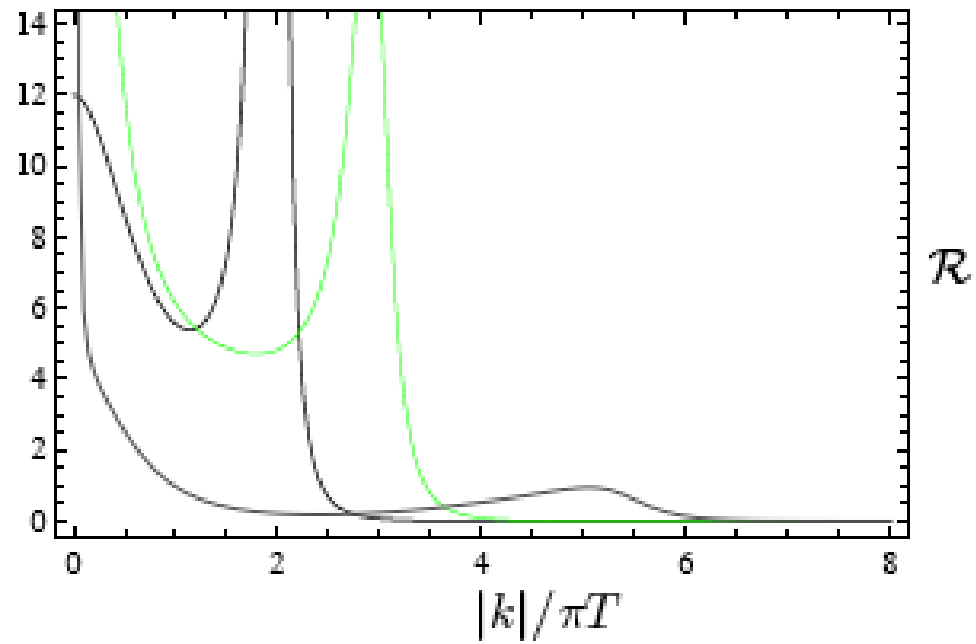
- reproduces (multiple) Fermi surfaces
- non-Fermi liquid with critical exponents $z = 1,\ \alpha = 2$
- qualitatively very similar to Liu et al

Superfluid phase: 3D Spectral measure

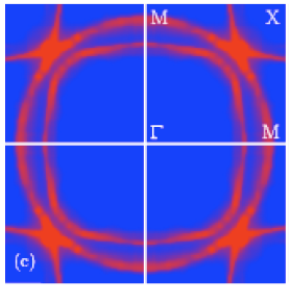


Superfluid phase: Anisotropic Fermi surface

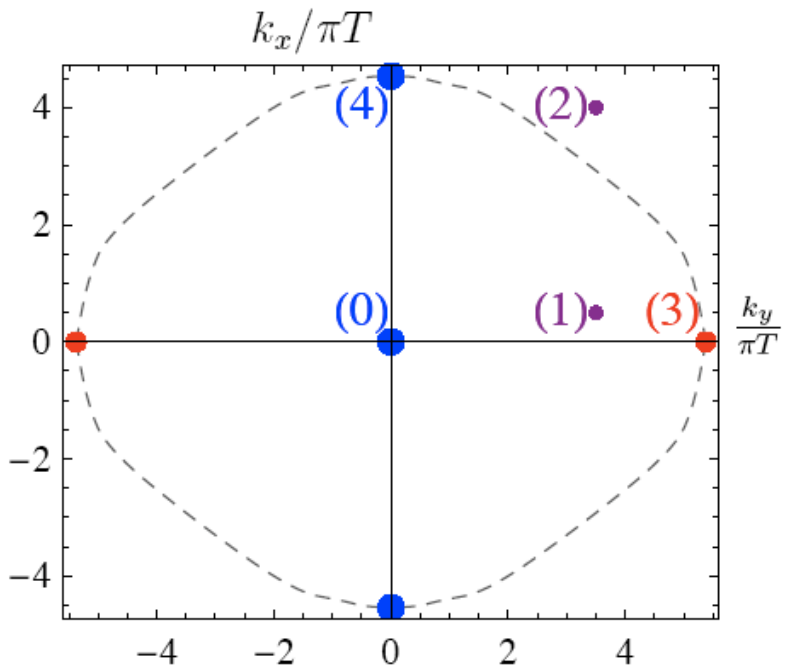




Recall:

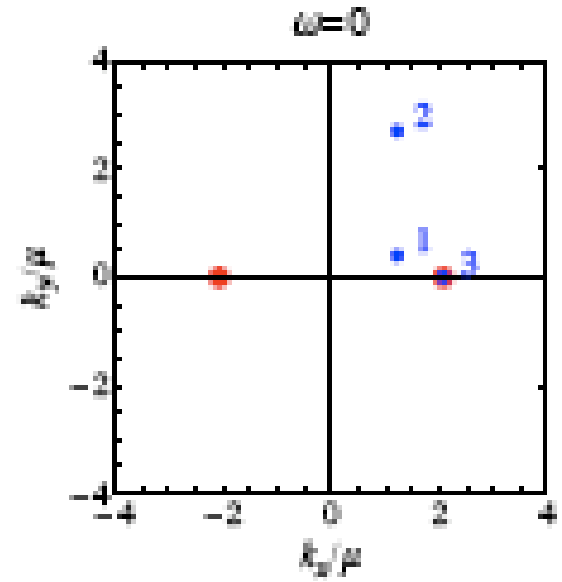


Superfluid phase: Reduced Fermi surface



(3+1)dim gravity & SU(2) field (fundamental fermions)

[Gubser et al. 1002.4416]

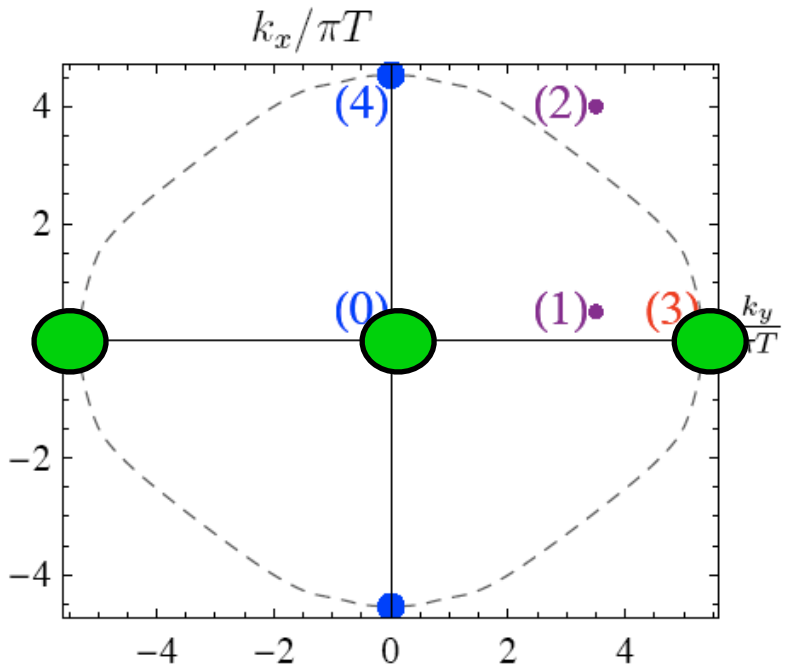


T=0: solve bulk Dirac equation exactly.

Green's function has poles at roots of the fermion representation.

$$k_x = qA_x^1(u_H)$$

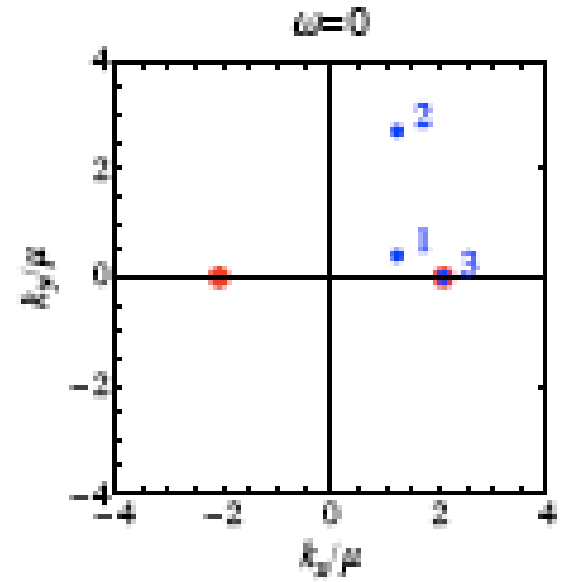
Superfluid phase: Reduced Fermi surface



at T=0 the Fermi surface apparently reduces to 3 points (adjoint fermions)

(3+1)dim gravity & SU(2) field (fundamental fermions)

[Gubser et al. 1002.4416]



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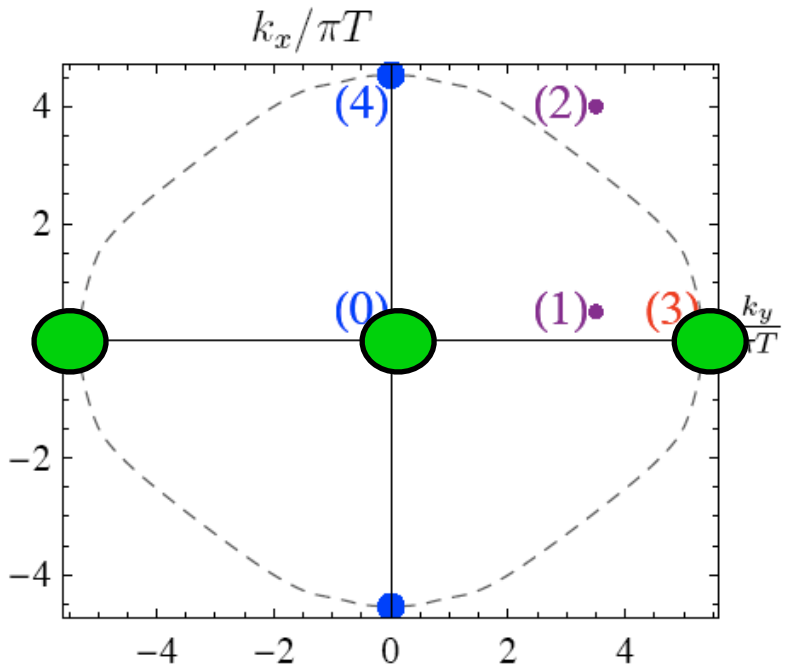
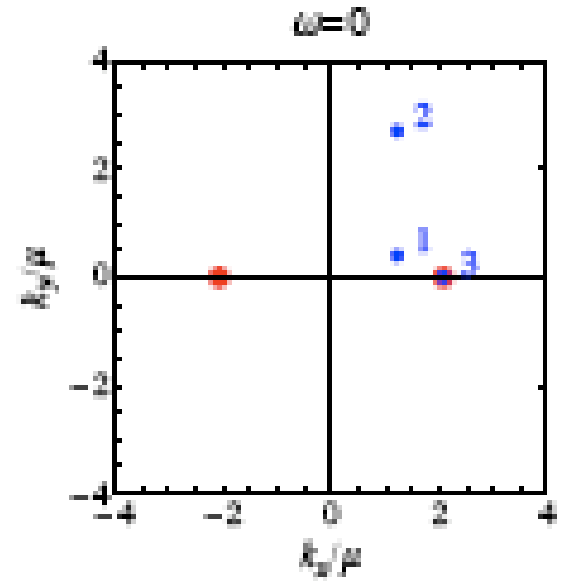
Superfluid phase: Reduced Fermi surface



at T=0 the Fermi surface apparently reduces to 3 points (adjoint fermions)

(3+1)dim gravity & SU(2) field (fundamental fermions)

[Gubser et al. 1002.4416]

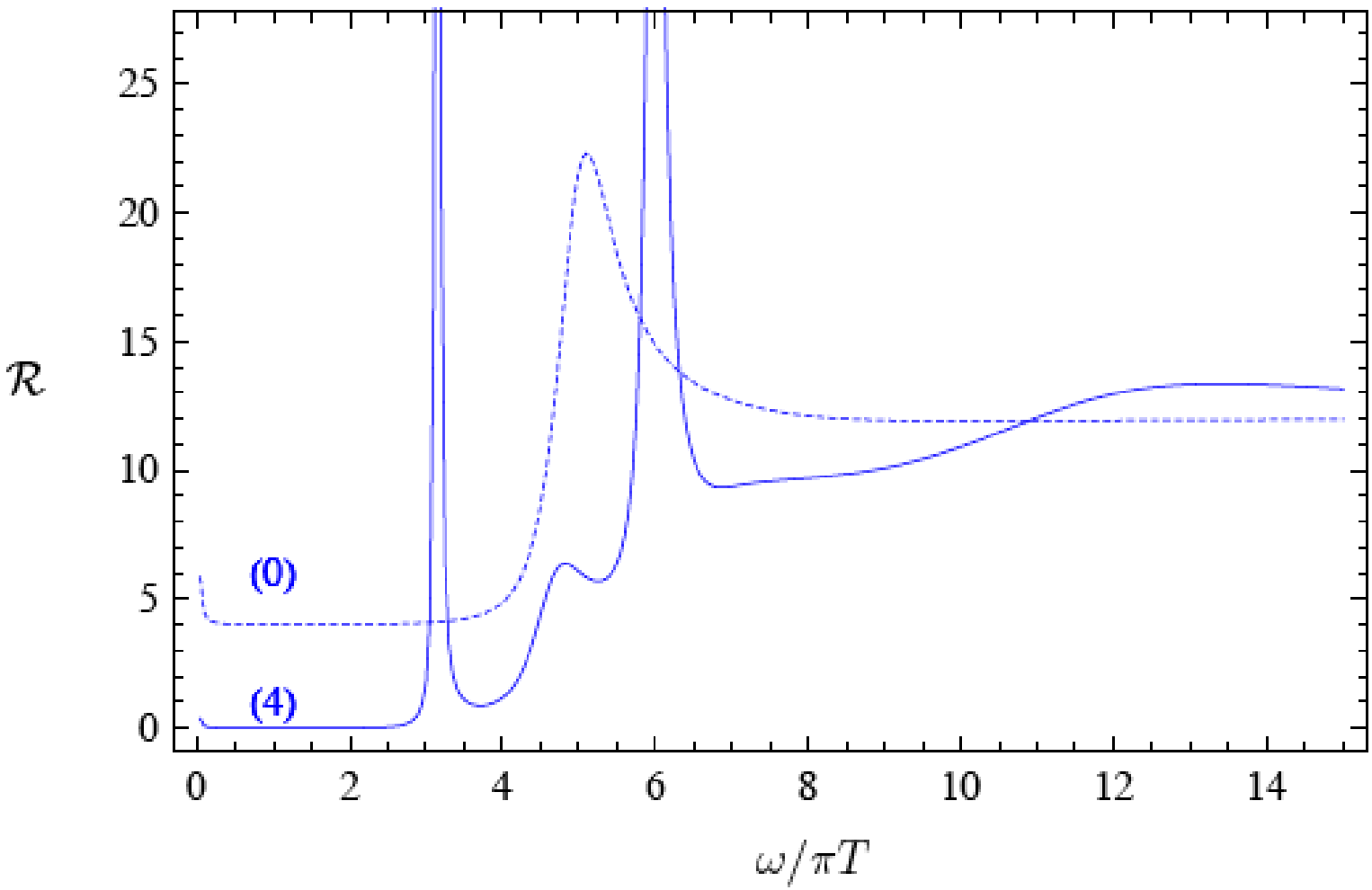


T=0: solve bulk Dirac equation exactly.

Green's function has poles at roots of the fermion representation.

$$k_x = qA_x^1(u_H)$$

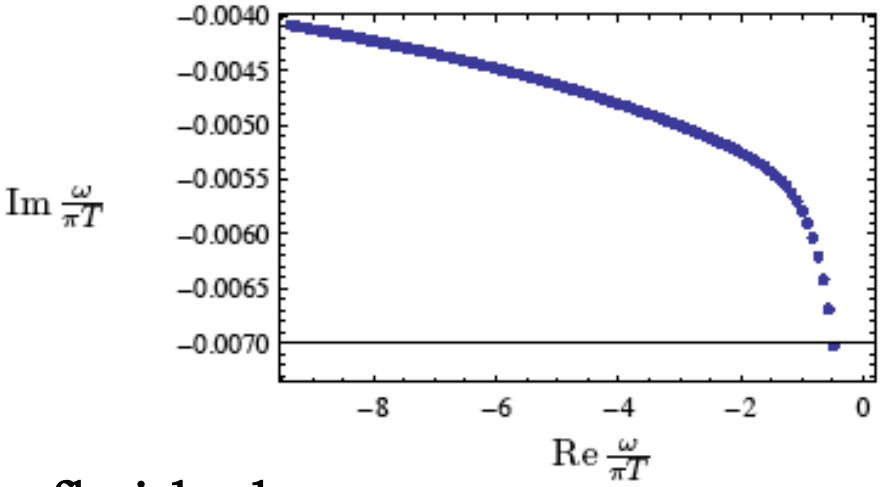
Superfluid phase: Spectrum



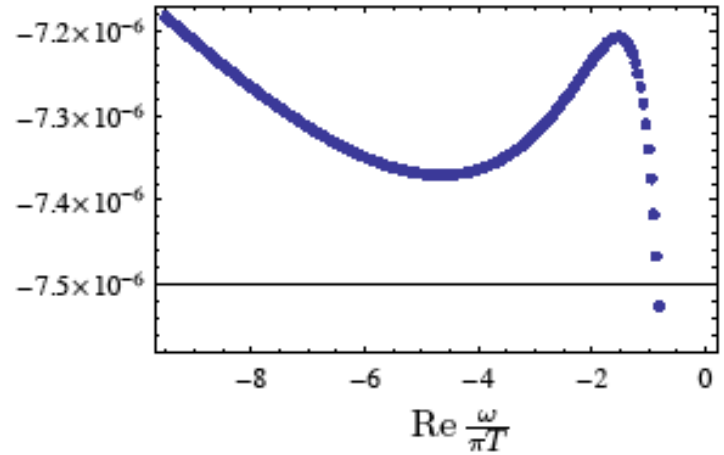
Low frequency poles in G (QNMs)

$q = -1$

wrong phase: $0.9T_c$ $0.4T_c$

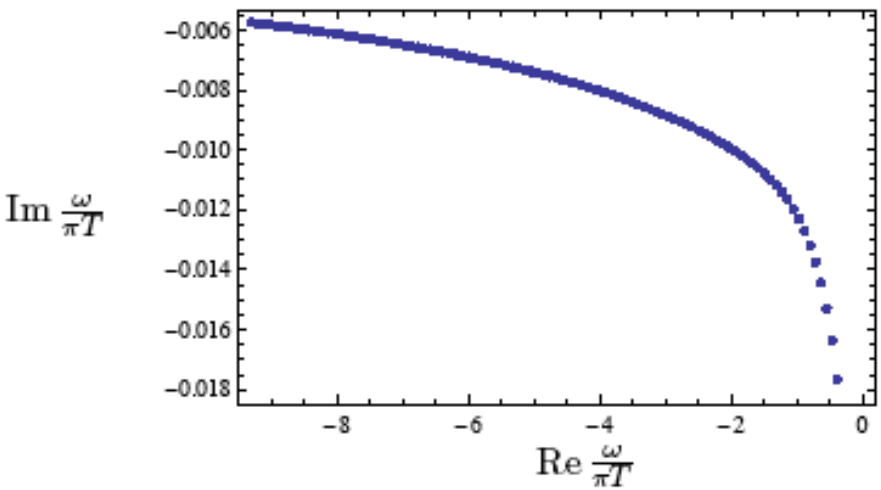


(a)

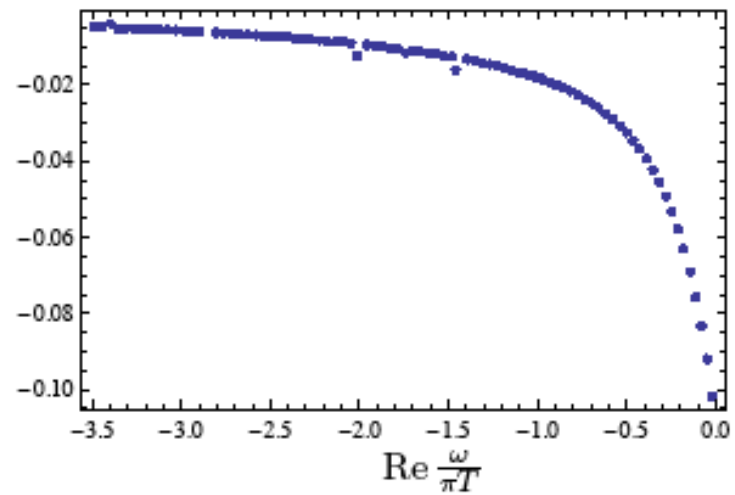


(b)

superfluid phase:

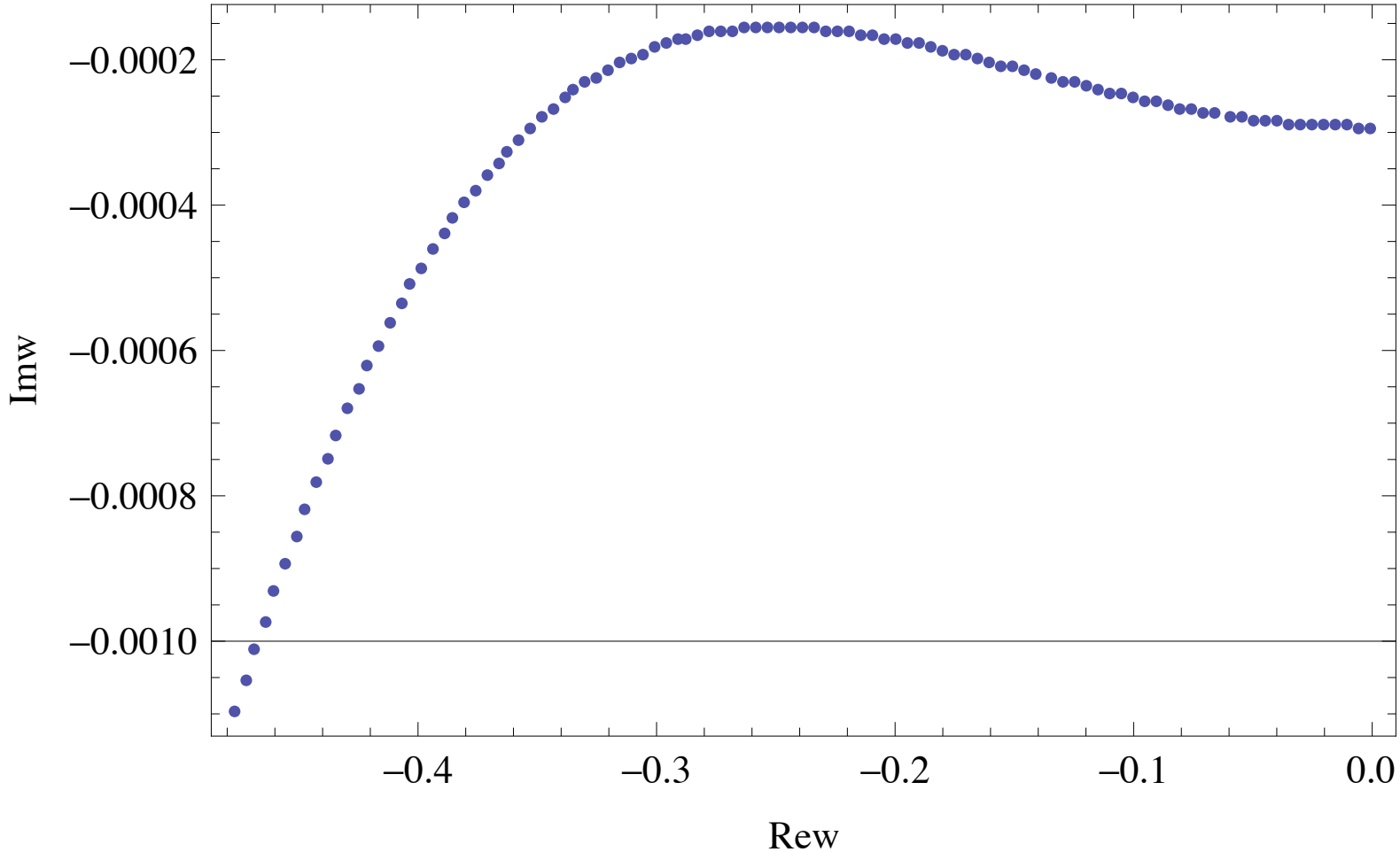


(c)



(d)

QNM at vanishing momentum



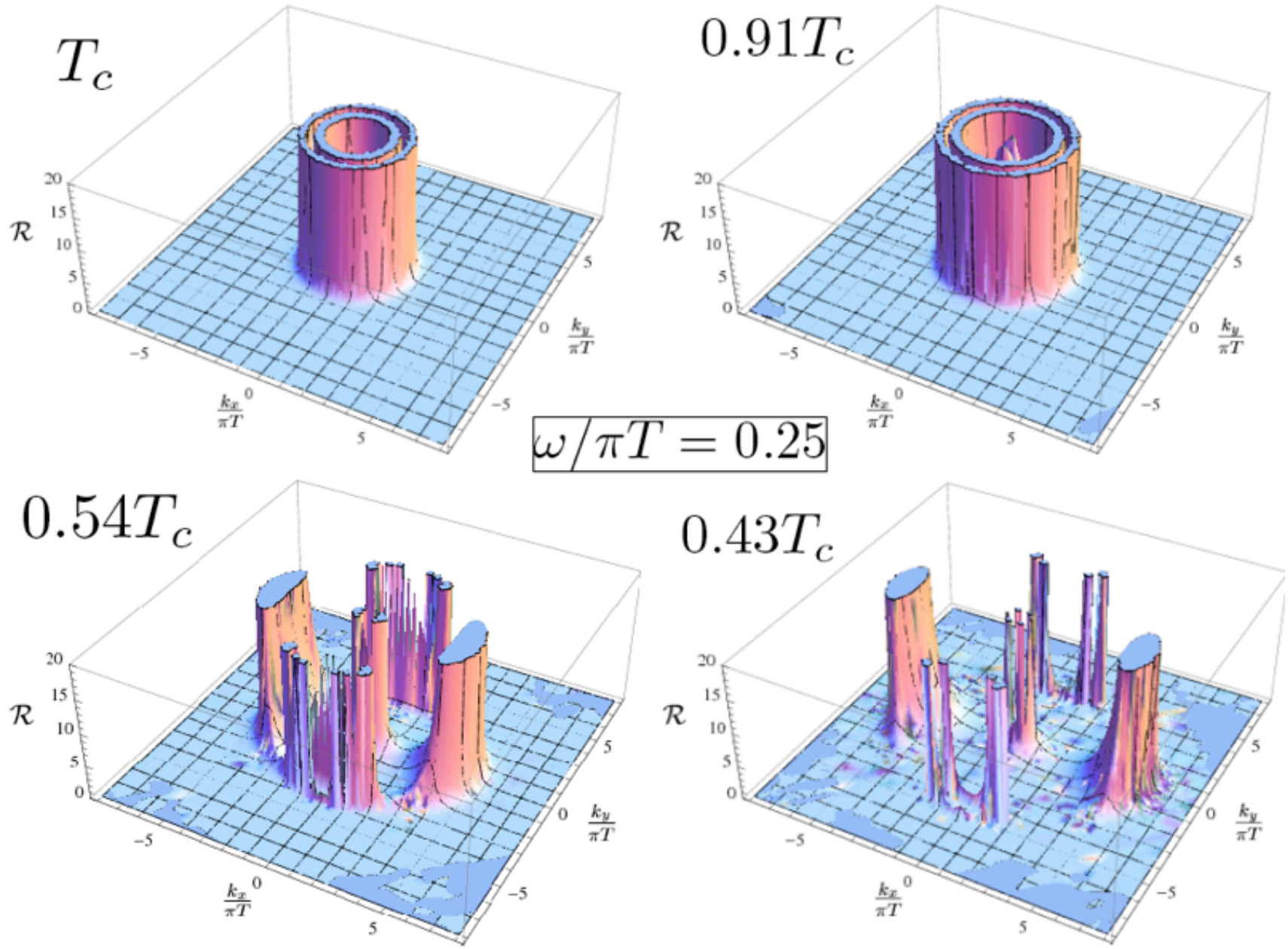
preliminary analysis suggests: $z = 1,\ \alpha = ?$

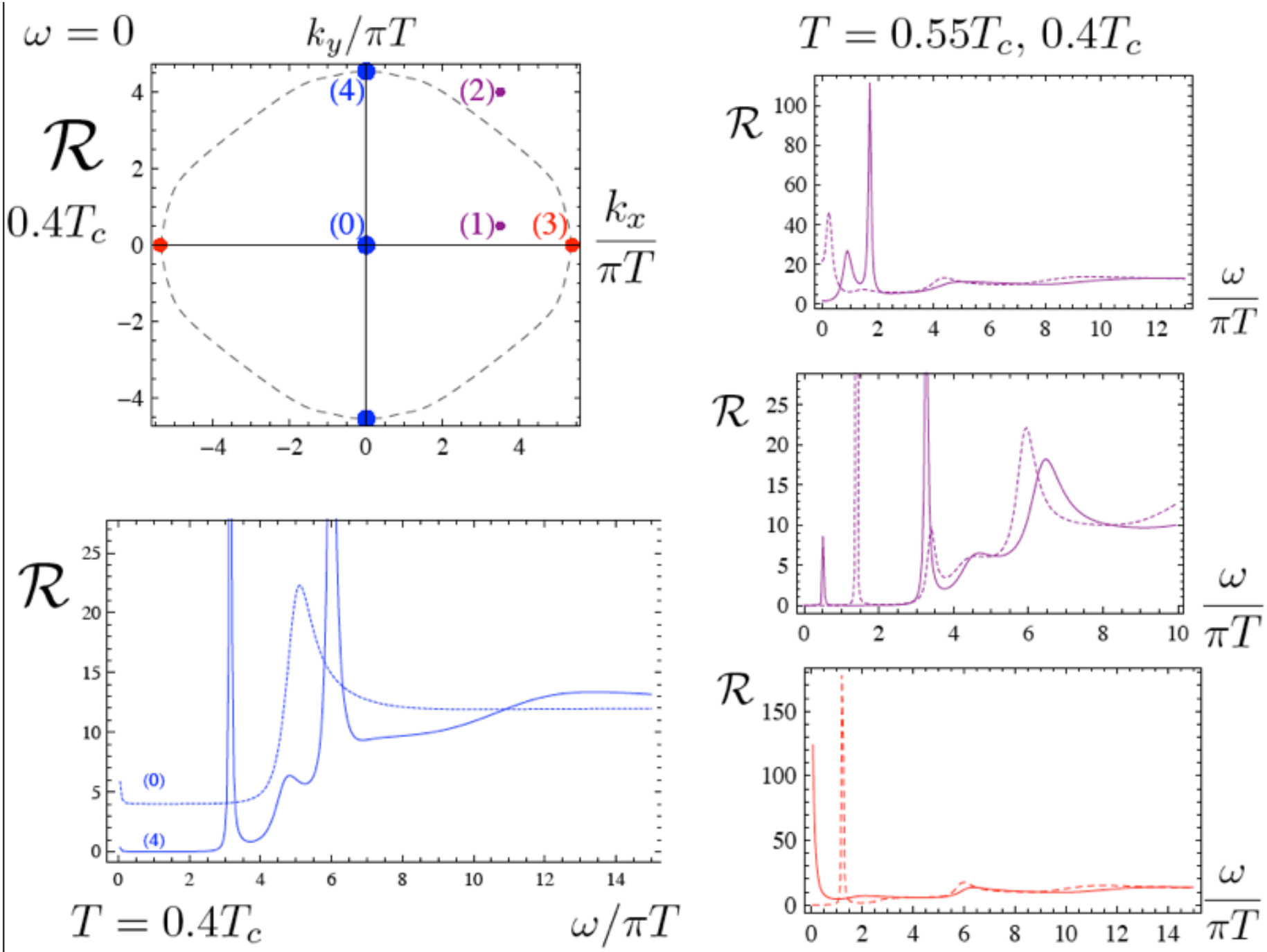
IV. Discussion

- ✓ generic dual to (non)Fermi liquids fixing parameters & considering condensation
- ✓ Fermi surface reduces to patches at low T
- ✓ left out: holographic renormalization with fermions, correlation functions for mixing operators

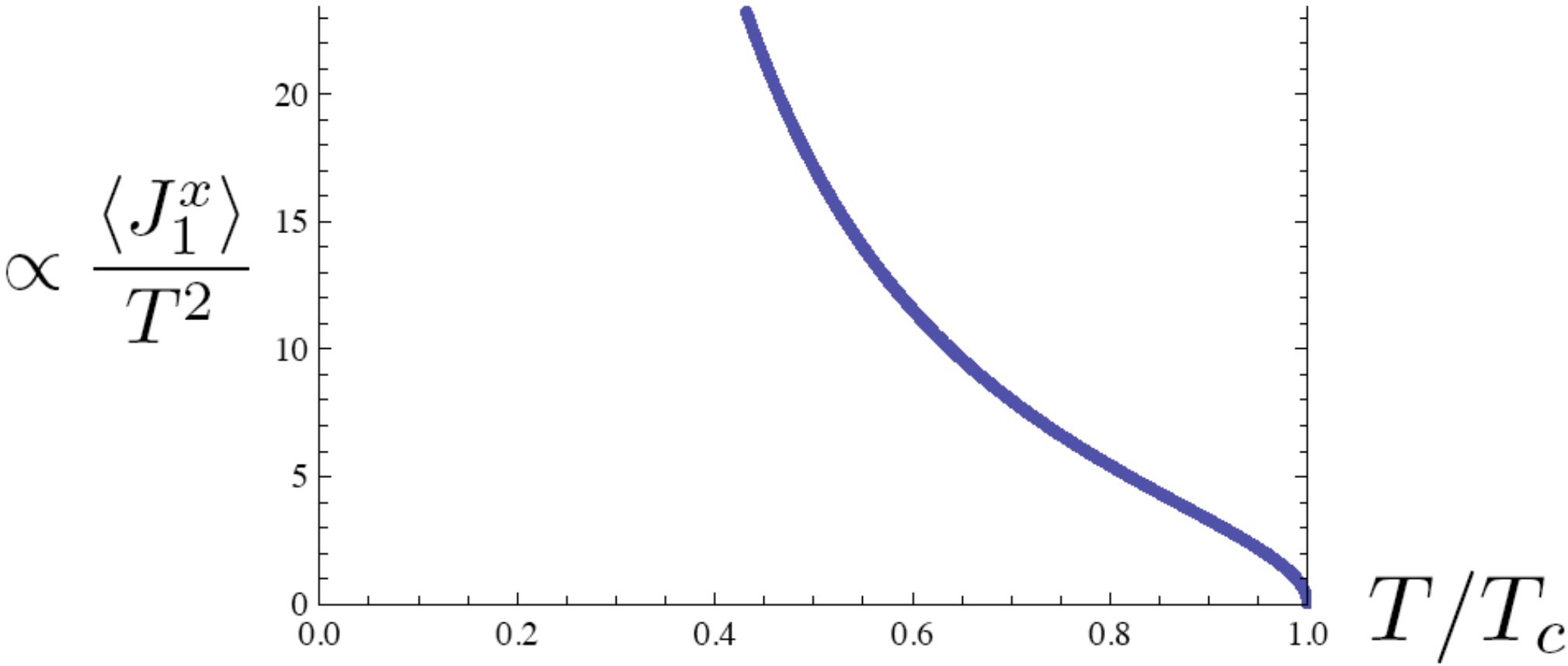
- ☐ critical exponents, (non) Fermi liquid?
- ☐ backreaction of fermions on metric
- ☐ zero T: backreaction of flavor branes on metric (speed of sound)
- ☐ massive fermions

APPENDIX: Spectral measure




$\omega = 0$
$k_y/\pi T$
$\mathcal{R}$
$0.4T_c$
(4)
(2)
(0)
(1)
(3)
$k_x/\pi T$
$T = 0.55T_c,\ 0.4T_c$
$\mathcal{R}$
$\omega/\pi T$
$\mathcal{R}$
$\omega/\pi T$
$\mathcal{R}$
$\omega/\pi T$
$\mathcal{R}$
(0)
(4)
$T = 0.4T_c$
$\omega/\pi T$

APPENDIX: Limits of Probe Limit



APPENDIX: Determinant Method

Quasinormal modes of coupled systems

$$a_t = \mathfrak{b}_1 a_t{}^I + \mathfrak{b}_2 a_t{}^{II}$$

$$a_x = \mathfrak{b}_1 a_x{}^I + \mathfrak{b}_2 a_x{}^{II}$$

where I and II are distinct sets of boundary conditions.

QNM condition at AdS boundary

$$\lim_{r\to\infty} a_t = 0 = \lim_{r\to\infty} a_x$$

Normalize and plug in for coeff

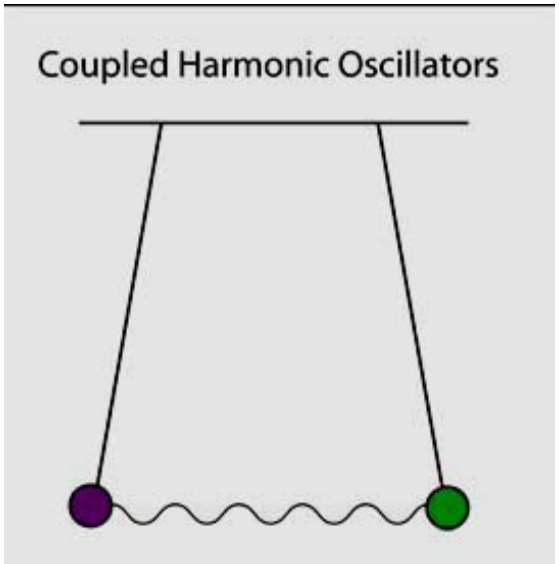
$$0 = -\frac{a_t{}^{II}}{a_t{}^I} a_x{}^I + a_x{}^{II}\Big|_{r\to\infty}$$

$$0 = det \begin{pmatrix} a_t{}^I & a_t{}^{II} \\ a_x{}^I & a_x{}^{II} \end{pmatrix}_{r\to\infty}$$

Example: $a_t{}^{II} = -\omega\alpha\,, \quad a_x{}^{II} = k\alpha$

$$0 = \omega\alpha\, a_x{}^I + k\alpha\, a_t{}^I \equiv \alpha E_x$$

Analog



Coupling an oscillator with eigenfrequency ω_1 to another with frequency ω_2, there are two new eigenmodes of the coupled system, not assigned to only one of the oscillators.

APPENDIX: Determinant Method (2)

General formalism switching on specific operators

$$\begin{pmatrix} c_1^1 & \dots & c_n^1 \\ & \dots & \\ c_1^n & \dots & c_n^n \end{pmatrix} \cdot \underbrace{\begin{pmatrix} {\phi_1}^I & \dots & {\phi_1}^{I_n} \\ & \dots & \\ {\phi_n}^I & \dots & {\phi_n}^{I_n} \end{pmatrix}}_{\mathfrak{F}}{}_{r\to\infty} = \mathrm{diagonal}(1,\dots,1) \quad \Rightarrow$$

retarded Green function

Set right hand side to zero:

General QNM condition

$$det\,\mathfrak{F}_{r\to\infty} = 0$$

QNM corresponding to poles in holographic Green functions are zeroes of the determinant of field values at the AdS boundary for a maximal set of linearly independent solutions (infalling at horizon).

Advantages

- no explicit trafo to gauge-invariant fields needed
- elegant formulation including all fields simultaneously
- works for any coupled system of gravitational fluctuations
- need not compute full Green function (but possible)
- coupled EOMs along r dual to operator mixing along RG-flow, cutoff!

APPENDIX: Holographic Operator Mixing

$$\Psi = \begin{pmatrix} 0 \\ \Psi_{+u} \\ 0 \\ \Psi_{+d} \end{pmatrix} + \begin{pmatrix} \Psi_{-u} \\ 0 \\ \Psi_{-d} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \Psi_{1u} \\ \Psi_{1d} \end{pmatrix} + \begin{pmatrix} \Psi_{2u} \\ \Psi_{2d} \\ 0 \\ 0 \end{pmatrix}$$

$$\gamma^u \Psi_\pm = \pm \Psi_\pm$$

$$\Psi_\pm = c_\pm(k)\, u^{\frac{d}{2}\pm m} + O\left(u^{\frac{d}{2}+1\pm m}\right)$$

$$S_{ren} = \int d^d x\, \bar{c}_+\, c_-,$$

$$\langle \mathcal{O}\, \bar{\mathcal{O}} \rangle_{ren} = -\frac{\delta^2 S_{ren}}{\delta c_- \delta \bar{c}_-} = -\frac{\delta c_+}{\delta c_-}.$$

$$\left[\tilde{P}^-_{aj}(u,\omega,k)\right] = \begin{pmatrix} \Psi^{(1)}_{1-} & \Psi^{(2)}_{1-} & \dots & \Psi^{(2N)}_{1-} \\ \Psi^{(1)}_{2-} & \Psi^{(2)}_{2-} & \dots & \Psi^{(2N)}_{2-} \\ \dots & \dots & & \dots \\ \Psi^{(1)}_{N-} & \Psi^{(2)}_{N-} & \dots & \Psi^{(2N)}_{N-} \end{pmatrix},$$

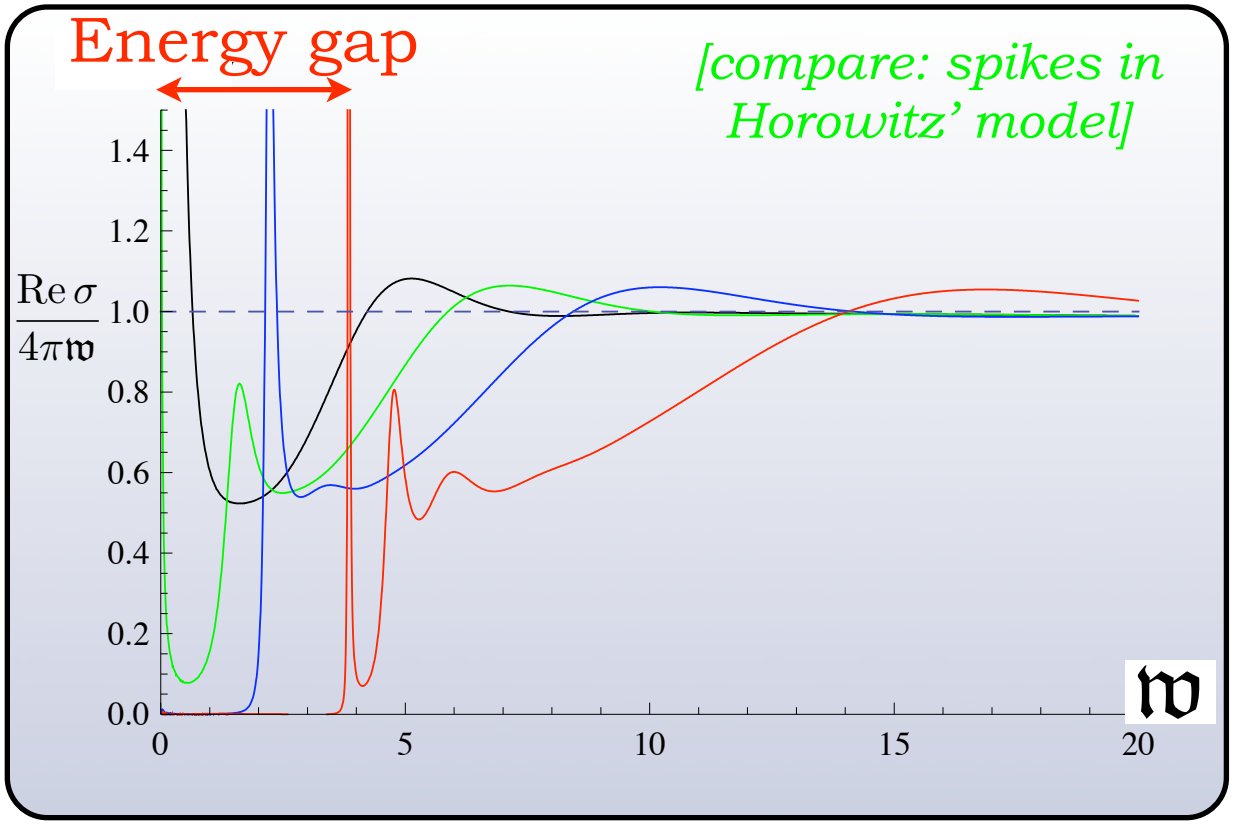
$$G^R_{ab}(\omega,k) = \lim_{\epsilon\to 0}\left(P^+(\epsilon)_{aj}\, P^-(\epsilon)^{-1}_{jb}\right)$$

APPENDIX: Conductivity

Conductivity:

$$\sigma = \frac{J}{E} = \frac{A^{(2)}}{\partial_t A^{(0)}} \sim \frac{i A^{(2)}}{\omega A^{(0)}} = -\frac{i}{\omega}\frac{\rho^3 A'}{2\,A} = \frac{i}{\omega} G^{\rm ret}(\omega, \mathbf{q}=0)$$

with flavorelectric current $J_m \longleftrightarrow A_m \in U(1)$



Only the first of these peaks was seen to second order in F.

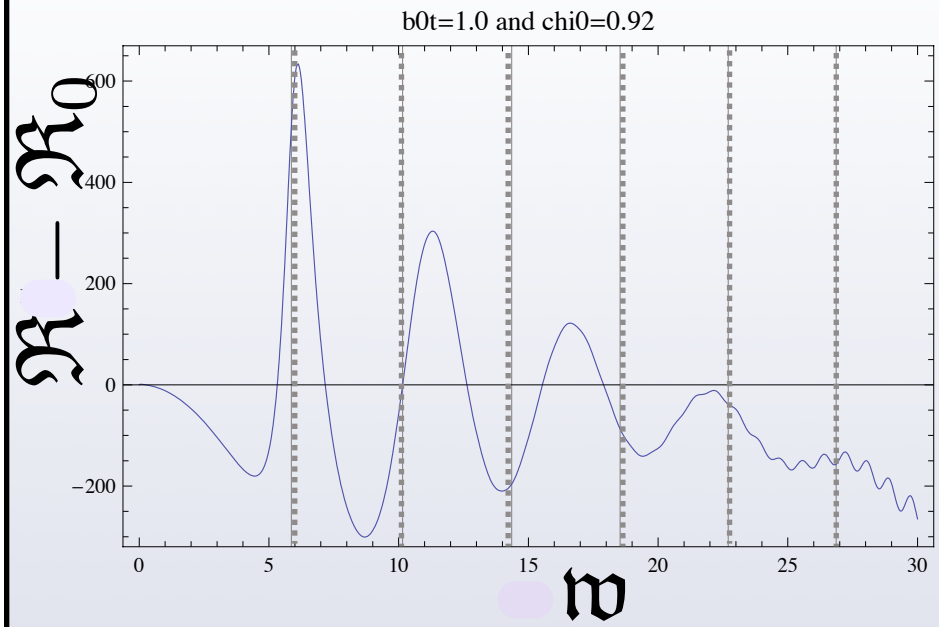
[Basu et al. 0810.3970

Our expansion to fourth order shows all peaks (zero quark m).

Peaks are higher order effect in F .

APPENDIX: Higgs mechanism & Meissner effect

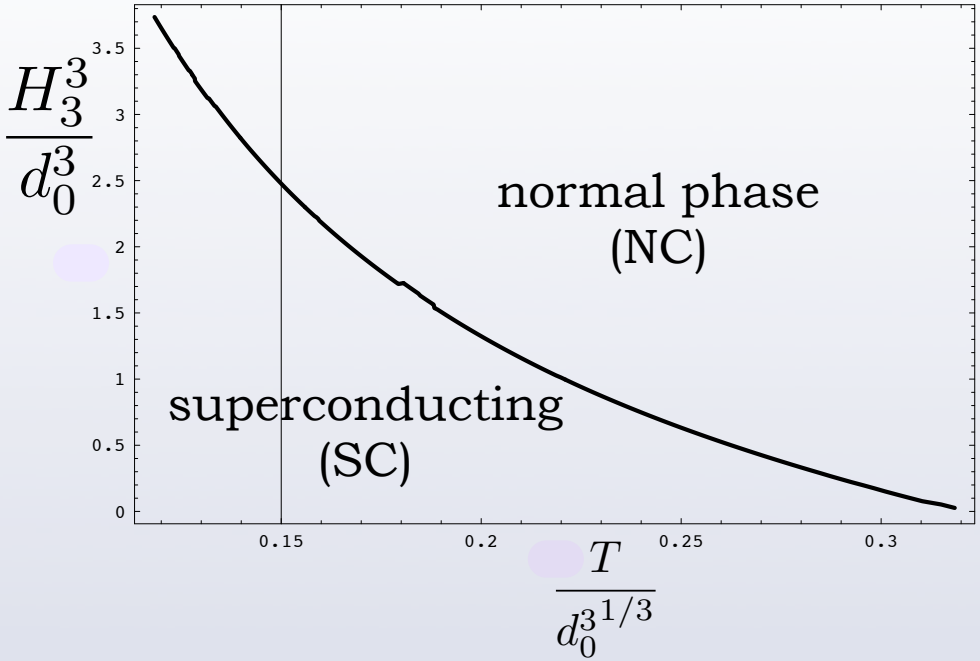
Peaks at finite mass:



Peaks in conductivity/ spectral function approach SUSY vector meson spectrum at large quark mass.

Bulk Higgs Mechanism

Finite magnetic field:



Add background field component: $H_3^3 = F_{12}^3 = \partial_1 A_2^3$

Induced currents in SC phase with H