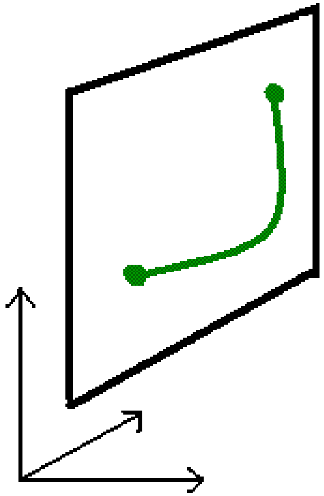
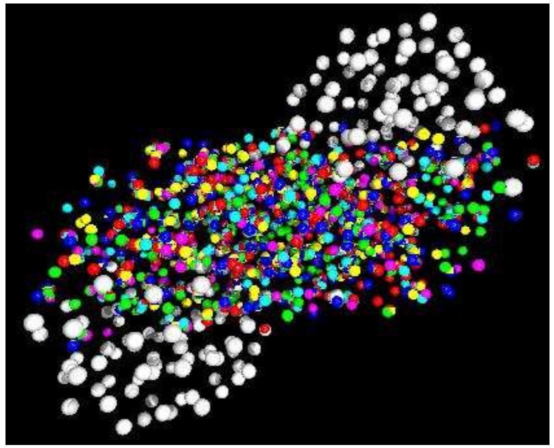
AdS/CFT and thermal properties of strongly coupled systems



$\longleftrightarrow$



[Frankfurt Group]

Kolloquium, Johann Wolfgang Goethe-Universität Frankfurt am Main

19 $^{\rm th}$ of April 2007

Max-Planck-Institut
für Physik
(Werner-Heisenberg-Institut)

Matthias Kaminski

Motivation by results

- still *only* strongly coupled Yang-Mills with $N \to \infty$
- realistic properties: chiral symmetry breaking, SUSY breaking, confinement,...
- flavor effects: meson-mass spectra, B-meson mass
- non-equilibrium: transport coefficients (universal η/s, κ_T, ...) [Policastro et al.,hep-th/0205052]
- thermal spectral functions [Kovtun,Starinets,hep-th/0602059]
- thermal phase transitions [Babington et al,hep-th/0306018]
- photon/dilepton production in plasma [Huot et al.,hep-th/0607237]

Overview

1. Overview

2. Introduction to concepts of AdS/CFT with flavor

3. Apply AdS-methods to find diffusion poles (Examples)

4. Understanding current results of AdS/CFT at finite temperature

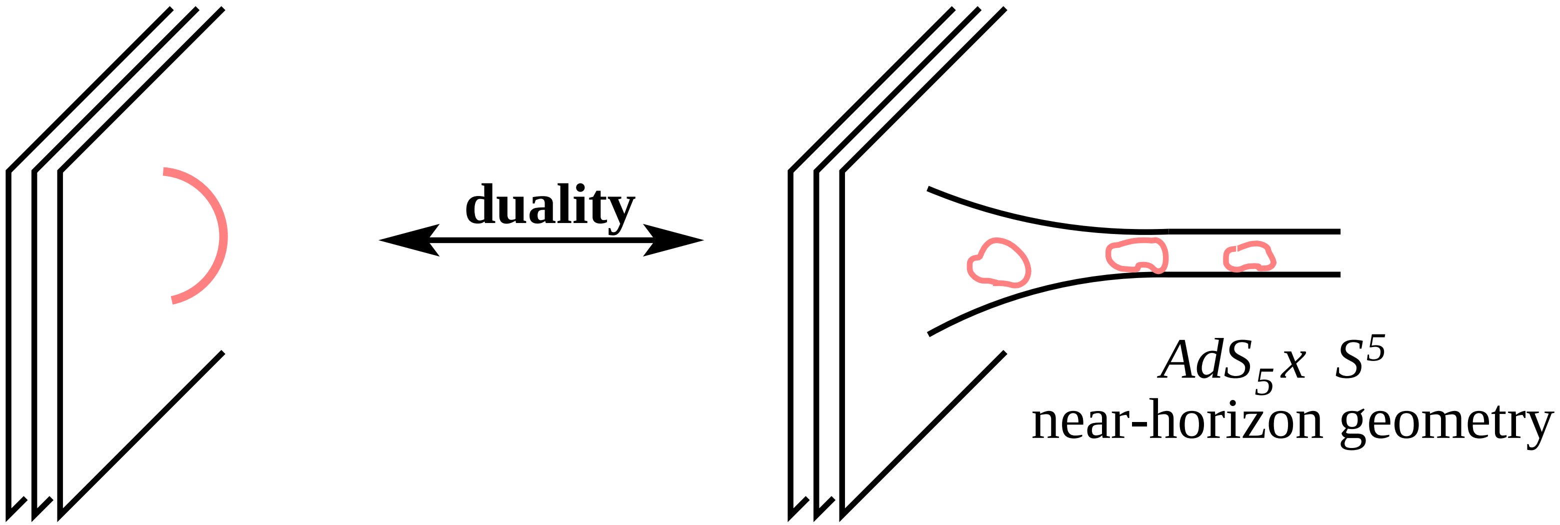
5. Summary and Outlook

2. Introduction to methods of the AdS/CFT correspondence

How (not why) do features on the *gravity-side* affect features on the *gauge-side*?

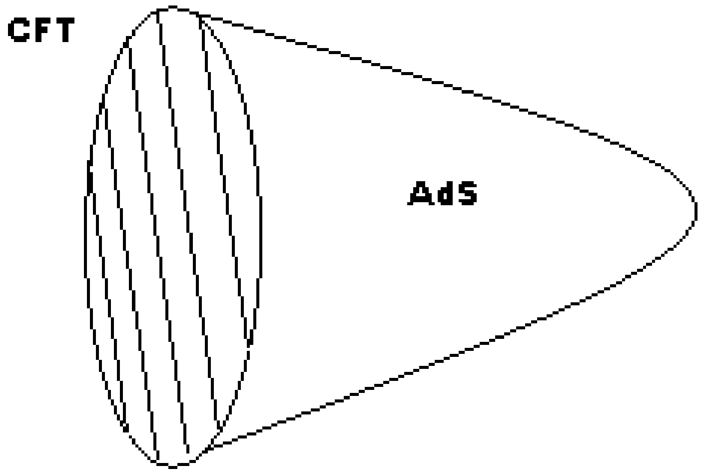
2.1 – Basic AdS/CFT correspondence

Super Yang-Mills theory in Minkowski space $\leftrightarrow$ supergravity in $AdS_5 \times S^5$
stack of N D3-branes in 10d



open strings ending on D3-branes $\leftrightarrow$ closed strings near the throat-region

What is AdS?



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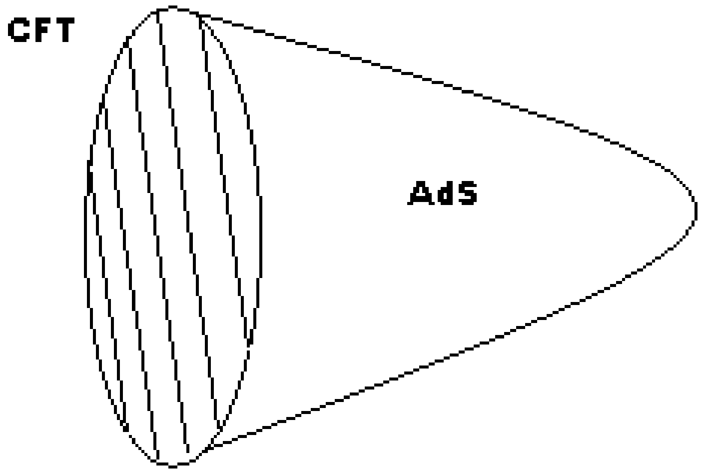
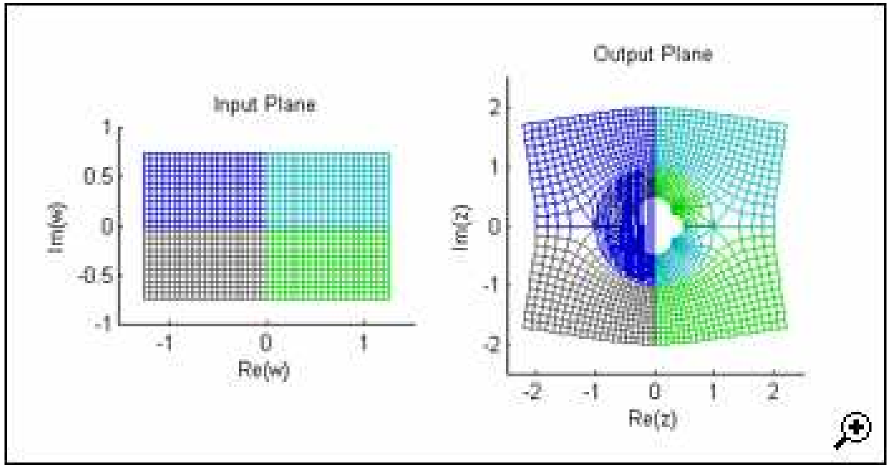


- ➡ Anti-de Sitter space
- ➡ the 5d space with constant negative curvature
- ➡ $AdS_5 \times S^5$:

  S^5 is the five-sphere
- ➡ AdS_5 has 4d boundary (Minkowski space)

What is CFT?



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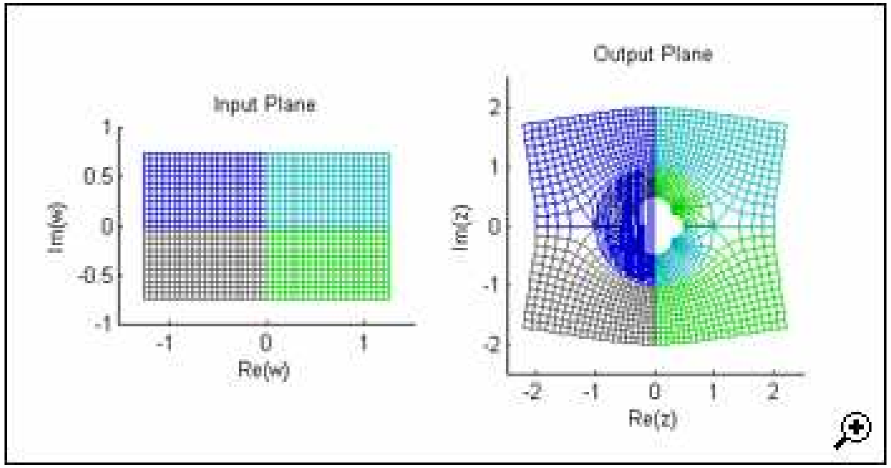
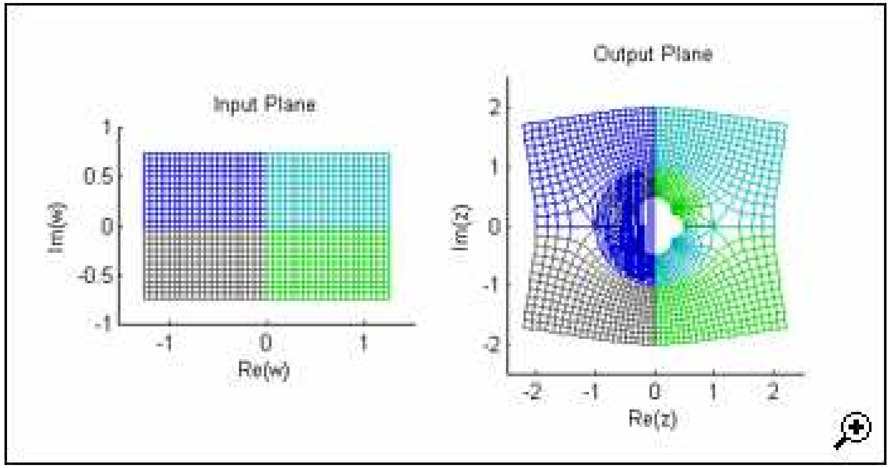


➡ **C**onformal **F**ield **T**heory

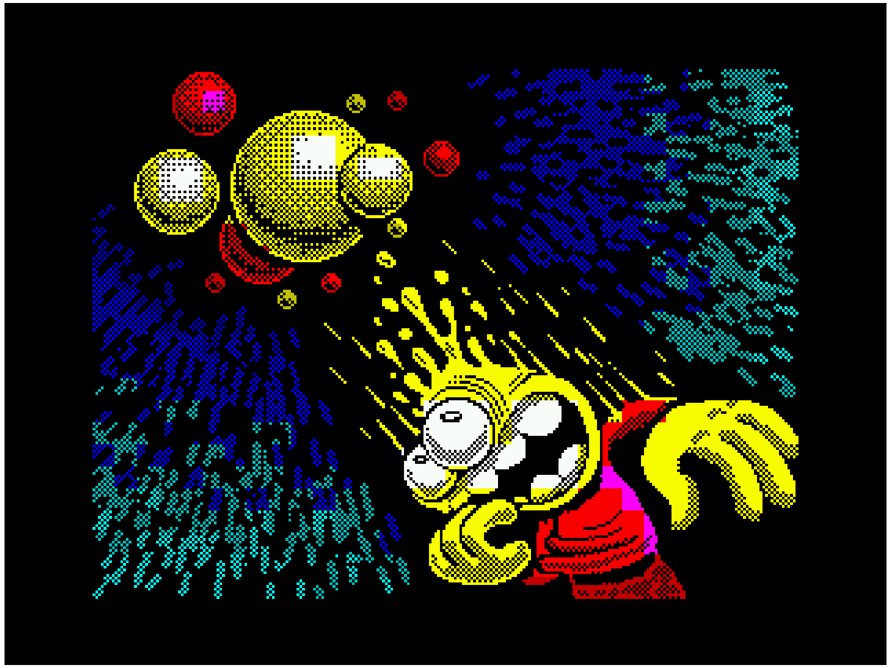
➡ invariant under rescaling of metric

What is CFT?



- ➡ **C**onformal **F**ield **T**heory
- ➡ invariant under rescaling of metric
- ➡ Yang-Mills theory in 4d
- ➡ gauge group $SU(N)$
- ➡ supersymmetric
- ➡ *quantum* field theory
- ➡ lives on the 4d worldvolume of D3-branes

What is supergravity?



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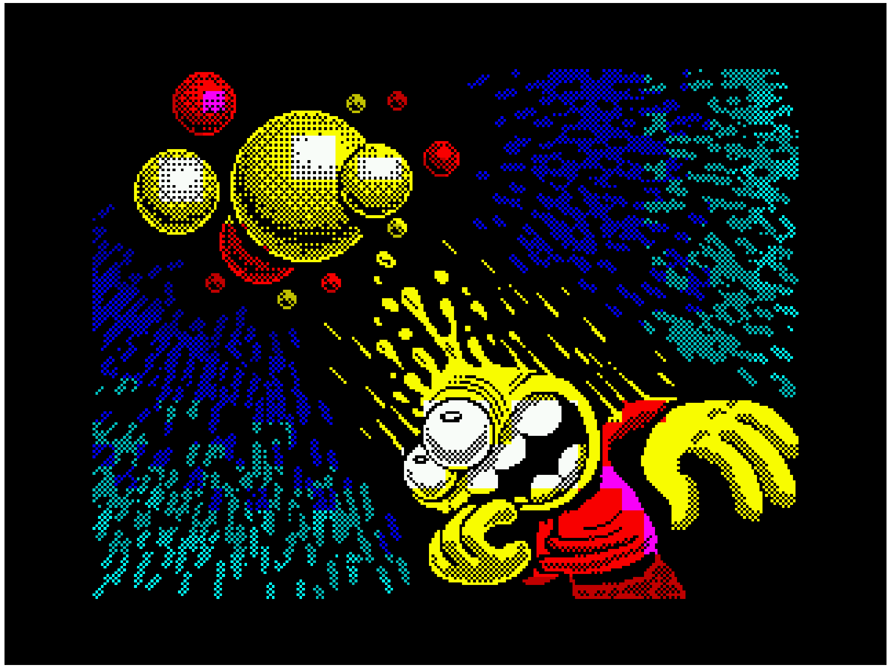
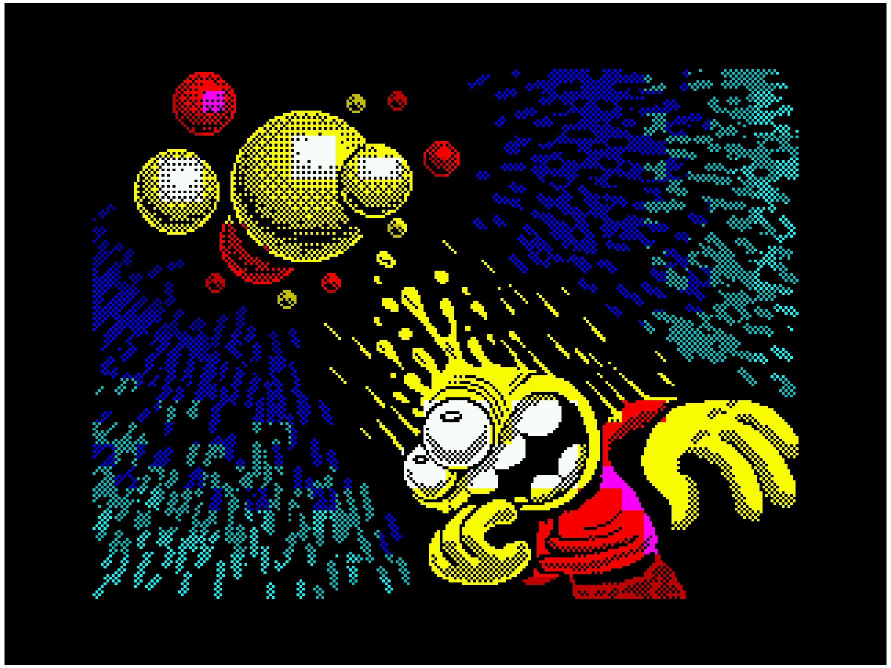


[Exocet, Demotopia]

What is supergravity?

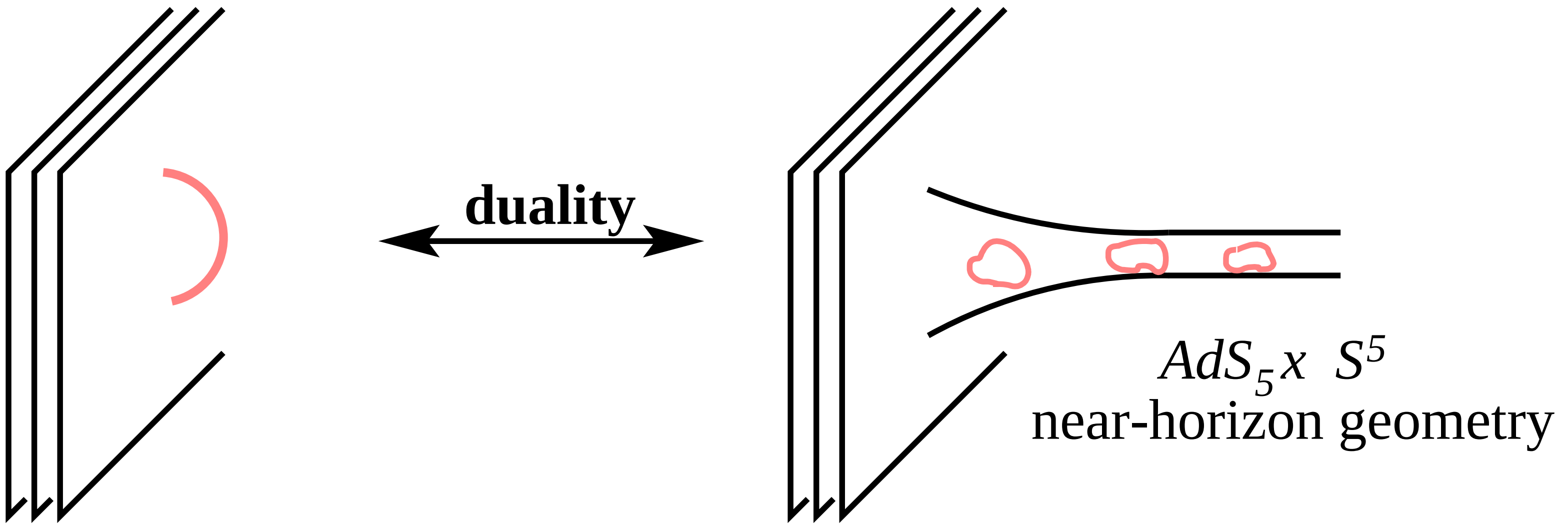


[Exocet, Demotopia]

- ➡ gravity-side of correspondence
- ➡ *classical* theory in 10 dimensions
- ➡ low-energy limit of string theory
- ➡ supersymmetric
- ➡ contains graviton as a *classical* field
- ➡ calculations involve *classical* actions, equations of motion, . . .
- ➡ contains branes as solitonic solutions to eom

Holographic dictionary of the AdS/CFT correspondence

operators $\mathcal{O}$ in Super Yang-Mills theory (Minkowski) $\leftrightarrow$ supergravity fields A (AdS)



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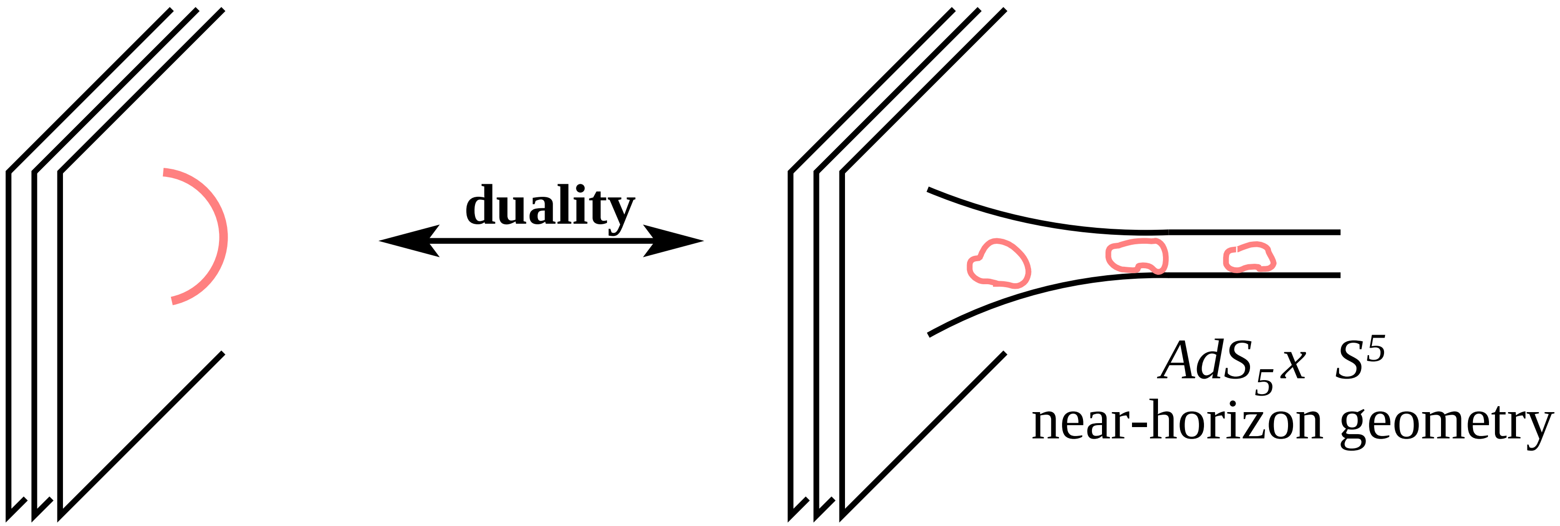

➡ scaling dimension Δ of operator $\mathcal{O}$ $\leftrightarrow$ mass m of the sugra-field A

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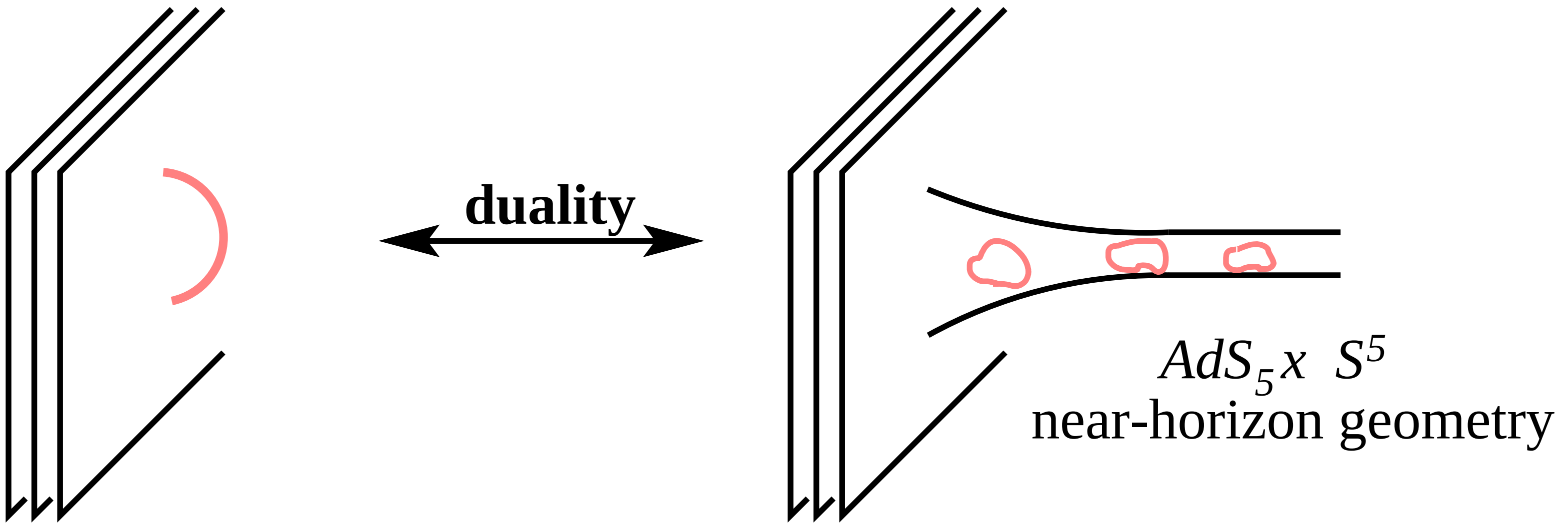

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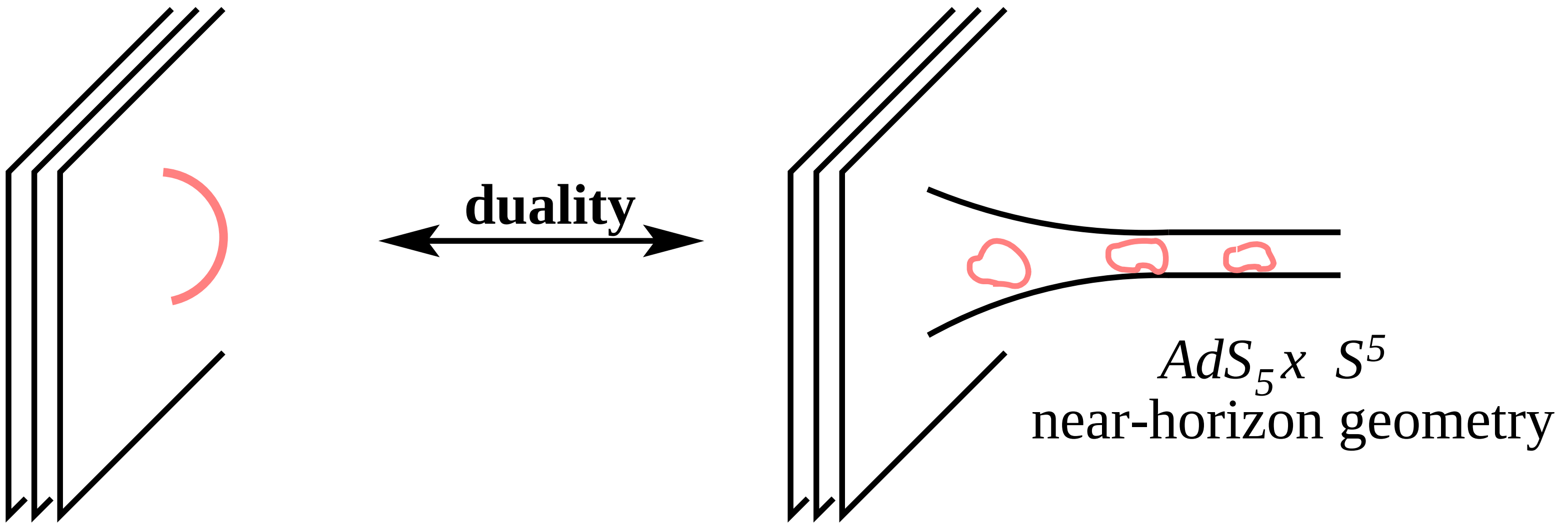

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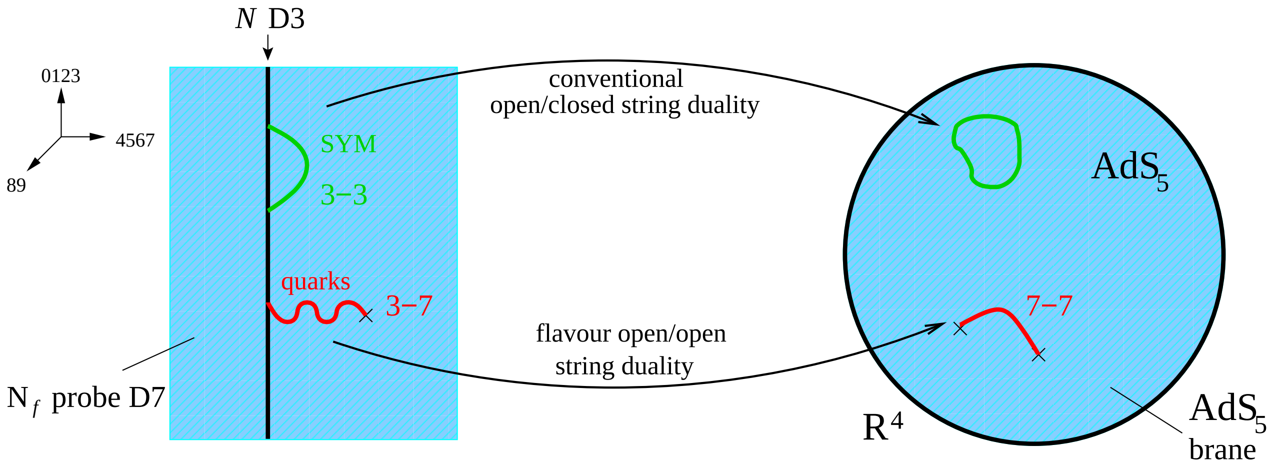
➡ conformal symmetries $\leftrightarrow$ AdS-isometries

➡ $SU(4)$: R-symmetry $\leftrightarrow S^5$ symmetry

2.2 – AdS/CFT with flavor

Adding probe D7-branes [Graña, Polchinski 2001] [Karch, Katz 2002]

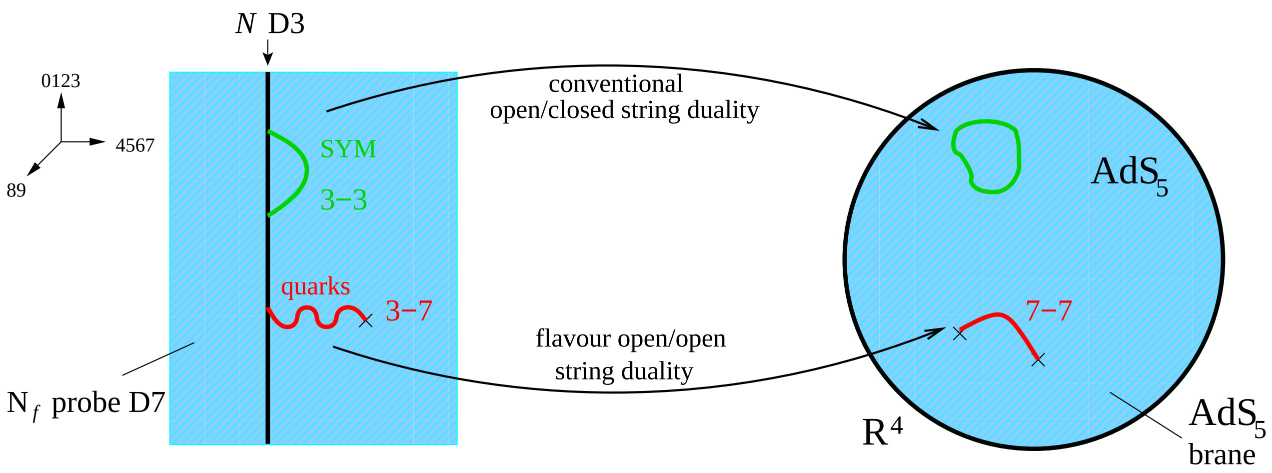


limit: $N \to \infty$ (standard Maldacena limit), N_f small (quenched approximation)

Duality acts twice!

2.2 – AdS/CFT with flavor

Adding probe D7-branes [Graña, Polchinski 2001] [Karch, Katz 2002]



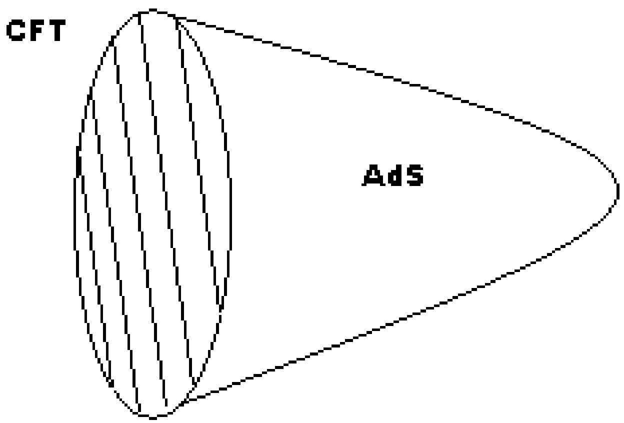
limit: $N \to \infty$ (standard Maldacena limit), N_f small (quenched approximation)

Duality acts twice!

4d $\mathcal{N} = 4$ SU(N) Super Yang-Mills theory		type IIB SUGRA on $AdS_5 \times S^5$
coupled to	$\longleftrightarrow$	+
4d $\mathcal{N} = 2$ fundamental hypermultiplet		Dirac-Born-Infeld theory on $AdS_5 \times S^3$

2.3 – Deformed backgrounds

Pure AdS-background (this means *space-time metric*):



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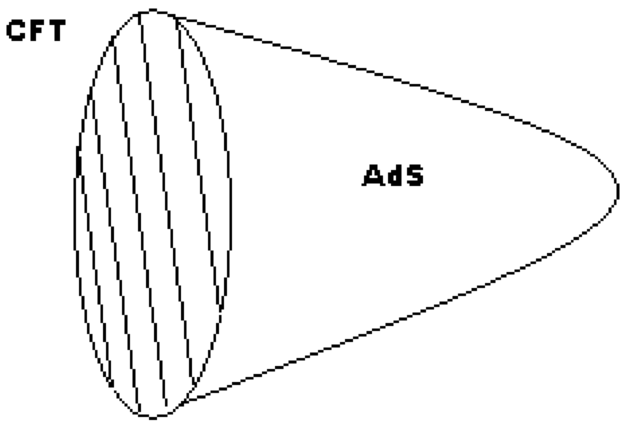
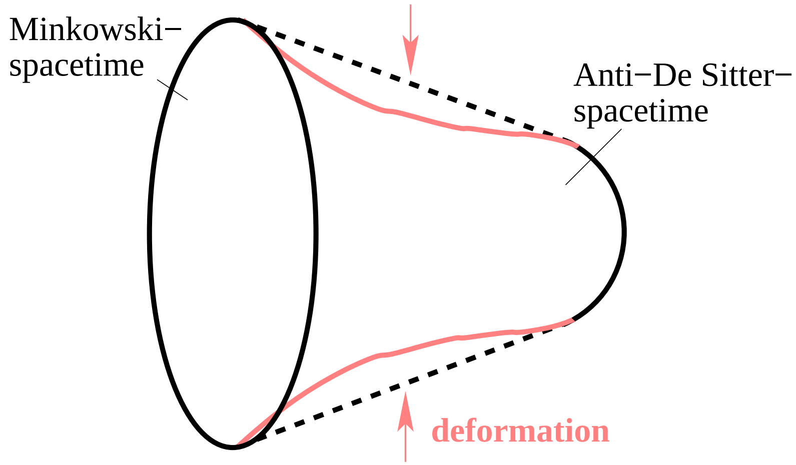


Deformed gravity background changes dual field theory:



➡ e.g. all SUSY broken, dual field theory at finite temperature

2.4 – Minkowski correlators from AdS/CFT

	AdS_5					S^3
coord. names	x^0	x^1	x^2	x^3	u	$-$
indices	$\mu, \nu \ldots$					
	$i, j \ldots$				u	
		α				

[Son, Starinets,hep-th/0205051]

$$S_{\rm cl} = \int \mathrm{d}u\mathrm{d}^4x B(u)(\partial_u A)^2 \tag{1}$$

Step 1: Extract the factor $B(u)$.

Step 2: Find solution $f(u, \vec{k})$ to mode equation of motion

with boundary conditions at $u = 0$

$$A(u, \vec{k}) = f(u, \vec{k}) A^{boundary}(\vec{k}) \tag{2}$$

and incoming wave b.c. at horizon $u = 1$.

Step 3: Find the retarded correlator as

$$G^R(\vec{k}) = -2B(u) f(u, -\vec{k}) \partial_u f(u, \vec{k}) \Big|_{u \to 0} \tag{3}$$

Summary of concepts:

- ➥ AdS/CFT is a holographic duality
- ➥ AdS/CFT relates operators from field theory to supergravity-fields
- ➥ flavor can be added by adding D7-branes
- ➥ realistic QFT features by deforming supergravity background
- ➥ QFT-correlators at strong coupling can be computed by dual calculation in weakly coupled supergravity

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1. Overview
2. Introduction to concepts of AdS/CFT with flavor
   - ➤ General setup (branes, supergravity,. . .) and adding flavor
   - ➤ Deformed backgrounds add realistic features
   - ➤ Correlation functions
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3. Apply AdS-methods to find diffusion poles

What can we calculate explicitly?

3.1 – Example #1: Super-Maxwell theory

[Policastro, Son, Starinets, '02]

Task: Calculate the R-charge diffusion constant ($\rightarrow$ correlator)!

Background:

$$\mathrm{d}s^2 = \frac{b^2 R^2}{u}\left(-f(u)\,\mathrm{d}x_0^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2\right) + \frac{R^2}{4u^2 f(u)}\,\mathrm{d}u^2 + R^2\mathrm{d}\Omega_5^2, \quad (4)$$

$$0 \leq u \leq 1, \qquad x_i \in \mathrm{R},$$

with

$$f(u) = 1 - u^2, \qquad R^4 = 4\pi g_s N_c \alpha'^2, \qquad b = \pi T. \quad (5)$$

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Classical action (AdS side):

$$S_{\mathrm{Super-Maxwell}} = -\frac{N^2}{16\pi^2}\int \mathrm{d}u\mathrm{d}^4x\sqrt{-g(u)}F_{\mu\nu}F^{\mu\nu} \qquad (6)$$

Now fix the gauge $A_u \equiv 0$, choose frame $\vec{k} = (\omega, 0, 0, q)$ and use Fourier transform of A_μ.

Applying the recipe for correlators

Step 1: Factor

$$B(u) = -\frac{N^2}{16\pi^2}\sqrt{-g(u)}g^{\mu\mu'}g^{\nu\nu'} \tag{7}$$

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Step 2: Solve equations of motion (5 Maxwell equations in 5d position-space)

$$\frac{1}{\sqrt{-g}}\partial_\nu[\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}(\partial_\rho A_\sigma - \partial_\sigma A_\rho)] = 0\,, \tag{8}$$

➡ Fourier transform fields

➡ Use dimensionless $\mathfrak{w} = \omega/(2\pi T)$, $\mathfrak{q} = q/(2\pi T)$

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Small frequencies, long wavelengths justify e.g.

$$\begin{aligned} A_t'(u) &= (1-u)^{-\frac{i\mathfrak{w}}{2}}\frac{\mathfrak{q}^2 A_t^{boundary} + \mathfrak{w}\mathfrak{q}A_z^{boundary}}{i\mathfrak{w}-\mathfrak{q}^2} \\ &\quad\times\left[1 + \frac{i\mathfrak{w}}{2}\ln\frac{2u^2}{1+u} + \mathfrak{q}^2\ln\frac{1+u}{2u} + \mathcal{O}(\mathfrak{w}^2,\mathfrak{q}^4,\mathfrak{w}\mathfrak{q}^2)\right] \end{aligned} \tag{9}$$

Correlator

Step 3: Formula

$$G^R(\vec{k}) = -2B(u)f(u,-\vec{k})f'(u,\vec{k})\Big|_{u\to 0} \tag{10}$$

identify $A(u,\vec{k}) = f(u,\vec{k})A^{boundary}(\vec{k})$.

Time components correlator

$$G^{ab}_{tt} = \frac{N^2 T q^2}{16\pi(i\omega - Dq^2)} + \cdots \tag{11}$$

exhibits a pole with

$$D = \frac{1}{2\pi T} \tag{12}$$

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Compare to the diffusion equation in position space

$$\partial_t \rho + D \nabla^2 \rho = 0, \tag{13}$$

and in momentum space

$$(i\omega - Dq^2)\rho = 0. \tag{14}$$

Interpretation of result

Question: What exactly have we computed?

Answer: The correlator of Super-Maxwell gauge fields A (a 5d generalization of the electromagnetic vector potential). This is dual to the retarded R-current correlator

$$G^R = -i \int \mathrm{d}^4 x \mathrm{e}^{-i\vec{k}\cdot\vec{x}} \Theta(t) \langle [J(\vec{x}), J(0)] \rangle \tag{15}$$

➡ J is the R-current (analog to ’electromagnetic’ current) of the Abelian Super Yang-Mills QFT.

➡ The R-charge Q coupling to this current undergoes a diffusion process with $D = 1/(2\pi T)$.

3.2 – Example #2: Isospin flavor theory

[Erdmenger, Kaminski, Rust '02]

Task: Compute the diffusion constant of non-Abelian isospin charge!

Background: AdS black hole **as before**.

Classical supergravity action $\rightarrow$ effective Dirac-Born-Infeld action on D7-brane.

What's new?

➡ Non-Abelian flavor indices

➡ Field strength $F^a_{\mu\nu} = 2\partial_{[\mu} A^a_{\nu]} + f^{abc} A^b_\mu A^c_\nu$

➡ Introduce $SU(2)$-chemical potential μ as background

➡ Consider fluctuations A_ν

$\mathbf{N_f = 2}$:

➡ 3 $\times$ 5 component equations of motion,

➡ coupled through flavor indices and

➡ coupled through Lorentz indices

Solution

Decouple equations by trafo

$$X_i = A_i^1 + iA_i^2 , \quad \widetilde{X}_i = A_i^1 - iA_i^2 . \tag{16}$$

Performing **three steps** as before gives

$$G_{t\tilde{t}} = -\frac{N_c T}{8\pi\sqrt{2\mu}} \frac{q^2\sqrt{\omega}}{\omega + q^2 D(\omega)} , \tag{17}$$

$$G_{\tilde{t}t} = \frac{N_c T}{8\pi\sqrt{2\mu}} \frac{q^2\sqrt{\omega}}{i\omega - q^2 D(\omega)} , \tag{18}$$

where $D(\omega)$ is given by

$$D(\omega) = \sqrt{\frac{\omega}{2\mu}} \frac{1}{2\pi T} . \tag{19}$$

Interpretation of result

Question: What exactly have we computed?

Answer: The correlator of Isospin gauge fields A (a generalization of the electromagnetic vector potential). This is dual to the retarded isospin current-current correlator

$$G^R = -i \int \mathrm{d}^4 x \mathrm{e}^{-i\vec{k}\cdot\vec{x}} \Theta(t) \langle [J(\vec{x}), J(0)] \rangle \tag{20}$$

➡ J is essentially the isospin charge current of the Abelian Super Yang-Mills QFT.

➡ The charge Q coupling to this current undergoes a diffusion process with diffusion coefficient

$$D = \sqrt{\frac{\omega}{2\mu}} \, \frac{1}{2\pi T} \, . \tag{21}$$

Summary of examples (diffusion poles):

- ➥ two examples for general method finding correlators in supergravity
- ➥ dual QFTs have hydrodynamic feature (diffusion pole in correlators)
- ➥ #1 is Super Maxwell theory with an ’Abelian’ charge undergoing diffusion with $D = 1/(2\pi T)$
- ➥ #2 is non-Abelian isospin flavor ($N_f = 2$) on D7-brane with dual isospin current and charge undergoing diffusion with $D = \sqrt{\frac{\omega}{2\mu}}\,\frac{1}{2\pi T}$.

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4. Understanding AdS/CFT results

What can we learn about thermal field theory at strong coupling?

4.1 – Universal viscosity bound

Kubo-formula relates viscosity η to energy-momentum T_{ij} correlator

$$\eta = \frac{\beta}{5} \int d^3x' \int_{-\infty}^{t} dt' \mathrm{e}^{\epsilon(t'-t)} \langle T_{\alpha\beta}(t,\vec{x}), T^{\alpha\beta}(t',\vec{x'}) \rangle_{\text{retarded}}, \tag{22}$$

with $\alpha, \beta = 1, 2$.

Dual (source) field is graviton $h_{\mu\nu}$

$$\langle T_{ij}(t,\vec{x}), T_{mn}(t',\vec{x'}) \rangle_{\text{retarded}} \rightsquigarrow \langle h_{ij}(t,\vec{x}), h_{mn}(t',\vec{x'}) \rangle_{\text{retarded}}. \tag{23}$$

Picture: graviton 'scattered' on black hole

$\rightarrow$ The only universal bound from AdS/CFT

$$\frac{\eta}{s} = \frac{1}{4\pi} \tag{24}$$

For all thermal theories with a gravity dual.

Case study of theories with universal bound reviewed in [Son, Starinets, 0704.0240]

Common features:

➡ the thermal field theory has a gravity dual

➡ Supergravity background is asymptotically AdS

Distinct features:

➡ supersymmetry

➡ conformal symmetry

➡ flavor/ fundamental matter

➡ nonzero chemical potential

Hydrodynamic poles and (quasi)normal modes

Alternatively, examine poles of T_{ij}-correlator (hydro limit)

$$\langle T_{ij}, T_{mn} \rangle \propto \frac{1}{i\omega - Dq^2} \tag{25}$$

with diffusion constant

$$D = \frac{\eta}{\epsilon + P} = \frac{1}{4\pi T}. \tag{26}$$

Generalization beyond hydrodynamics (to arbitrary wave-vector $\vec{k}$):

➡ Supergravity correlators have infinitely many poles

➡ Solutions in flat spacetime: normal modes (ω real)

➡ Solutions in curved spacetime: quasinormal modes (ω complex $\rightarrow$ dissipative mode)

➡ quasinormal modes $\leftrightarrow$ poles of QFT-correlators

➡ hydrodynamic poles are identical to the lowest quasinormal mode frequencies

➡ what are the higher ones?

4.2 – AdS-view on a thermal phase transition

Consider $\mathcal{N} = 4\ SU(N)$ SYM at finite temperature [Witten, hep-th/9803131]

- ➡ Dual string theory background is Euclidean AdS-Schwarzschild metric.
- ➡ add one D7-brane
- ➡ D7 is dynamical object with equations of motion
- ➡ $x_8 \equiv 0$, $x_9 = x_9(x_4)$, x_4 radial AdS coordinate

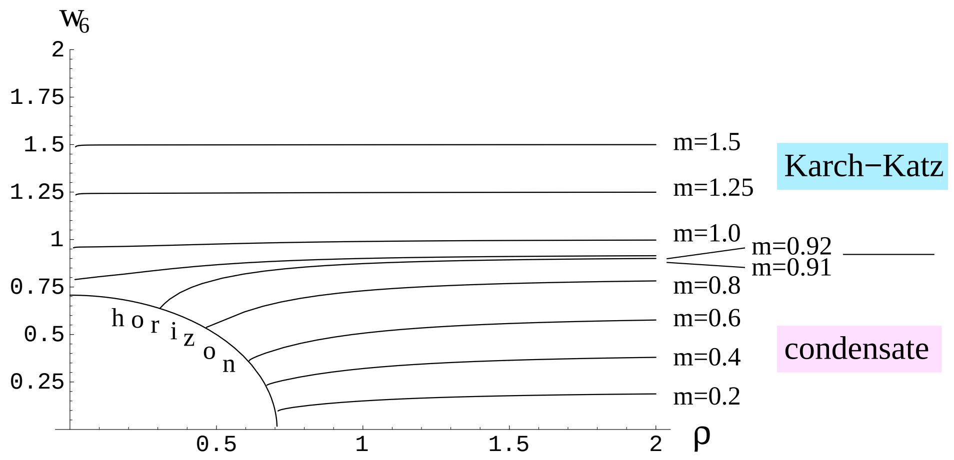
	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
$D7$	X	X	X	X	X	X	X	X		
$D3$	X	X	X	X						

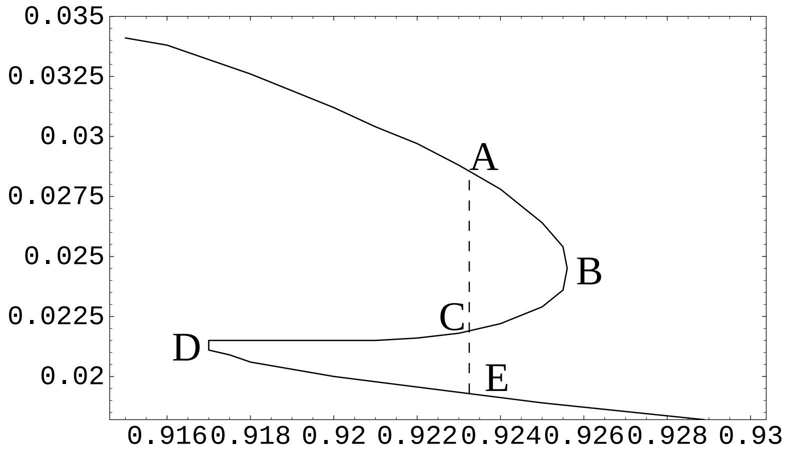
(27)

Geometry change vs. phase change

[Babington, Erdmenger, Evans, Guralnik, Kirsch, hep-th/0306018]

For the regular solutions the D7-brane either ends at the horizon, or ends at a point outside the horizon, ⇒Two classes of regular solutions in the AdS black hole background:





[Ingo Kirsch, PhD thesis]

First order phase transition in type II B AdS black hole background

Summary & Outlook

[Son, Starinets, 0704.0240]

- ✓ Gauge/gravity correspondence relates supergravity in AdS black hole backgrounds at weak coupling to a thermal field theory at strong coupling.
- ✓ Correlators can be computed.
- ✓ Universal viscosity bound in all theories with gravity dual. (not from conformal symmetry or SUSY)
- ✓ Diffusion constants from poles of correlators/ lowest quasinormal mode
- ✓ Thermal phase transition in toy model dual to change of geometry.

- ✍ Why is η/s universal in a class of theories?
- ✍ What do higher quasinormal modes correspond to?
- ✍ Is QCD in the same universality class as the known theories?
- ✍ What is the QCD gravity dual?
- ✍ What is the thermal FT meaning of the two correlators connected to isospin diffusion?

APPENDIX A1: Applying the recipe for correlators

Step 1: Factor

$$B(u) = -\frac{N^2}{16\pi^2}\sqrt{-g(u)}g^{\mu\mu'}g^{\nu\nu'} \tag{28}$$

Step 2: Equations of motion (5 Maxwell equations in 5d position-space)

$$\frac{1}{\sqrt{-g}}\partial_\nu[\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}(\partial_\rho A_\sigma - \partial_\sigma A_\rho)] = 0\,, \tag{29}$$

reduce to (in momentum-space)

$$\mathfrak{w}A_t' + \mathfrak{q}f\,A_z' = 0\,, \tag{30a}$$

$$A_t'' - \frac{1}{uf}\left(\mathfrak{q}^2 A_t + \mathfrak{w}\mathfrak{q}A_z\right) = 0\,, \tag{30b}$$

$$A_z'' + \frac{f'}{f}A_z' + \frac{1}{uf^2}\left(\mathfrak{w}^2 A_z + \mathfrak{w}\mathfrak{q}A_t\right) = 0\,, \tag{30c}$$

$$A_\alpha'' + \frac{f'}{f}A_\alpha' + \frac{1}{uf}\left(\frac{\mathfrak{w}^2}{f} - \mathfrak{q}^2\right)A_\alpha = 0\,, \tag{30d}$$

with dimensionless $\mathfrak{w} = \omega/(2\pi T)$, $\mathfrak{q} = q/(2\pi T)$ and $A' = \partial_u A$.

APPENDIX A2: Equation of motion and boundary conditions

Combining the first two equations gives

$$A_t''' + \frac{(uf)'}{uf} A_t'' + \frac{\mathfrak{w}^2 - \mathfrak{q}^2 f(u)}{uf^2} A_t' = 0 \,, \tag{31}$$

Singular coefficients! $\rightarrow$ Indicial ansatz with regular $F(u)$

$$A_t' = (1-u)^{\beta} F(u) \tag{32}$$

gives

$$\beta = \pm \frac{i\mathfrak{w}}{2} \tag{33}$$

Define $\ln(1-u) = -r$, with $0 \leq r \leq \infty$, then

$$A_t' \propto \mathrm{e}^{-\beta r} \tag{34}$$

$\longrightarrow$ Pick incoming wave $\beta = -i\mathfrak{w}/2$.

APPENDIX A3: Hydrodynamic limit and solution

Small frequencies, long wavelengths justify

$$F(u) = F_0 + \mathfrak{w} F_1 + \mathfrak{q}^2 G_1 + \mathfrak{w}^2 F_2 + \mathfrak{w}\mathfrak{q}^2 H_1 + \mathfrak{q}^4 G_2 + \cdots \, . \tag{35}$$

Simple solution

$$F_0 = C \, , \quad F_1 = \frac{iC}{2} \ln \frac{2u^2}{1+u} \, , \quad G_1 = C \ln \frac{1+u}{2u} \, . \tag{36}$$

Fix constant by boundary values (b.c. at $u = 0$)

$$C = \frac{\mathfrak{q}^2 A_t^{boundary} + \mathfrak{w}\mathfrak{q} A_z^{boundary}}{i\mathfrak{w} - \mathfrak{q}^2} \, , \tag{37}$$

APPENDIX A4: Correlator

Step 3: Formula

$$G^R(\vec{k}) = -2B(u) f(u, -\vec{k}) f'(u, \vec{k}) \Big|_{u \to 0} \quad (38)$$

identify $A(u, \vec{k}) = f(u, \vec{k}) A^{boundary}(\vec{k})$.

Boundary behavior of our solution e.g. in time-components

$$A'_t = (\mathfrak{q}^2 A_t^0 + \mathfrak{w}\mathfrak{q} A_z^0) \ln \epsilon + \frac{\mathfrak{q}^2 A_t^0 + \mathfrak{w}\mathfrak{q} A_z^0}{i\mathfrak{w} - \mathfrak{q}^2}, \quad (39)$$

And the corresponding correlator

$$G_{tt}^{ab} = \frac{N^2 T q^2}{16\pi(i\omega - Dq^2)} + \cdots \quad (40)$$

exhibits a difusion pole with diffusion constant

$$D = \frac{1}{2\pi T} \quad (41)$$

APPENDIX B1: Examples for deformed backgrounds

1. Constable-Myers background:

➥ dual field theory confining

➥ all SUSY broken

APPENDIX B1: Examples for deformed backgrounds

1. Constable-Myers background:

➥ dual field theory confining

➥ all SUSY broken

2. AdS black hole background:

➥ horizon

➥ $\mathcal{N} = 4$ SYM at finite temperature

➥ dual QFT also confining

➥ cf. thermal examples in next section

In UV limit, both geometries return to $AdS_5 \times S^5$ with D7 probe wrapping $AdS_5 \times S^3$.

APPENDIX B2: Constable-Myers background

The Myers-Constable background is given by the metric

$$ds^2 = H^{-1/2}\left(\frac{w^4+b^4}{w^4-b^4}\right)^{\delta/4} dx_4^2 + H^{1/2}\left(\frac{w^4+b^4}{w^4-b^4}\right)^{(2-\delta)/4} \frac{w^4-b^4}{w^4}\sum_{i=1}^{6} dw_i^2,$$

where

$$H = \left(\frac{w^4+b^4}{w^4-b^4}\right)^{\delta} - 1 \qquad (\Delta^2+\delta^2=10)$$

and the dilaton and four-form

$$e^{2\phi} = e^{2\phi_0}\left(\frac{w^4+b^4}{w^4-b^4}\right)^{\Delta}, \qquad C_{(4)} = -\frac{1}{4}H^{-1} dt\wedge dx\wedge dy\wedge dz$$

This background has a singularity at $w=b$

APPENDIX B3: Schwarzschild black hole background

$$ds^2 = \left(w^2 + \frac{b^4}{4w^2}\right) d\vec{x}^2 + \frac{(4w^4 - b^4)^2}{4w^2(4w^4 + b^4)} d\tau^2 + \frac{1}{w^2} \sum_{i=1}^{6} dw_i^2 \qquad (42)$$

with radial coordinate $w^2 = \rho^2 + w_5^2 + w_6^2$ and b a deformation parameter, τ periodic (period $\pi b = T^{-1}$)
horizon: S^1 collapses at $w = \frac{1}{2}b$

APPENDIX C: Correlation functions from AdS/CFT

Formally stated for Euclidean metric:

$$\langle e^{\int d^4x A^{boundary}(\vec{x}) \mathcal{O}(\vec{x})} \rangle_{CFT} = Z_{\mathrm{SUGRA}}[A(z, \vec{x})] \Big|_{A(0,\vec{x}) = A^{boundary}(\vec{x})} \tag{43}$$

APPENDIX C: Correlation functions from AdS/CFT

Formally stated for Euclidean metric:

$$\langle e^{\int d^4x\, A^{boundary}(\vec{x})\mathcal{O}(\vec{x})}\rangle_{CFT} = Z_{\mathrm{SUGRA}}[A(z,\vec{x})]\Big|_{A(0,\vec{x})=A^{boundary}(\vec{x})} \tag{43}$$

and by functional differentiation on both sides:

$$\langle\mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\rangle_{CFT} = \frac{\delta}{\delta A^{boundary}(\vec{x})}\frac{\delta}{\delta A^{boundary}(\vec{y})} Z_{\mathrm{SUGRA}}\Big|_{A=A^{boundary}} \tag{44}$$

The fields A act as sources for the operators $\mathcal{O}$.