Flavor Superconductivity/ Superfluidity

Fifth Aegean Summer School, Adamas, Milos, September 2009

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[arXiv: 0810.2316]
[arXiv: 0903.1864]
Outline

I. Invitation: Superconductivity & Holography

II. Review: Holographic Concepts

III. Details: Flavored Plasma (D3/D7)

IV. Results: Flavor Superconducting Phase (D3/D7)

V. Discussion

Remarks on references:
All references are hyperlinked in this document.
Publications quoted here are chosen because they review or explain certain aspects in a (pedagogical) way which is accessible to the unexperienced reader.
I. Invitation: Conventional Superconductors

**Examples**
- Color superconducting phase at high densities
  - [Alford, Rajagopal, Wilczek ‘97]
- Higgs mechanism: Superconductivity of vacuum
  - [e.g. Weinberg]

**Weak coupling concepts**
- Charged condensate of Cooper-pairs
- (gauge) symmetry: electromagnetic $U(1)_{em}$
- Local symmetry spontaneously broken
- Goldstone bosons eaten
  - (photons in SC become massive $\Rightarrow$ Meissner effect)

**Theory**: BCS (Bardeen-Cooper-Schrieffer) well established

**Superfluidity**: global symmetry spontaneously broken,
  Goldstone bosons survive (become hydro modes)
  $\Rightarrow$ weakly gauge boundary theory
I. Invitation: Unconventional Superconductors
[see talk by Panagopoulos]

Typical signatures

- magnetic field expulsion (c)
- energy gap (peak at edge)
- pseudo gap
- underdoped: strong coupl.

Figure: Tunneling spectra measured in high temperature superconductor $Bi_2Sr_2CaCu_2O_{8+\delta}$.

[Renner et al., Phys. Rev. Lett. 80, 149 - 152 (1998)]

Theory? Pairing mechanism? Meissner effect?
[see lectures by Sadchdev]
I. Invitation: Unconventional Superconductors

[see talk by Panagopoulos]

Holographic result

Energy gap

Theory? Pairing mechanism? Meissner effect?

[see lectures by Sadchdev]
I. Invitation: Building a Holographic SC Gravity

What do we need?
- charged condensate (vev)
- no source
- condensate of charge carriers
- finite temperature
- AdS-boundary
- curvature
- electromag.
- horizon
- gravity
- introduce normalizable mode
- no non-normalizable mode
- condensate hovers over horizon
- black hole

Is this stable?

[Gubser, Pufu 0805.2960]
I. Invitation: Get some intuition

**Field Theory**

\[ \mathcal{L} \sim D_\nu \phi D^\nu \phi \sim (M_q^2 - \mu_{\text{isospin}}^2) \phi^2 \]

charged particles condense at large enough chemical potential

\[ \mu_{\text{isospin}} \sim M_q \]

**Gravity**

strings (D3-D7) give FT charges

cannot put infinitely many

second brane is important

Why do we need a non-Abelian structure?
I. Invitation: Get some intuition

**Field Theory**

\[ \mathcal{L} \sim D_\nu \phi D^\nu \phi \sim (M_q^2 - \mu_{\text{isospin}}^2) \phi^2 \]

- charged particles condense at large enough chemical potential
  \[ \mu_{\text{isospin}} \sim M_q \]

**Gravity**

- strings (D3-D7) give FT charges
- cannot put infinitely many
- second brane is important

Why do we need a non-Abelian structure?
I. Invitation: Why so complicated?

**Bottom-up**

String Theory

? Phenomenological Gravity Dual of SC

Phenomenological requirements, SC Field Theory

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**Top-down**

String Theory

**Geometric construction** (e.g. Dp/Dq-branes)

Holographic Dual of SC

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string theory derived
identification of FT degrees o.f.
‘dirty’
many effects at once

**Pairing mechanism!**

---

study effects in clean setup
separate effects

---

[Gubser, Pufu 0805.2960]
[Hartnoll, Herzog, Horowitz 0803.3295]

[see lectures by Horowitz]
Navigator

✓ Invitation: Superconductivity & Holography

II. Review: Holographic Concepts

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also reviewed in [M.K. 0808.1114]
II. Review: Boundary Asymptotics

non-normalizable (source) \[ A = A^{(0)} + \frac{A^{(2)}}{\rho^2} + \ldots \]

(normalizable (vev) \( \rho \to \infty \))

[see lectures by Argyres]

Dictionary

QFT FEATURE \( \xleftrightarrow{\text{geometry}} \)

(energy scale) \( \text{(radial coord. } \rho \text{ )} \)

operator \( J_\mu \) \( \xleftrightarrow{\text{field} A_\mu} \) (gauge)

vev \( A^{(2)} \) (charge)

source \( A^{(0)} \) (chem. pot.)

[see lectures by Argyres]
II. Review: Correlators & Spectral Functions

\[ \tilde{J}_\mu \]

Gauge Theory
Problem (strong): Find retarded two-point function of flavor current in YM-plasma.

Gravity problem (weak): Find solution for equation of motion of vector field in SUGRA.

\[ J_\mu \leftrightarrow A_\mu \]

\[ G^{\text{ret}}(\omega, q) = -i \int d^4x \, e^{i\vec{k}\vec{x}} \, \theta(x^0) \langle [J(\vec{x}), J(0)] \rangle \leftrightarrow \frac{\delta^2}{\delta A_{\text{bdy}} \delta A_{\text{bdy}}} S_{\text{Sugra}} \]

Recipe:
\[ S_{\text{Sugra}} \sim \int \partial_\rho A \partial_\rho A \rightarrow S_{\text{on-shell}} \sim \int A \partial_\rho A \rightarrow G^{\text{ret}} \sim \lim_{\rho \to \infty} \frac{A \partial_\rho A}{A} \]

Thermal spectral function:
\[ \mathcal{R}(\omega, q) = -2 \Im G^{\text{ret}}(\omega, q) \]

[Son, Starinets hep-th/0205051]
II. Review: Quasinormal modes

Special frequencies: $\omega_n \in \mathbb{C}$; \[ \lim_{\rho \to \rho_{bdy}} |\tilde{A}(\omega_n)|^2 = 0 \]

Example:

$G^{\text{ret}} = \frac{N_f N_c T^2}{8} \lim_{\rho \to \rho_{bdy}} \left( \rho^3 \frac{\partial_\rho \tilde{A}(\rho)}{\tilde{A}(\rho)} \right)$

\[ e^{-i\omega r} = e^{-i\text{Re}\{\omega\} r} e^{i\text{Im}\{\omega\} r} \]

Gravity:

quasinormal frequencies

Gauge theory:

poles of correlator

(energy, damping, stability of mesonic excitations)
II. Review: D3/D7-Brane Setup (1)

**Flavor Probe Branes (D7)**

![Diagram of D3/D7-brane setup with AdS5 and flavor open/open string duality]

**Dirac-Born-Infeld (DBI) action**

\[
S_{\text{DBI}} = -T_7 \int d^8 \sigma \left( \sqrt{-\det(P[G + B]_{\mu\nu} + (2\pi \alpha')^2)F_{\mu\nu}} \right)
\]

\[B \equiv 0\]
II. Review: D3/D7-Brane Setup (2)

- $N_C$ D3-branes
II. Review: D3/D7-Brane Setup (2)

- $N_f$ D7-branes
- $N_c$ D3-branes
II. Review: D3/D7-Brane Setup (2)

$N_f$ D$_7$-branes

$N_c$ D$_3$-branes (black)
II. Review: D3/D7-Brane Setup (2)
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\( \hat{A}_\nu \subset U(N_f) \)

\( N_c \) D3-branes (black)

\( N_f \) D7-branes
II. Review: D3/D7-Brane Setup (2)

\[ \hat{A}_\nu \subset U(N_f) \]

\[ L(\rho) \]

\[ \rho \]

\[ N_f \text{ D}_7 \text{-branes} \]

\[ N_c \text{ D}_3 \text{-branes (black)} \]

\[ \begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
D_3 & x & x & x & x & & & & & \\
D_7 & x & x & x & x & x & x & x & & \\
\end{array} \]
II. Review: D3/D7-Brane Setup (2)

\[ \hat{A}_\nu \subset U(N_f) \]

\[ L(\rho) \]

\[ N_c \text{ D3-branes} \]

\[ \rho \]

\[ N_f \text{ D7-branes} \]

Chemical potential:

\[ \hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu \]

(cf. therm. FT)

[Kobayashi et al. hep-th/0611099]

[Mateos et al. 0709.1225]
Chemical potential: \[ \hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu \]

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\[ \hat{A}_\nu \subset U(N_f) \]

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0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
D_3 & x & x & x & x & x & x & x & x & x \\
D_7 & x & x & x & x & x & x & x & x & x \\
\end{array} \]
II. Review: D3/D7-Brane Setup (2)

\[
\hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu \\
(\text{cf. therm. FT})
\]

Chemical potential:

\[
L(\rho) = \rho^4 (8,9) \times (4,5,6,7)
\]

\[
\hat{A}_\nu \subset U(N_f)
\]

\[
N_f \text{D}_7\text{-branes}
\]

\[
N_c \text{D}_3\text{-branes} \quad \text{(black)}
\]

\[
\begin{array}{cccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
\text{D}_3 & x & x & x & x & x \\
\text{D}_7 & x & x & x & x & x & x & x & x & x
\end{array}
\]

Mesons

[Mateos et al. 0709.1225]

[Kobayashi et al. hep-th/0611099]
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III. Abelian Chemical Potential: Background

[Myers et al. hep-th/0611099]

AdS black hole metric

\[ ds^2 = \frac{1}{2} \left( \frac{\varrho}{L} \right)^2 \left[ -\frac{f^2}{f} dt^2 + f dx_3^2 \right] + \frac{L^2}{\varrho^2} \left[ \frac{1 - \chi^2 + \varrho^2 (\partial_\varrho \chi)^2}{1 - \chi^2} \right] d\varrho^2 + L^2 (1 - \chi^2) d\Omega_3^2 \]

\[ f(\varrho) = 1 - \frac{\varrho H^4}{\varrho^4}, \quad \tilde{f}(\varrho) = 1 + \frac{\varrho H^4}{\varrho^4}, \quad \chi = \cos(\theta), \quad \varrho^2 = r^2 + \sqrt{r^4 - r H^4} \]

DBI action

\[ I_{D7} = -N_f T_{D7} \int d^8 \sigma \frac{\varrho^3}{4} f \tilde{f}(1 - \chi^2) \sqrt{1 - \chi^2 + \varrho^2 (\partial_\varrho \chi)^2 - 2(2\pi \ell_s^2)^2 \frac{\tilde{f}}{f^2} (1 - \chi^2) F_{\varrho t}^2 } \]

Can be rewritten as constant of motion \( \tilde{\rho} \).
III. Abelian Chemical Potential: Fluctuations

DBI action:

\[ S_{D7} = \int d^8 x \sqrt{\det \left\{ [g + F] + \tilde{F} \right\}} G, \quad F_{\mu\nu} = \partial_{[\mu} A_{\nu]} \]

Equation of motion:

\[ 0 = \tilde{A}'' + \frac{\partial_{\rho} [\sqrt{\det G} G^{22} G^{44}] \tilde{A}'}{\sqrt{\det G} G^{22} G^{44}} - \frac{G^{00}}{G^{44}} \varrho_H^2 \omega^2 \tilde{A} \]
III. Abelian Chemical Potential: Fluctuations

[Myers, Starinets, Thomson 0710.0334]

DBI action:

\[
S_{D7} = \int d^8 x \sqrt{|\det \{ [g + F] + \tilde{F} \} |}, \quad F_{\mu \nu} = \partial_{[\mu} A_{\nu]} \]

Equation of motion:

`Curved’ Maxwell equations:

\[
\partial_\mu F^{\mu \nu} = 0 \\
\partial_\mu \left( \sqrt{-G} G^{\mu \nu} G^{\rho \sigma} F_{\nu \sigma} \right) = 0 \\
\partial_\mu \left( \sqrt{-G} G^{\mu \nu} G^{\rho \sigma} \partial_{[\nu} \tilde{A}_{\sigma]} \right) = 0
\]
III. Abelian Chemical Potential: Fluctuations

DBI action:

\[ S_{D7} = \int d^8 x \sqrt{\left| \det \left\{ [g + F] + \tilde{F} \right\} \right|} G \]

\[ F_{\mu \nu} = \partial_{[\mu} A_{\nu]} \]

Equation of motion:

\[ 0 = \ddot{A}'' + \frac{\partial_{\rho} \left[ \sqrt{\left| \det \tilde{G} \right| G_{22} G_{44}^2} \right]}{\sqrt{\left| \det \tilde{G} \right| G_{22} G_{44}^2}} \dot{A}' - \frac{G_{00}^0}{G_{44}^0} \varrho_H^2 \omega^2 \tilde{A} \]

Boundary conditions:

\[ \tilde{A} = (\varrho - \varrho_H)^{-i\omega} \left[ 1 + \frac{i\omega}{2} (\varrho - \varrho_H) + \ldots \right] \]

Translation to gauge theory by duality:

\[ \tilde{A} \leftrightarrow J^\mu \]

Gauge correlator:

\[ G_{\text{ret}}^{\text{AdS/CFT}} = \frac{N_f N_c T^2}{8} \lim_{\mu \rightarrow \mu \text{bdy}} \left( \rho^3 \frac{\partial_{\rho} \tilde{A}(\rho)}{\tilde{A}(\rho)} \right) \]
III. Abelian Chemical Potential: Spectral F’s

### Finite baryon density:

\[
\tilde{d} = 0.25
\]

\[\chi_0 = 0.1 \quad \chi_0 = 0.5 \quad \chi_0 = 0.7 \quad \chi_0 = 0.8\]

\[\tilde{d} = 0.25\]

\[\tilde{d} = 0\]

\[\tilde{d} = 0.00315\]

\[L(\varrho) = \varrho \chi(\varrho)\]

\[\chi_0 = \chi(\rho)\big|_{\rho \to \rho_H} \sim \frac{m_{\text{quark}}}{T}\]

\[\chi = \chi(\tilde{d}, \rho)\]
III. Abelian Chemical Potential: Spectral F’s

[Karch, O'Bannon '07]

Finite baryon density:

Lower temperature

Phase diagram:

\[
L(\rho) = \rho \chi(\rho)
\]

\[
\chi_0 = \chi(\rho)\bigg|_{\rho \to \rho_H} \sim \frac{m_{\text{quark}}}{T}
\]

\[
\chi = \chi(\tilde{d}, \rho)
\]
### III. Abelian Chemical Potential: Spectral F’s

**Finite baryon density:**

\[ \tilde{d} = 0.25 \]

\[ \chi_0 = 0.8 \]

\[ \chi_0 = 0.94 \]

\[ \chi_0 = 0.962 \]

**Phase diagram:**

\[ L(\varrho) = \varrho \chi(\varrho) \]

\[ \chi_0 = \chi(\rho) \bigg|_{\rho \to \rho_H} \sim \frac{m_{\text{quark}}}{T} \]

\[ \chi = \chi(\tilde{d}, \rho) \]
III. Isospin Chemical Potential: Spectral F’s

[Erdmenger, M.K., Rust 0710.0334]

Finite isospin density

What has changed?

Background action contains

$$\text{tr}(\sqrt{g + F^a T^a}) \sim N_f \times \text{Abelian}$$

Diagonalize action in flavor space (X, Y, E)

Fluctuations in three flavor directions, so two new:

X, Y (orthogonal to isospin)

$$0 = X^\prime \prime + \ldots X^\prime + \ldots (\omega - \mu)X$$

$$0 = Y^\prime \prime + \ldots Y^\prime + \ldots (\omega + \mu)Y$$

triplet splitting

analog to Rho-vector meson

analytical results & interpretation:

[M.K. 0808.1114]
III. High isospin densities: Instability!

New (lowest) mesonic excitation:

\[
m = 3 \quad \bar{d} = 8 \quad \bar{d} = 14
\]

Stability:

New phase:

\[
\frac{\mu}{M_q} \text{ vs } \frac{1}{m} \propto T/M_q
\]

unstable
black hole
Minkowski
First summary: QGP Phenomenology

✓ stable mesons survive deconfinement

✓ meson mass changes with temperature

✓ vec-mesons are isospin triplets (QCD’s Rho-meson)

✓ new excitation/phase: instability

☐ stabilize the new phase, new ground state
Navigator

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IV. Flavor Superconducting Phase

General idea

\[ A^3_0 = \mu + \frac{d}{\rho^2} + \ldots \]

[Ammon, Erdmenger, M.K., Kerner 0810.2316]

\[ A^3_0 = \mu + \frac{d^3}{\rho^2} + \ldots \]

[Ammon, Erdmenger, M.K., Kerner 0810.2316]

\[ A^1_3 = \frac{d^1_3}{\rho^2} + \ldots \]

[Ammon, Gubser, Pufu 0805.2960]
IV. Field Theory Picture

Gravity field

\[ A_0^3 = \mu + \frac{d_0^3}{\rho^2} + \ldots \]

dual to current

\[ J_0^3 \propto \bar{\psi} \tau^3 \gamma_0 \psi + \phi \tau^3 \partial_0 \phi = n_u - n_d \]

explicitly breaks

\[ U(2) \sim U(1)_B \times SU(2)_I \rightarrow U(1)_B \times U(1)_3 \]

---

New field

\[ A_3^1 = \frac{d_3^1}{\rho^2} + \ldots \]

dual to current

\[ J_3^1 \propto \bar{\psi} \tau^1 \gamma_3 \psi + \phi \tau^1 \partial_3 \phi \]

\[ = \bar{\psi}_u \gamma_3 \psi_d + \bar{\psi}_d \gamma_3 \psi_u + \text{bosons} \]

spontaneously breaks

\[ U(1)_3 \leftrightarrow U(1)_{\text{em}} \]
Exercise 1: Derive the background EOMs.

\[ S_{DBI} = - T_{D7} \int d^8 \xi \text{Str} \left( \sqrt{\det Q} \sqrt{\det (P_{ab} [E_{\mu\nu} + E_{\mu i}(Q^{-1} - \delta)^{ij} E_{j\nu}] + 2\pi \alpha' F_{ab})} \right) \]

\[ Q^i_j = \delta^i_j + i2\pi \alpha' [\Phi^i, \Phi^k] E_{kj}, \quad E_{\mu\nu} = g_{\mu\nu} + B_{\mu\nu} \]

(\( B \equiv 0 \))

\( \mu, \nu = 0, \ldots, 9; \quad a, b = 0, \ldots, 7; \quad i, j = 8, 9 \)

8,9 rotation: \( \Phi^9 \equiv 0 \Rightarrow Q^i_j = \delta^i_j \)

Choose: \( A = A_0^3 d\tau^3 + A_1^3 d\tau^1 \) and \( \Phi^8 \parallel \tau^0 \)

\( \iff \) FT: charge eigenstates are also mass eigenstates

\( \implies \) scalars \( \Phi^8, \Phi^9 \) decouple from vectors \( A \)

(set to zero from now on, i.e. massless quarks)
Exercise 1: Derive the background EOMs.

\[ S_{\text{DBI}} = -T_{D7} \int d^8 \xi \text{Str}\{ \sqrt{\text{det}[g + 2\pi \alpha' F]} \} \]

\[ = -T_{D7} N_f \int d^8 \xi \sqrt{-g} \text{Str} \sqrt{1 + g^{tt} g^{\rho \rho} (F_\rho^3)^2 (\sigma^3)^2 + g^{33} g^{44} (F_\rho^1)^2 (\sigma^1)^2 - c^2 g^{tt} g^{33} (F_{03}^2)^2 (\sigma^2)^2} \]

\[ F = dA + [A, A] \]

(e.g. [Myers et al. hep-th/0611099])
Exercise 1: Derive the background EOMs.

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\[ F = dA + [A, A] \]

\[ = \partial_\rho A_0^3 \tau^3 \, d\rho \wedge dt + \partial_\rho A_3^1 \tau^1 \, d\rho \wedge dz + i \epsilon^{231} A_0^3 A_3^1 \, dt \wedge dz \]

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Exercise 1: Derive the background EOMs.

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Problem 1: How to evaluate the symmetrized trace of the square root exactly?

Problem 2: Non-Abelian DBI-action only known to fourth order in \( \alpha' \).

(e.g. [Myers et al. hep-th/0611099])
Exercise 1: Derive the background EOMs.

\[ S_{\text{DBI}} = -T_{D7} \int d^8 \xi \text{Str}\{\sqrt{\text{det}[g + 2\pi \alpha' F]}\} \]

\[ = -T_{D7} N_f \int d^8 \xi \sqrt{-g} \text{Str}\left(1 + g^{tt} g^{\rho\rho} (F_{\rho0}^3)^2 (\sigma^3)^2 + g^{33} g^{44} (F_{\rho3}^1)^2 (\sigma^1)^2 - c^2 g^{tt} g^{33} (F_{03}^2)^2 (\sigma^2)^2\right) \]

Problem 1: How to evaluate the symmetrized trace of the square root exactly?

Solution: Set commutators zero, set \((\sigma^i)^2 = 1\) inside symmetrized trace.

Problem 2: Non-Abelian DBI-action only known to fourth order in \(\alpha'\).

Solution: Expand square root to fourth order in \(\alpha'\).

(e.g. [Myers et al. hep-th/0611099])
Exercise 1: Derive the background EOMs.

\[ S_{\text{DBI}} = -T_{D7} \int d^8\xi \, \text{Str}\{\sqrt{\det[g + 2\pi \alpha' F]}\} \]

\[ = -T_{D7} N_f \int d^8\xi \sqrt{-g} \text{Str}\sqrt{1 + g^{tt}g^{\rho\rho}(F^3_{\rho0})^2(\sigma^3)^2 + g^{33}g^{44}(F^1_{\rho3})^2(\sigma^1)^2 - c^2 g^{tt}g^{33}(F^2_{03})^2(\sigma^2)^2} \]

\[ F = dA + [A, A] \]

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\[ = -T_{D7} N_f \int d^8 \xi \sqrt{-g} \text{Str} \sqrt{1 + g^{tt} g^{\rho\rho} (F^{3}_\rho)_0^2 (\sigma^3)^2 + g^{33} g^{44} (F^{1}_\rho)_3^2 (\sigma^1)^2 - c^2 g^{tt} g^{33} (F^{2}_{03})^2 (\sigma^2)^2} \]

\[ = -T_{D7} N_f \int d^8 \xi \sqrt{-g} \sqrt{1 + g^{tt} g^{\rho\rho} (\partial_{\rho} A^3_0)^2 + g^{33} g^{44} (\partial_{\rho} A^1_3)^2 - c^2 g^{tt} g^{33} (A^3_0 A^1_3)^2} \]

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(e.g. [Myers et al. hep-th/0611099])
Exercise 1: Derive the background EOMs.

\[
S_{\text{DBI}} = -T_{D7} \int d^8 \xi \text{Str}\{ \sqrt{\det[g + 2\pi \alpha' F]} \}
\]

\[
= -T_{D7} N_f \int d^8 \xi \sqrt{-g} \text{Str} \sqrt{1 + g^{\rho \rho} (F^3_\rho_0)^2 (\sigma^3)^2 + g^{33} g^{44} (F^1_\rho_3)^2 (\sigma^1)^2 - c^2 g^{tt} g^{33} (F^2_{03})^2 (\sigma^2)^2}
\]

\[
= -T_{D7} N_f \int d^8 \xi \sqrt{-g} \sqrt{1 + g^{tt} g^{\rho \rho} (\partial_{\rho} A^3_0)^2 + g^{33} g^{44} (\partial_{\rho} A^1_3)^2 - c^2 g^{tt} g^{33} (A^3_0 A^1_3)^2}
\]

\[\implies \text{equations of motion}\]

\[
F = dA + [A, A]
\]

\[
= \partial_{\rho} A^3_0 \tau^3 \, d\rho \wedge dt + \partial_{\rho} A^1_3 \tau^1 \, d\rho \wedge dz + i \epsilon^{231} A^3_0 A^1_3 \, dt \wedge dz
\]

(e.g. [Myers et al. hep-th/0611099])
Exercise 1: Derive the background EOMs.

\[ S_{\text{DBI}} = -T_{D7} \int d^8 \xi \text{Str}\{\sqrt{\det[g + 2\pi \alpha' F]}\} \]

\[ = -T_{D7} N_f \int d^8 \xi \sqrt{-g} \text{Str} \sqrt{1 + g^{tt} g^{\rho\rho} (F_{\rho 0}^3)^2 (\sigma^3)^2 + g^{33} g^{44} (F_{\rho 3}^1)^2 (\sigma^1)^2 - c^2 g^{tt} g^{33} (A_{0}^3 A_{3}^1)^2} \]

\[ \Rightarrow \text{equations of motion} \]

\[ F = dA + [A, A] \]

\[ = \partial_\rho A_{\rho 0}^3 \tau^3 \, d\rho \wedge dt + \partial_\rho A_{\rho 3}^1 \tau^1 \, d\rho \wedge dz + i \epsilon^{231} A_{0}^3 A_{3}^1 \, dt \wedge dz \]

Legendre transformed:

\[ \hat{S}_{\text{DBI}} = -N_f T_{D7} \int d^8 \xi \sqrt{-g} \left[ (1 - \frac{2c^2 (A_{0}^3 A_{3}^1)^2}{\pi^2 \rho^4 f^2})(1 + \frac{8(p_{0}^3)^2}{\rho^6 f^3} - \frac{8(p_{3}^1)^2}{\rho^6 f^2}) \right]^{\frac{1}{2}} \]

\[ = \text{factor} \times \text{grand potential} \]

(e.g. [Myers et al. hep-th/0611099])
IV. Thermodynamics

Grand potential:

Specific heat:

Order parameter:

Critical exponent is 1/2.

Vanishes linearly.

SC density:

Vanishes linearly.
IV. Background field configuration

Gravity fields:

\[ A_0^3 \rightarrow \mu_0^3 \]

Conjugate momenta:

\[ p_0^3 \rightarrow d_0^3 \]

\[ A_3^1 \rightarrow \rho \]

\[ p_3^1 \rightarrow d_3^1 \]
Exercise 2: Derive the fluctuation EOMs.

\[ A = A_0^3 \int dt \tau^3 + A_3^1 \int dz \tau^1 + \tilde{A}_{\alpha^m} \int dx^m \tau^\alpha \]

Linearized fluctuation equation of motion:

\[ (\tilde{A}_2')'' + \frac{\partial \rho H}{H} (\tilde{A}_2') - \left[ \frac{4 \rho_4}{R^4} \left( \frac{\mathcal{G}^{33}}{\mathcal{G}^{44}} (m_3^1)^2 + \frac{\mathcal{G}^{00}}{\mathcal{G}^{44}} w^2 \right) + 16 \frac{\partial \rho \left( \frac{H}{\rho^4 f^2} A_0^3 (\partial \rho A_0^3) (m_3^1)^2 \right)}{H \left( 1 - \frac{2c^2}{\pi^2 \rho^4 f^2} (A_3^1 \tilde{A}_0^3)^2 \right) \right] \tilde{A}_2^3 = 0 \]

\[ m_3^1 = \frac{c}{2 \sqrt{2\pi}} A_3^1 \]
IV. Stability

Poles of $X, Y, \tilde{A}_2^3$ in complex plane:

$n = 1$
$n = 0$
$n = 0$
IV. Conductivity

Conductivity:

\[ \sigma = \frac{J}{E} = \frac{A^{(2)}}{\partial_t A^{(0)}} \sim \frac{i A^{(2)}}{\omega A^{(0)}} = -\frac{i \rho^3 A'}{\omega 2 A} = \frac{i}{\omega} G^{\text{ret}}(\omega, q = 0) \]

with flavorelectric current \( J_m \leftrightarrow A_m \in U(1) \)

Only the first of these peaks was seen to second order in \( F \).

[Basu et al. 0810.3970]

Our expansion to fourth order shows all peaks \textbf{at zero quark mass}!

Peaks are higher order effect in \( F \).
IV. Higgs mechanism & Meissner effect

Peaks at finite mass:

- Peaks in conductivity/spectral function approach
- SUSY vector meson spectrum at large quark mass.

Bulk Higgs Mechanism generates meson mass

Finite magnetic field:

- Add background field component: $H_3^3 = F_{12}^3 = \partial_1 A_2^3$

Induced currents in SC phase with H
IV. String Theory Picture

7-7 strings generate $A^1_3$ i.e. they break the U(1) and are thus dual to Cooper pairs.

$E^3_\rho = F^3_{\rho 0} = \partial_\rho A^3_0$

$B^1_{\rho 3} = F^1_{\rho 3} = \partial_\rho A^1_3$

$E^2_3 = F^2_{30} = A^1_3 A^3_0$

charged horizon (3-7 strings generate $A^3_0$)
IV. Discussion

✓ Rich strong coupling phenomenology (QGP)
  • Resonances are vector mesons (analogous to rho-meson)
  • Vector mesons survive deconfinement

✓ Top-down approach: direct identification of d.o.f.
✓ Energy gap, Meissner effect, Higgs mechanism
✓ Stringy picture of pairing mechanism

☐ Critical exponents
☐ Speed of second, fourth sound (backreact)
☐ Drag on D7-D7 strings
☐ Fermi surface?
APPENDIX: Embeddings

\[ \tilde{d} = 0.25 \]

\[ \tilde{d} = \frac{10^{-4}}{4} \]
APPENDIX: Parameters

The mass parameter $m$ depending on the parameter $\chi_0$.

\[
\chi_0 = \chi(\rho) \bigg|_{\rho \to \rho_H}
\]

\[
m = \lim_{\rho \to \rho_{bdy}} \rho \chi(\rho) = \frac{2m_{\text{quark}}}{\sqrt{\lambda T}}
\]

Near-boundary expansions:

\[
\chi(\rho) = m \chi_0 + \frac{c}{\rho^3} + \ldots
\]

\[
A_0 = \mu - \frac{1}{\rho^2} \frac{\tilde{d}}{2\pi \alpha'} + \ldots
\]

Other relations:

\[
L(\rho) = \rho \chi(\rho) , \quad \rho = \frac{\rho}{\rho_H}
\]
APPENDIX: Fluctuations

Equation of motion written out:

\[ 0 = \tilde{A}'' + \partial_\rho \ln \left( \frac{1}{8} \tilde{f}^2 f \rho^3 (1 - \chi^2 + \rho^2 \chi'^2)^{3/2} \times \sqrt{1 - \frac{2 \tilde{f}(1 - \chi^2)(\partial_\rho A_0)^2}{f^2(1 - \chi^2 + \rho^2 \chi'^2)}} \right) \tilde{A}' + 8w^2 \frac{\tilde{f}}{f^2} \frac{1 - \chi^2 + \rho^2 \chi'^2}{\rho^4(1 - \chi^2)} \tilde{A} \]

\[ \rho = \frac{\varrho}{\varrho_H}, \quad \tilde{f}(\varrho) = 1 + \frac{\varrho_H^4}{\varrho^4}, \quad f(\varrho) = 1 - \frac{\varrho_H^4}{\varrho^4}, \quad L(\varrho) = \varrho \chi(\varrho), \quad w = \frac{\omega}{2\pi T} \]

The D7-brane embedding \( \chi(\rho) \) and gauge field component \( A_0(\rho) \) are given numerically.

\[ \partial_\rho A_t = 2\tilde{d} \frac{f^2 \sqrt{1 - \chi^2 + \rho^2 \chi'^2}}{\sqrt{\tilde{f}(1 - \chi^2)[\rho^6 \tilde{f}^3(1 - \chi^2)^3 + 8\tilde{d}^2]}} \]

\[ A_0 \equiv A_t \]
### APPENDIX: Extension of the correspondence

<table>
<thead>
<tr>
<th>Universality</th>
<th>Original AdS/CFT correspondence</th>
<th>AdS Schwarzschild black hole (D3/D7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge</td>
<td>QCD $N = 4$ SuperYangMills</td>
<td>thermal Yang-Mills Type II Sugra in AdS Schwarzschild b.h.</td>
</tr>
<tr>
<td>Gravity</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>Gauge theory</td>
<td>non-conf. ✓</td>
<td></td>
</tr>
<tr>
<td>symmetry</td>
<td>non-SUSY ✓</td>
<td></td>
</tr>
<tr>
<td>Relations</td>
<td>$g_{YM}^2 = g_s$</td>
<td>$R^4 (\alpha')^2 = 4\pi N_c g_s \equiv \lambda$</td>
</tr>
</tbody>
</table>

$T \leftrightarrow \text{horizon}$

$$\mu_B, \mu_I \leftrightarrow A_0(\rho)$$