AdS/CFT and thermal properties of strongly coupled systems

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Motivation by results

➤ still only strongly coupled Yang-Mills with $N \to \infty$

➤ realistic properties: chiral symmetry breaking, SUSY breaking, confinement, . . .

➤ flavor effects: meson-mass spectra, B-meson mass

➤ non-equilibrium: transport coefficients (universal $\eta/s$, $\kappa_T$, . . .) [Policastro et al., hep-th/0205052]

➤ thermal spectral functions [Kovtun, Starinets, hep-th/0602059]

➤ thermal phase transitions [Babington et al, hep-th/0306018]

➤ photon/dilepton production in plasma [Huot et al., hep-th/0607237]
1. Overview

2. Introduction to concepts of AdS/CFT with flavor

3. Apply AdS-methods to find diffusion poles (Examples)

4. Understanding current results of AdS/CFT at finite temperature

5. Summary and Outlook
2. Introduction to methods of the AdS/CFT correspondence

How (not why) do features on the *gravity-side* affect features on the *gauge-side*?
2.1 – Basic AdS/CFT correspondence

Super Yang-Mills theory in Minkowski space $\leftrightarrow$ supergravity in $AdS_5 \times S^5$

stack of $N$ D3-branes in 10d

duality

open strings ending on D3-branes $\leftrightarrow$ closed strings near the throat-region

$AdS_5 \times S^5$

near-horizon geometry
What is AdS?
What is AdS?

- Anti-de Sitter space
- the 5d space with constant negative curvature
- \( AdS_5 \times S^5 \):
  - \( S^5 \) is the five-sphere
- \( AdS_5 \) has 4d boundary (Minkowski space)
What is CFT?
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- Conformal Field Theory
- invariant under rescaling of metric
What is CFT?

- Conformal Field Theory
- invariant under rescaling of metric
- Yang-Mills theory in 4d
- gauge group $SU(N)$
- supersymmetric
- quantum field theory
- lives on the 4d worldvolume of D3-branes
What is supergravity?
What is supergravity?
What is supergravity?

- gravity-side of correspondence
- *classical* theory in 10 dimensions
- low-energy limit of string theory
- supersymmetric
- contains graviton as a *classical* field
- calculations involve *classical* actions, equations of motion, …
- contains *branes as solitonic solutions* to eom

[Exocet, Demotopia]
Holographic dictionary of the AdS/CFT correspondence

operators $\mathcal{O}$ in Super Yang-Mills theory (Minkowski) $\leftrightarrow$ supergravity fields $A$ (AdS)

AdS$_5 \times S^5$

near-horizon geometry
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$\Rightarrow$ correlators of operators $\langle [\mathcal{O}, \mathcal{O}] \rangle$ $\sim$ correlators of the sugra-fields $\langle [A, A] \rangle$

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$\Rightarrow$ conformal symmetries $\leftrightarrow$ AdS-isometries

$\Rightarrow$ $SU(4)$: R-symmetry $\leftrightarrow S^5$ symmetry

AdS$_5 \times S^5$

near-horizon geometry
2.2 – AdS/CFT with flavor

Adding probe D7-branes [Graña, Polchinski 2001]  [Karch, Katz 2002]

\[ \text{limit: } N \rightarrow \infty \text{ (standard Maldacena limit), } N_f \text{ small (quenched approximation)} \]

Duality acts twice!
2.2 – AdS/CFT with flavor

Adding probe D7-branes [Graña, Polchinski 2001] [Karch, Katz 2002]

\[ \text{limit: } N \rightarrow \infty \text{ (standard Maldacena limit), } N_f \text{ small (quenched approximation)} \]

Duality acts twice!

\[
\begin{align*}
4d \mathcal{N} = 4 \text{ SU}(N) \text{ Super Yang-Mills theory} & \quad \text{coupled to} \quad 4d \mathcal{N} = 2 \text{ fundamental hypermultiplet} \\
\text{type IIB SUGRA on } AdS_5 \times S^5 & \quad + \quad \text{Dirac-Born-Infeld theory on } AdS_5 \times S^3
\end{align*}
\]
Pure AdS-background (this means *space-time metric*):
2.3 – Deformed backgrounds

Pure AdS-background (this means *space-time metric*):

Deformed *gravity background changes dual field theory*:

- e.g. all SUSY broken, dual field theory at *finite temperature*
2.4 – Minkowski correlators from AdS/CFT

coord. names

\begin{array}{cccc}
  x^0 & x^1 & x^2 & x^3 & u \\
\end{array}

\begin{array}{c}
  \mu, \nu \ldots \\
\end{array}

\begin{array}{c}
  i, j \ldots \\
\end{array}

\begin{array}{c}
  \alpha \\
\end{array}

\begin{array}{c}
  \text{indices} \\
\end{array}

\begin{array}{c}
  \text{AdS}_5 \quad S^3 \\
\end{array}

[Son, Starinets, hep-th/0205051]

\[ S_{\text{cl}} = \int dud^4x B(u)(\partial_u A)^2 \]  \hfill (1)

Step 1: Extract the factor \( B(u) \).

Step 2: Find solution \( f(u, \vec{k}) \) to mode equation of motion

with boundary conditions at \( u = 0 \)

\[ A(u, \vec{k}) = f(u, \vec{k}) A^{\text{boundary}}(\vec{k}) \]  \hfill (2)

and incoming wave b.c. at horizon \( u = 1 \).

Step 3: Find the retarded correlator as

\[ G^R(\vec{k}) = -2B(u)f(u, -\vec{k})\partial_u f(u, \vec{k}) \bigg|_{u \to 0} \]  \hfill (3)
Summary of concepts:

- AdS/CFT is a holographic duality
- AdS/CFT relates operators from field theory to supergravity-fields
- Flavor can be added by adding D7-branes
- Realistic QFT features by deforming supergravity background
- QFT-correlators at strong coupling can be computed by dual calculation in weakly coupled supergravity
1. Overview

2. Introduction to concepts of AdS/CFT with flavor
   ➤ General setup (branes, supergravity, . . . ) and adding flavor
   ➤ Deformed backgrounds add realistic features
   ➤ Correlation functions

3. Apply AdS-methods to find diffusion poles (Examples)

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3. Apply AdS-methods to find diffusion poles

What can we calculate explicitly?
3.1 – Example #1: Super-Maxwell theory

[Policastro, Son, Starinets, ’02]

Task: Calculate the R-charge diffusion constant (→ correlator!)

Background:

\[ ds^2 = \frac{b^2 R^2}{u} (-f(u) \, dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{R^2}{4u^2 f(u)} \, du^2 + R^2 d\Omega_5^2, \]

\[ 0 \leq u \leq 1, \quad x_i \in \mathbb{R}, \]

with

\[ f(u) = 1 - u^2, \quad R^4 = 4\pi g_s N_c \alpha'^2, \quad b = \pi T. \]
3.1 – Example #1: Super-Maxwell theory

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\]

\[0 \leq u \leq 1, \quad x_i \in \mathbb{R},\]

with

\[f(u) = 1 - u^2, \quad R^4 = 4\pi g_s N_c \alpha'{}^2, \quad b = \pi T.\]

Classical action (AdS side):

\[
S_{\text{Super–Maxwell}} = -\frac{N^2}{16\pi^2} \int du d^4 x \sqrt{-g(u)} F_{\mu\nu} F^{\mu\nu}
\]

Now fix the gauge \(A_u \equiv 0\), choose frame \(\vec{k} = (\omega, 0, 0, q)\) and use Fourier transform of \(A_\mu\).
Applying the recipe for correlators

Step 1: Factor

\[ B(u) = -\frac{N^2}{16\pi^2} \sqrt{-g(u)} g^{\mu\mu'} g^{\nu\nu'} \]  

(7)
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(7)

Step 2: Solve equations of motion (5 Maxwell equations in 5d position-space)

\[ \frac{1}{\sqrt{-g}} \partial_\nu [ \sqrt{-g} g^{\mu\rho} g^{\nu\sigma} (\partial_\rho A_\sigma - \partial_\sigma A_\rho) ] = 0 , \]  

(8)

➡ Fourier transform fields

➡ Use dimensionless \( \omega = \omega/(2\pi T) \), \( q = q/(2\pi T) \)
Applying the recipe for correlators

Step 1: Factor

\[ B(u) = - \frac{N^2}{16\pi^2} \sqrt{-g(u)} g^{\mu\nu} g^{\nu'\nu'} \]  \hspace{1cm} (7)

Step 2: Solve equations of motion (5 Maxwell equations in 5d position-space)

\[ \frac{1}{\sqrt{-g}} \partial_v [\sqrt{-g} g^{\mu\rho} g^{\nu\sigma} (\partial_\rho A_\sigma - \partial_\sigma A_\rho)] = 0 , \]  \hspace{1cm} (8)

⇒ Fourier transform fields

⇒ Use dimensionless \( \omega = \omega / (2\pi T), q = q / (2\pi T) \)

Small frequencies, long wavelengths justify e.g.

\[ A'_t(u) = (1 - u) \left( \frac{-i\omega}{2} q^2 A^\text{boundary}_t + i\omega q A^\text{boundary}_z \right) \frac{i\omega - q^2}{i\omega - q^2} \times \left[ 1 + \frac{i\omega}{2} \ln \frac{2u^2}{1 + u} + q^2 \ln \frac{1 + u}{2u} + O(\omega^2, q^4, \omega q^2) \right] \]  \hspace{1cm} (9)

3. Apply AdS-methods to find diffusion poles

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**Correlator**

**Step 3: Formula**

\[
G_R^R(\vec{k}) = -2B(u)f(u, -\vec{k})f'(u, \vec{k}) \bigg|_{u \to 0} \tag{10}
\]

identify \(A(u, \vec{k}) = f(u, \vec{k})A^\text{boundary}(\vec{k})\).

Time components correlator

\[
G_{tt}^{ab} = \frac{N^2 T q^2}{16 \pi (i\omega - Dq^2)} + \cdots \tag{11}
\]

exhibits a pole with

\[
D = \frac{1}{2\pi T} \tag{12}
\]
Correlator

Step 3: Formula

\[ G^R(\vec{k}) = -2B(u) f(u, -\vec{k}) f'(u, \vec{k}) \bigg|_{u \to 0} \]  \hspace{1cm} (10)

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\[ D = \frac{1}{2\pi T} \] \hspace{1cm} (12)

Compare to the diffusion equation in position space

\[ \partial_t \rho + D \nabla^2 \rho = 0, \] \hspace{1cm} (13)

and in momentum space

\[ (i\omega - D q^2) \rho = 0. \] \hspace{1cm} (14)

3. Apply AdS-methods to find diffusion poles  

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**Interpretation of result**

*Question*: What exactly have we computed?

*Answer*: The correlator of Super-Maxwell gauge fields $A$ (a 5d generalization of the electromagnetic vector potential). This is dual to the retarded $R$-current correlator

$$G^R = -i \int d^4x e^{-i\vec{k}\cdot\vec{x}} \Theta(t) \langle [J(\vec{x}), J(0)] \rangle$$  \hspace{1cm} (15)

$\Rightarrow$ $J$ is the $R$-current (analog to ’electromagnetic’ current) of the Abelian Super Yang-Mills QFT.

$\Rightarrow$ The $R$-charge $Q$ coupling to this current undergoes a diffusion process with $D = 1/(2\pi T)$. 

3. Apply AdS-methods to find diffusion poles
3.2 – Example #2: Isospin flavor theory

[Erdmenger, Kaminski, Rust ’02]

**Task:** Compute the diffusion constant of non-Abelian isospin charge!

**Background:** AdS black hole as before.

Classical supergravity action $\rightarrow$ effective Dirac-Born-Infeld action on D7-brane.

**What’s new?**

- Non-Abelian flavor indices
- Field strength $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}^{a} + f^{abc}A_{\mu}^{b}A_{\nu}^{c}$
- Introduce $SU(2)$-chemical potential $\mu$ as background
- Consider fluctuations $A_{\nu}$

$N_{f} = 2$:

- $3 \times 5$ component equations of motion,
- coupled through flavor indices and
- coupled through Lorentz indices

3. Apply AdS-methods to find diffusion poles

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Solution

Decouple equations by trafo

\[ X_i = A_i^1 + iA_i^2, \quad \tilde{X}_i = A_i^1 - iA_i^2. \]  \hspace{1cm} (16)

Performing **three steps** as before gives

\[ G_{\tilde{t}\tilde{t}} = -\frac{N_c T}{8\pi \sqrt{2\mu}} \frac{q^2 \sqrt{\omega}}{\omega + q^2 D(\omega)}, \]  \hspace{1cm} (17)

\[ G_{\tilde{t}t} = \frac{N_c T}{8\pi \sqrt{2\mu}} \frac{q^2 \sqrt{\omega}}{i\omega - q^2 D(\omega)}, \]  \hspace{1cm} (18)

where \( D(\omega) \) is given by

\[ D(\omega) = \sqrt{\frac{\omega}{2\mu}} \frac{1}{2\pi T}. \]  \hspace{1cm} (19)

3. Apply AdS-methods to find diffusion poles
Interpretation of result

Question: What exactly have we computed?

Answer: The correlator of Isospin gauge fields $A$ (a generalization of the electromagnetic vector potential). This is dual to the retarded isospin current-current correlator

$$ G^R = -i \int d^4x e^{-i \vec{k} \cdot \vec{x}} \Theta(t) \langle [J(\vec{x}), J(0)] \rangle $$  \hspace{1cm} (20)

$\Rightarrow$ $J$ is essentially the isospin charge current of the Abelian Super Yang-Mills QFT.

$\Rightarrow$ The charge $Q$ coupling to this current undergoes a diffusion process with diffusion coefficient

$$ D = \sqrt{\frac{\omega}{2\mu}} \frac{1}{2\pi T}. \hspace{1cm} (21) $$
Summary of examples (diffusion poles):

- Two examples for general method finding correlators in supergravity
- Dual QFTs have hydrodynamic feature (diffusion pole in correlators)
- #1 is Super Maxwell theory with an ’Abelian’ charge undergoing diffusion with $D = \frac{1}{(2\pi T)}$
- #2 is non-Abelian isospin flavor ($N_f = 2$) on D7-brane with dual isospin current and charge undergoing diffusion with $D = \sqrt{\frac{\omega}{2\mu}} \frac{1}{2\pi T}$.
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What can we learn about thermal field theory at strong coupling?
4.1 – Universal viscosity bound

Kubo-formula relates viscosity $\eta$ to energy-momentum $T_{i,j}$ correlator

$$\eta = \frac{\beta}{5} \int d^3 x' \int_{-\infty}^{t} dt' e^{(t'-t)} \langle T_{\alpha\beta}(t, \vec{x}) , T_{\alpha\beta}(t', \vec{x}') \rangle_{\text{retarded}}, \quad (22)$$

with $\alpha, \beta = 1, 2$.

Dual (source) field is graviton $h_{\mu\nu}$

$$\langle T_{ij}(t, \vec{x}) , T_{mn}(t', \vec{x}') \rangle_{\text{retarded}} \sim \langle h_{ij}(t, \vec{x}) , h_{mn}(t', \vec{x}') \rangle_{\text{retarded}}. \quad (23)$$

*Picture*: graviton 'scattered' on black hole

$\rightarrow$ The only universal bound from AdS/CFT

$$\frac{\eta}{s} = \frac{1}{4\pi} \quad (24)$$

For all thermal theories with a gravity dual.
Case study of theories with universal bound reviewed in [Son, Starinets, 0704.0240]

Common features:

➡️ the thermal field theory has a gravity dual
➡️ Supergravity background is asymptotically AdS

Distinct features:

➡️ supersymmetry
➡️ conformal symmetry
➡️ flavor/ fundamental matter
➡️ nonzero chemical potential
Hydrodynamic poles and (quasi)normal modes

Alternatively, examine poles of $T_{ij}$-correlator (hydro limit)

$$\langle T_{ij}, T_{mn} \rangle \propto \frac{1}{i\omega - Dq^2}$$

with diffusion constant

$$D = \frac{\eta}{\epsilon + P} = \frac{1}{4\pi T}.$$  \hspace{1cm} (26)

Generalization beyond hydrodynamics (to arbitrary wave-vector $\vec{k}$):

- Supergravity correlators have infinitely many poles
- Solutions in flat spacetime: normal modes ($\omega$ real)
- Solutions in curved spacetime: quasinormal modes ($\omega$ complex → dissipative mode)
- quasinormal modes $\leftrightarrow$ poles of QFT-correlators
- hydrodynamic poles are identical to the lowest quasinormal mode frequencies
- what are the higher ones?
4.2 – AdS-view on a thermal phase transition

Consider $\mathcal{N} = 4 \ SU(N)$ SYM at finite temperature [Witten, hep-th/9803131]

- Dual string theory background is Euclidean AdS-Schwarzschild metric.
- add one D7-brane
- D7 is dynamical object with equations of motion
- $x_8 \equiv 0$, $x_9 = x_9(x_4)$, $x_4$ radial AdS coordinate

\[
\begin{array}{c|cccccccccc}
& x_0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \\
D7 & X & X & X & X & X & X & X & X & & \\
D3 & X & X & X & X & & & & & & \\
\end{array}
\]

(27)
Geometry change vs. phase change

For the regular solutions the D7-brane either ends at the horizon, or ends at a point outside the horizon, \( \Rightarrow \) Two classes of regular solutions in the AdS black hole background:

First order phase transition in type II B AdS black hole background
∑ Gauge/gravity correspondence relates supergravity in AdS black hole backgrounds at weak coupling to a thermal field theory at strong coupling.

∑ Correlators can be computed.

∑ Universal viscosity bound in all theories with gravity dual.
   (not from conformal symmetry or SUSY)

∑ Diffusion constants from poles of correlators/ lowest quasinormal mode

∑ Thermal phase transition in toy model dual to change of geometry.
Why is $\eta/s$ universal in a class of theories?

What do higher quasinormal modes correspond to?

Is QCD in the same universality class as the known theories?

What is the QCD gravity dual?

What is the thermal FT meaning of the two correlators connected to isospin diffusion?
APPENDIX A1: Applying the recipe for correlators

Step 1: Factor

\[ B(u) = -\frac{N^2}{16\pi^2} \sqrt{-g(u)} g^{\mu\nu'} g_{\nu'\nu} \]  \hspace{1cm} (28)

Step 2: Equations of motion (5 Maxwell equations in 5d position-space)

\[ \frac{1}{\sqrt{-g}} \partial_\nu [\sqrt{-g} g^{\mu\rho} g^{\nu\sigma} (\partial_\rho A_\sigma - \partial_\sigma A_\rho)] = 0, \]  \hspace{1cm} (29)

reduce to (in momentum-space)

\[ \omega A'_t + q f A'_z = 0, \]  \hspace{1cm} (30a)

\[ A''_t - \frac{1}{uf} (q^2 A_t + \omega q A_z) = 0, \]  \hspace{1cm} (30b)

\[ A''_z + \frac{f'}{f} A'_z + \frac{1}{uf^2} (\omega^2 A_z + \omega q A_t) = 0, \]  \hspace{1cm} (30c)

\[ A''_\alpha + \frac{f'}{f} A'_\alpha + \frac{1}{uf} \left( \frac{\omega^2}{f} - q^2 \right) A_\alpha = 0, \]  \hspace{1cm} (30d)

with dimensionless \[ \omega = \omega/(2\pi T), \quad q = q/(2\pi T) \] and \[ A' = \partial_u A. \]
APPENDIX A2: Equation of motion and boundary conditions

Combining the first two equations gives

\[ A'''' + \left(\frac{uf'}{u^2} \right) A''' + \frac{\omega^2 - q^2 f(u)}{u^2} A' = 0, \]  

(31)

Singular coefficients! → Indicial ansatz with regular \( F(u) \)

\[ A' = (1 - u)^\beta F(u) \]  

(32)

gives

\[ \beta = \pm \frac{i\omega}{2} \]  

(33)

Define \( \ln(1 - u) = -r \), with \( 0 \leq r \leq \infty \), then

\[ A' \propto e^{-\beta r} \]  

(34)

→ Pick incoming wave \( \beta = -i\omega/2 \).
APPENDIX A3: Hydrodynamic limit and solution

Small frequencies, long wavelengths justify

\[ F(u) = F_0 + \omega F_1 + \mathbf{q}^2 G_1 + \omega^2 F_2 + \omega \mathbf{q}^2 H_1 + \mathbf{q}^4 G_2 + \cdots . \]  

(35)

Simple solution

\[ F_0 = C, \quad F_1 = \frac{\mathbf{i} \mathbf{C}}{2} \ln \frac{2u^2}{1 + u}, \quad G_1 = C \ln \frac{1 + u}{2u}. \]  

(36)

Fix constant by boundary values (b.c. at \( u = 0 \))

\[ C' = \frac{\mathbf{q}^2 A_t^{\text{boundary}} + \omega \mathbf{q} A_z^{\text{boundary}}}{\mathbf{i} \omega - \mathbf{q}^2}, \]  

(37)
APPENDIX A4: Correlator

Step 3: Formula

\[ G^R(k) = -2B(u)f(u, -k)f'(u, k) \left|_{u \to 0} \right. \] (38)

identify \( A(u, k) = f(u, k)A_{\text{boundary}}(k) \).

Boundary behavior of our solution e.g. in time-components

\[ A'_t = (\mathbf{q}^2 A^0 + \mathbf{w}q A^0) \ln \epsilon + \frac{\mathbf{q}^2 A^0 + \mathbf{w}q A^0}{i\omega - \mathbf{q}^2}, \] (39)

And the corresponding correlator

\[ G^{ab}_{tt} = \frac{N^2 T q^2}{16\pi (i\omega - D q^2)} + \ldots \] (40)

exhibits a diffusion pole with diffusion constant

\[ D = \frac{1}{2\pi T} \] (41)
APPENDIX B1: Examples for deformed backgrounds

1. Constable-Myers background:

- dual field theory confining
- all SUSY broken
APPENDIX B1: Examples for deformed backgrounds

1. Constable-Myers background:
   - dual field theory confining
   - all SUSY broken

2. AdS black hole background:
   - horizon
   - $\mathcal{N} = 4$ SYM at finite temperature
   - dual QFT also confining

$\Rightarrow$ cf. thermal examples in next section

In UV limit, both geometries return to $AdS_5 \times S^5$ with D7 probe wrapping $AdS_5 \times S^3$. 
The Myers-Constable background is given by the metric

\[ ds^2 = H^{-1/2} \left( \frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta/4} \frac{dx_4^2}{w^4 - b^4} + H^{1/2} \left( \frac{w^4 + b^4}{w^4 - b^4} \right)^{(2-\delta)/4} \frac{w^4 - b^4}{w^4} \sum_{i=1}^{6} dw_i^2, \]

where

\[ H = \left( \frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta} - 1 \quad (\Delta^2 + \delta^2 = 10) \]

and the dilaton and four-form

\[ e^{2\phi} = e^{2\phi_0} \left( \frac{w^4 + b^4}{w^4 - b^4} \right)^{\Delta}, \quad C_4 = -\frac{1}{4} H^{-1} dt \wedge dx \wedge dy \wedge dz \]

This background has a singularity at \( w = b \)
APPENDIX B3: Schwarzschild black hole background

\[ ds^2 = \left( w^2 + \frac{b^4}{4w^2} \right) dx^2 + \frac{(4w^4 - b^4)^2}{4w^2(4w^4 + b^4)} d\tau^2 + \frac{1}{w^2} \sum_{i=1}^{6} dw_i^2 \]

(42)

with radial coordinate \( w^2 = \rho^2 + w_5^2 + w_6^2 \) and \( b \) a deformation parameter, \( \tau \) periodic (period \( \pi b = T^{-1} \))

horizon: \( S^1 \) collapses at \( w = \frac{1}{2} b \)
Formally stated for Euclidean metric:

\[
\langle e^{\int d^4 x A^{\text{boundary}}(\vec{x})} \mathcal{O}(\vec{x}) \rangle_{\text{CFT}} = Z_{\text{SUGRA}}[A(z, \vec{x})] \bigg|_{A(0, \vec{x}) = A^{\text{boundary}}(\vec{x})}
\]  (43)
APPENDIX C: Correlation functions from AdS/CFT

Formally stated for Euclidean metric:

\[ \langle e^{\int d^4x A^{\text{boundary}}(\vec{x})} \mathcal{O}(\vec{x}) \rangle_{CFT} = Z_{\text{SUGRA}} [A(z, \vec{x})] \bigg|_{A(0,\vec{x}) = A^{\text{boundary}}(\vec{x})} \]  \hspace{1cm} (43)

and by functional differentiation on both sides:

\[ \langle \mathcal{O}(\vec{x}) \mathcal{O}(\vec{y}) \rangle_{CFT} = \frac{\delta}{\delta A^{\text{boundary}}(\vec{x})} \frac{\delta}{\delta A^{\text{boundary}}(\vec{y})} Z_{\text{SUGRA}} \bigg|_{A = A^{\text{boundary}}} \]  \hspace{1cm} (44)

The fields \( \mathcal{A} \) act as sources for the operators \( \mathcal{O} \).