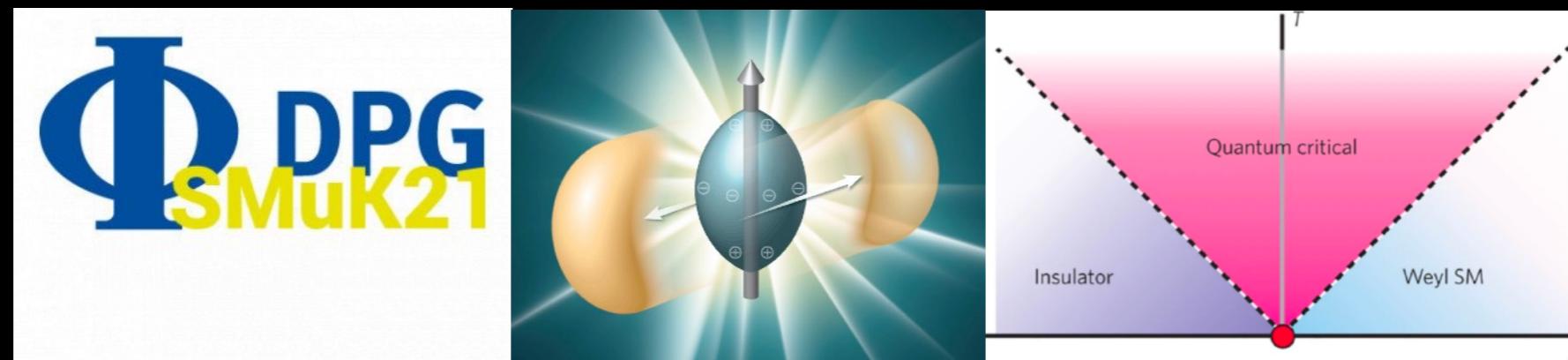


Inverted c-functions in thermal states

Matthias Kaminski
University of Alabama

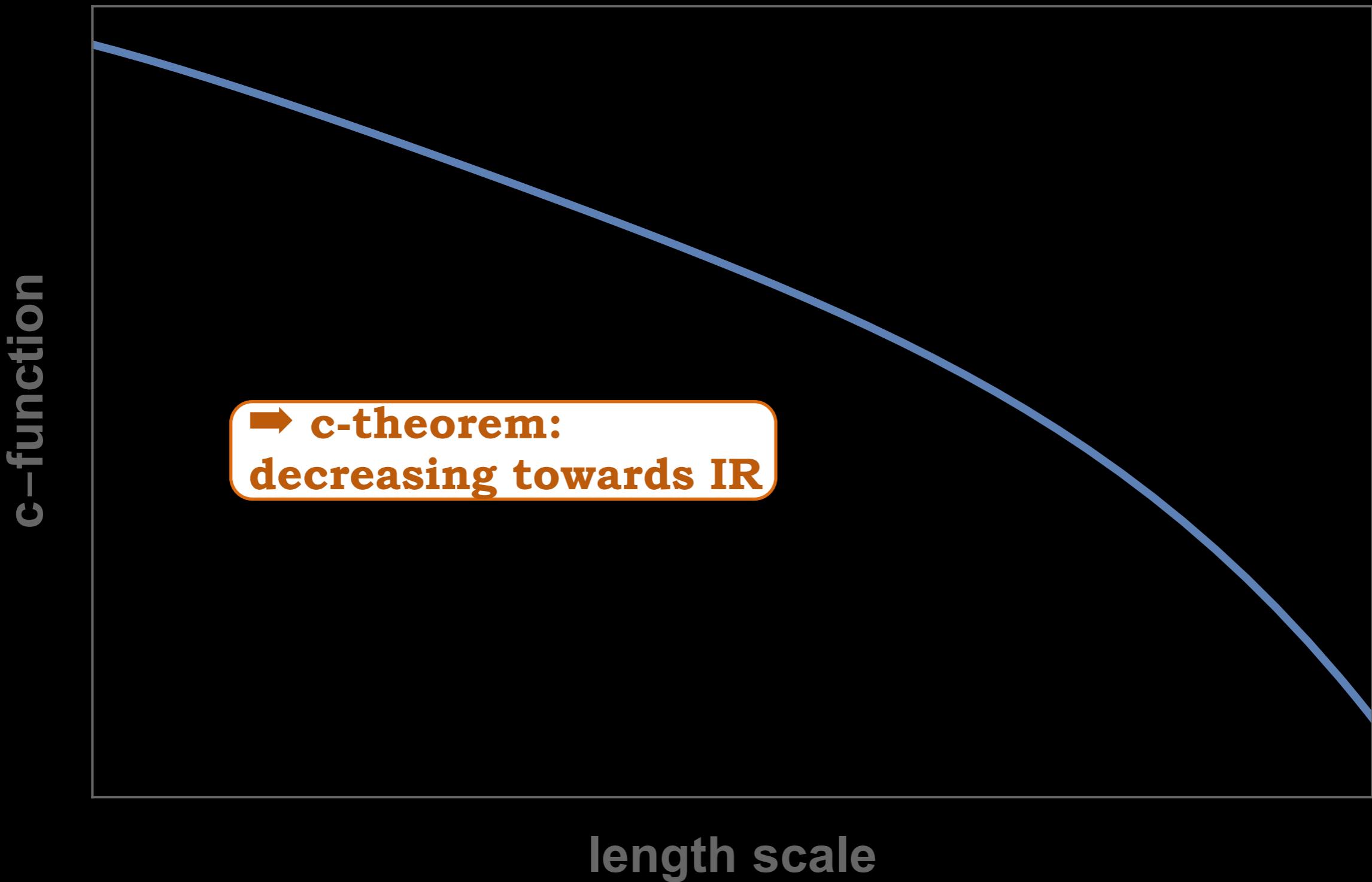
at DPG-Tagung der Sektion Materie und Kosmos, September 2nd, 2021



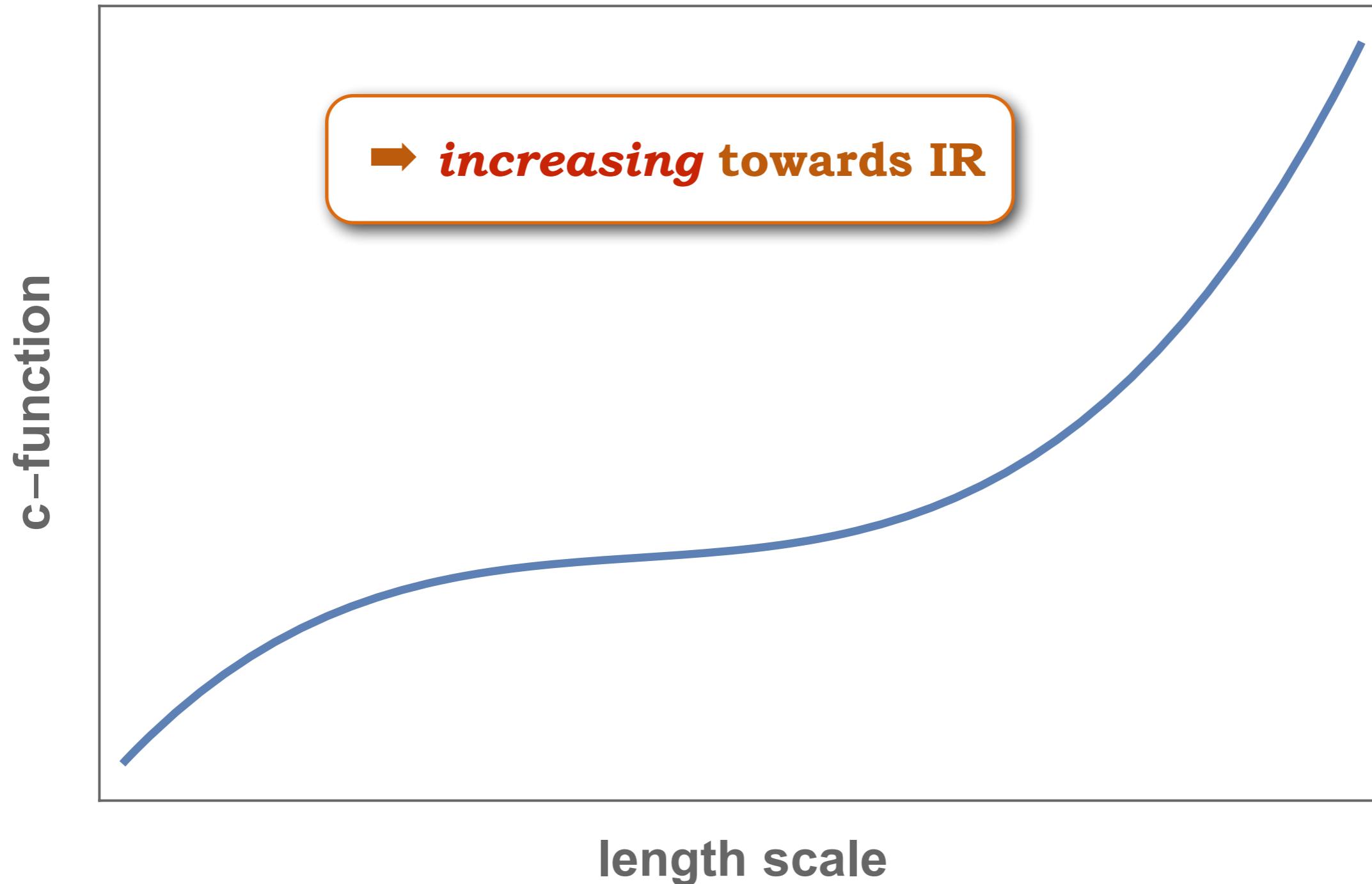
[Cartwright, Kaminski; arXiv: 2107.12409]



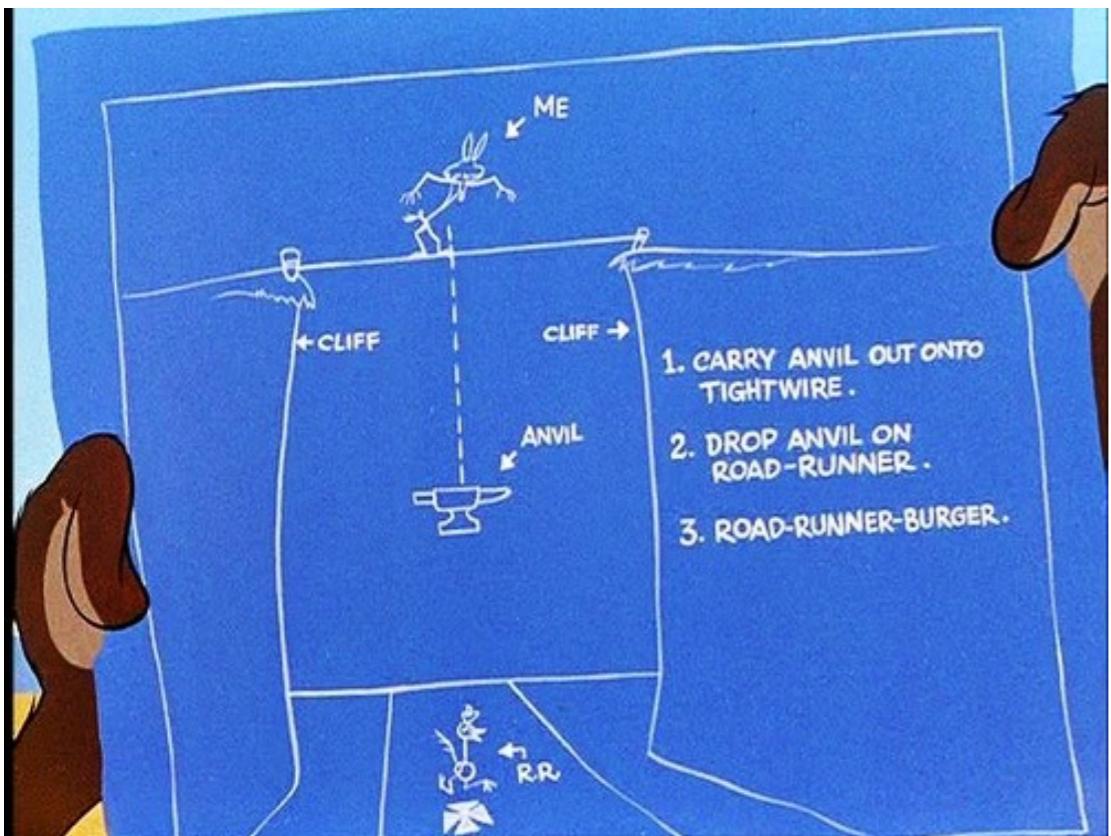
Entropic c-function in vacuum state



Entropic c-function in *thermal* state



Roadmap



1. c-function & entanglement entropy
2. Gauge/gravity correspondence
3. $N=4$ Super-Yang-Mills (CFT)
 - c-function in vacuum state
 - c-function in thermal states
 - effect of the chiral anomaly
4. Discussion

1.1 c-Function

Zamolodchikov's
c-theorem in 2D

$$(c)_{\text{UV}} \geq (c)_{\text{IR}}$$

energy-momentum tensor: $\langle T^a_a \rangle = -\frac{c}{12}R$
trace anomaly

[Zamolodchikov; JETP Lett.(1986)]

c-theorem in 4D
(the a-theorem)

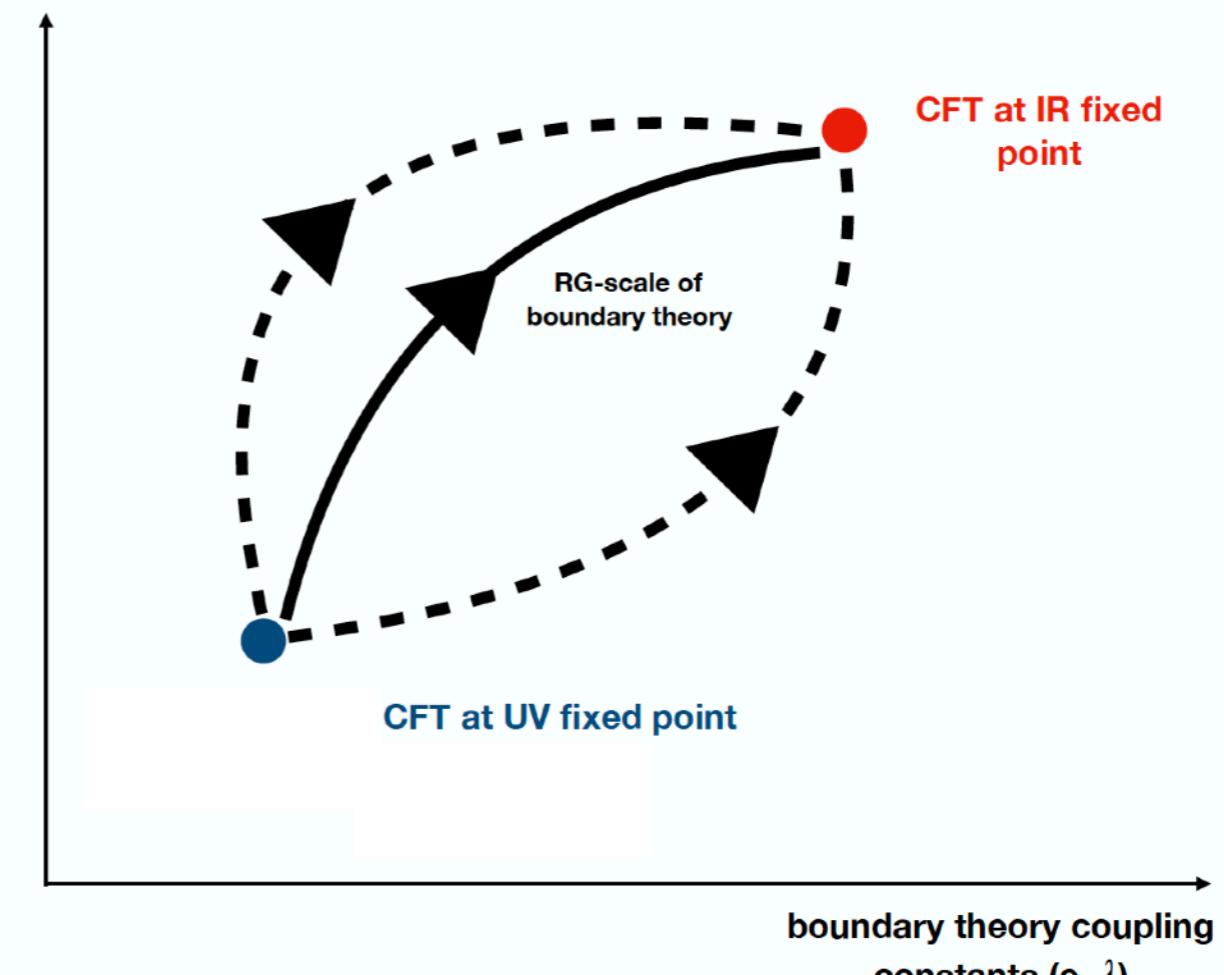
$$(a_4)_{\text{UV}} - (a_4)_{\text{IR}}$$

$$\langle T^b_b \rangle = \frac{c_{TT}}{16\pi^2}C^2 - \frac{a_4}{16\pi^2}\mathcal{E} - \frac{1}{4}F^2$$

[Komargodski, Schwimmer; (2011)]

[Cardy; Phys.Lett.B(1988)]

[Osborn; Phys.Lett.B(1988)]



- IR/UV fixed points: c-function equals central charge of IR/UV CFT
- c-function measures degrees of freedom
- take a CFT: c-function constant

1.2 *Entropic* c-function

2D

$$c_2 = 3\ell \frac{\delta S_a}{\delta \ell} \quad \begin{matrix} \text{entanglement} \\ \text{entropy} \end{matrix}$$

[Casini,Huerta; Phys.Lett.B (2004)]

ℓ : length scale (inverse energy scale)

4D

$$a_4 = \beta_4 \frac{\ell^3}{H^2} \frac{\partial S_a}{\partial \ell}$$

[Nishioka,Takayanagi; JHEP (2007)]

[Myers,Sinha; JHEP (2011)]

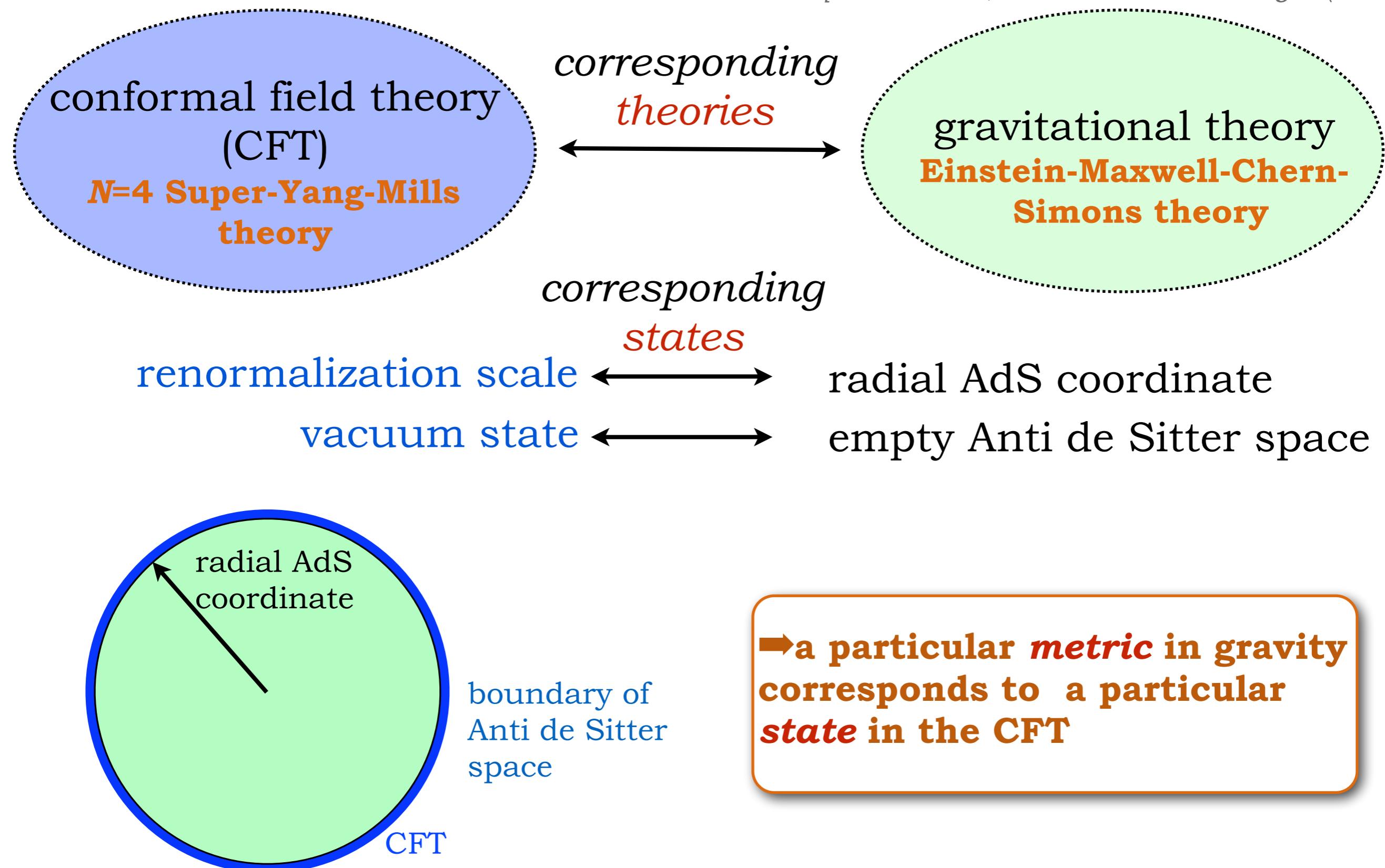
H : IR-regulator

β_4 : known constant

→ c-function defined by
entanglement entropy

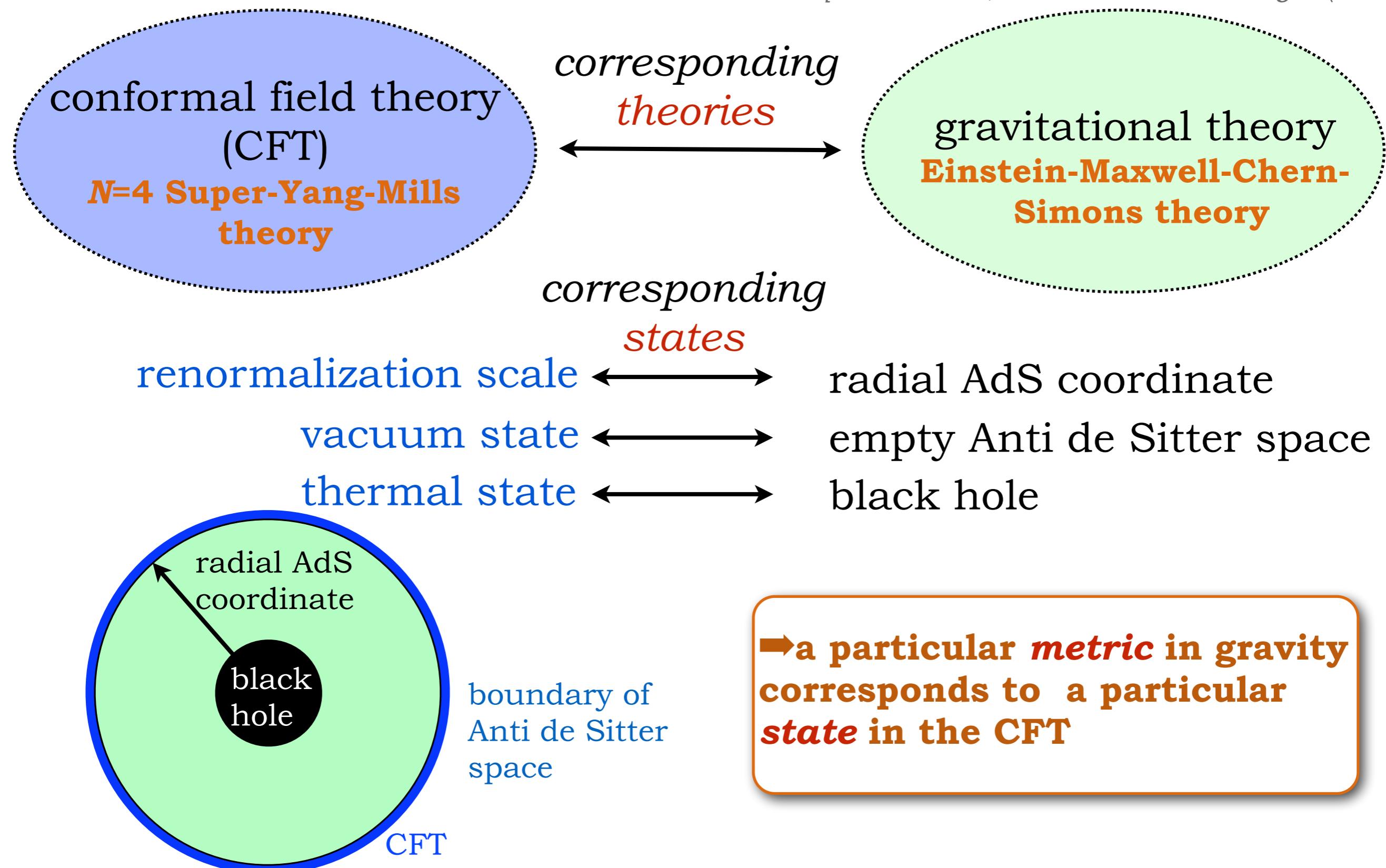
2. Holography: *theories & states*

[Maldacena; *Adv.Theor.Math.Phys.* (1998)]



2. Holography: *theories & states*

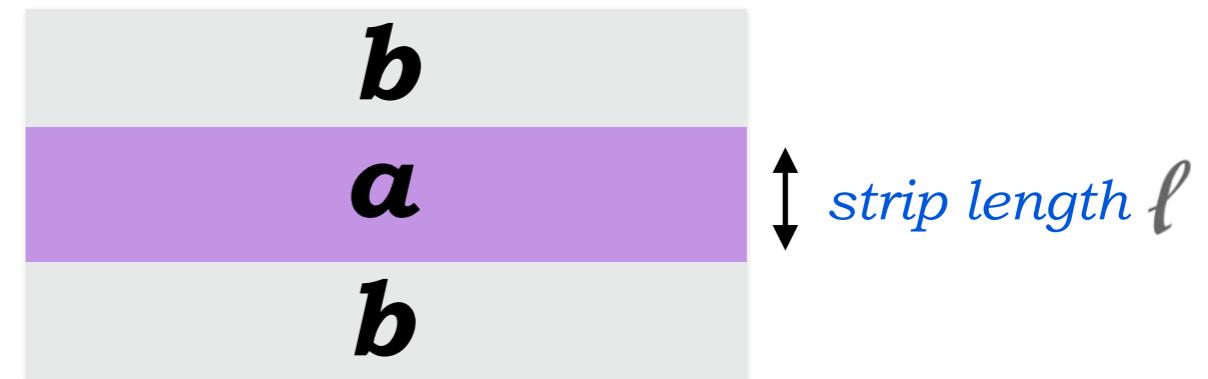
[Maldacena; *Adv.Theor.Math.Phys.* (1998)]



2.1 Holographic entanglement entropy

Definition

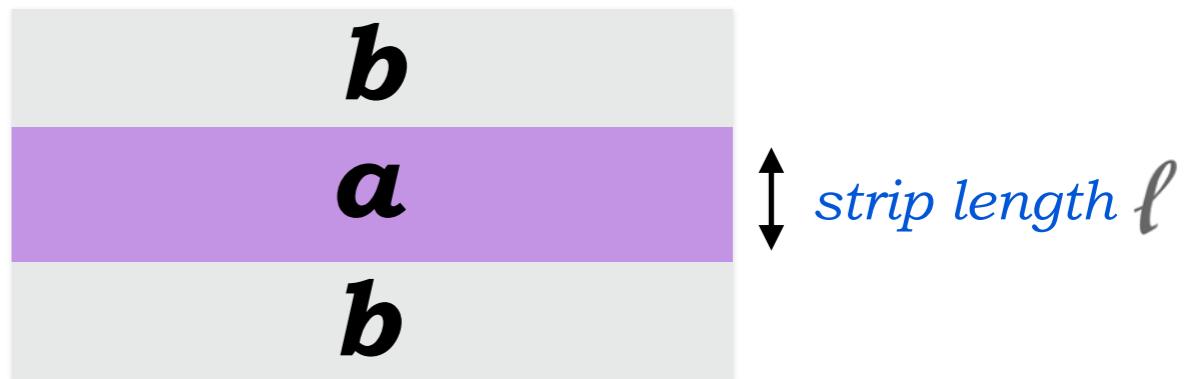
$$S_a = -\text{Tr} \rho_a \log \rho_a, \quad \rho_a = \text{Tr}_b |\psi\rangle \langle \psi|$$



2.1 Holographic entanglement entropy

Definition

$$S_a = -\text{Tr} \rho_a \log \rho_a, \quad \rho_a = \text{Tr}_b |\psi\rangle \langle \psi|$$

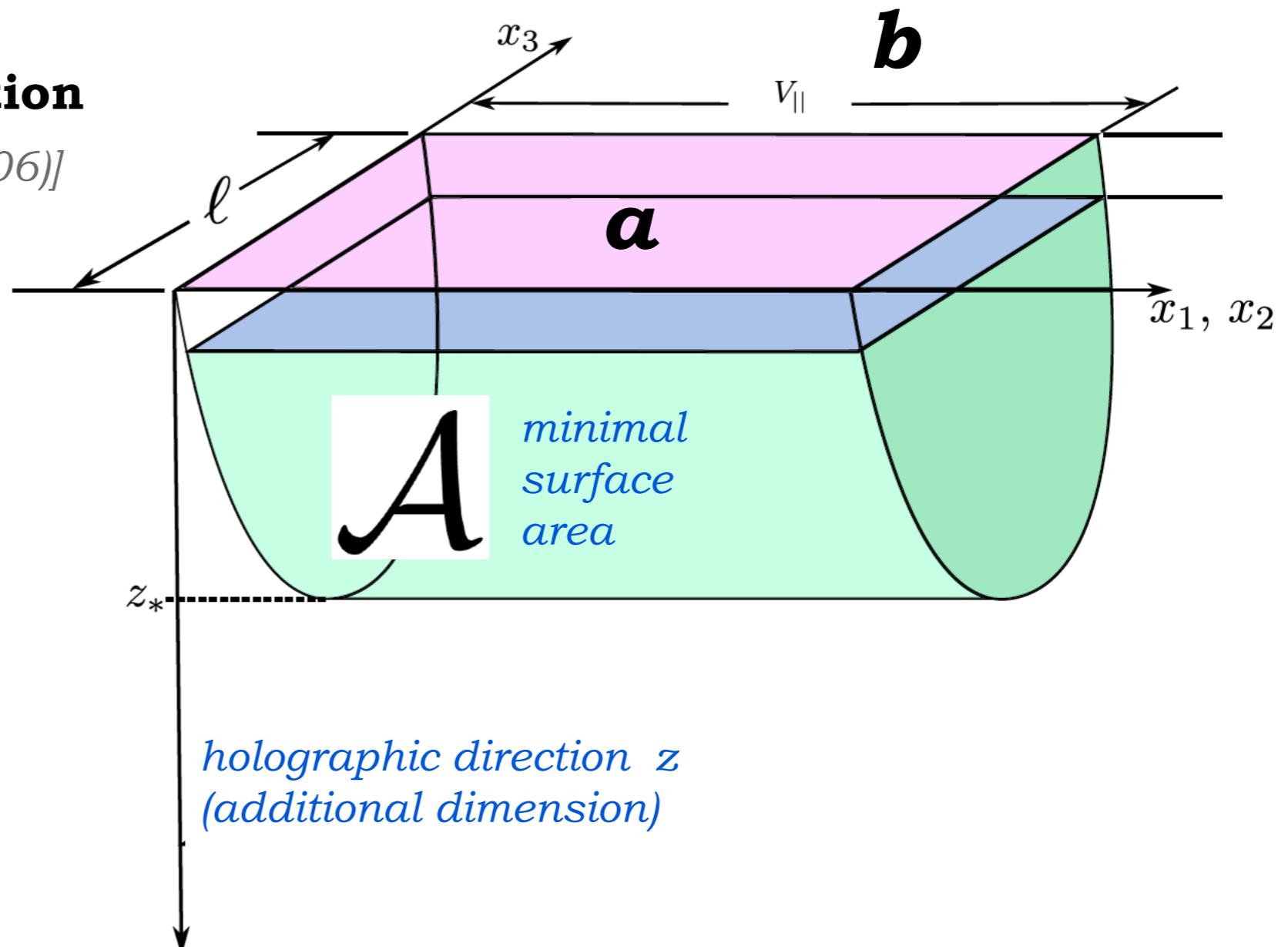


Holographically dual definition

[Ryu, Takayanagi; JHEP (2006)]

$$S_a = \frac{1}{4G_5} \mathcal{A}$$

G_5 is the 5-dimensional gravitational constant of Anti de Sitter spacetime



2.2 Gravity dual to $N=4$ SYM theory with magnetic field



Einstein-Maxwell-Chern-Simons action

$$S_{grav} = \frac{1}{2\kappa^2} \left[\int_{\mathcal{M}} d^5x \sqrt{-g} \left(R + \frac{12}{L^2} - \frac{1}{4} F_{mn} F^{mn} \right) - \frac{\gamma}{6} \int_{\mathcal{M}} A \wedge F \wedge F \right]$$

5-dimensional Einstein-Maxwell action encodes $N=4$ Super-Yang-Mills theory with axial **$U(1)$ gauge symmetry**

5-dimensional Chern-Simons term **encodes chiral anomaly**

Einstein-Maxwell equations

$$R_{\mu\nu} + 4g_{\mu\nu} = \frac{1}{2} \left(F_{\mu\alpha} F_{\nu}^{\alpha} - \frac{1}{6} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right)$$

$$\nabla_{\mu} F^{\mu\nu} = -\frac{\gamma}{8\sqrt{-g}} \epsilon^{\nu\alpha\beta\lambda\sigma} F_{\alpha\beta} F_{\lambda\sigma} .$$

Solution: charged magnetic black brane metric

[D'Hoker, Kraus; JHEP (2010)]

- magnetic extension of a (charged) Reissner-Nordstrom black brane

$$ds^2 = \frac{1}{z^2} \left(\frac{dz^2}{U(z)} - U(z) dt^2 + v(z)^2 (dx_1^2 + dx_2^2) + w(z)^2 (dx_3^2 + c(z) dt)^2 \right)$$

with numerically known solutions for U, v, w, c

2.3 Gravitational calculation

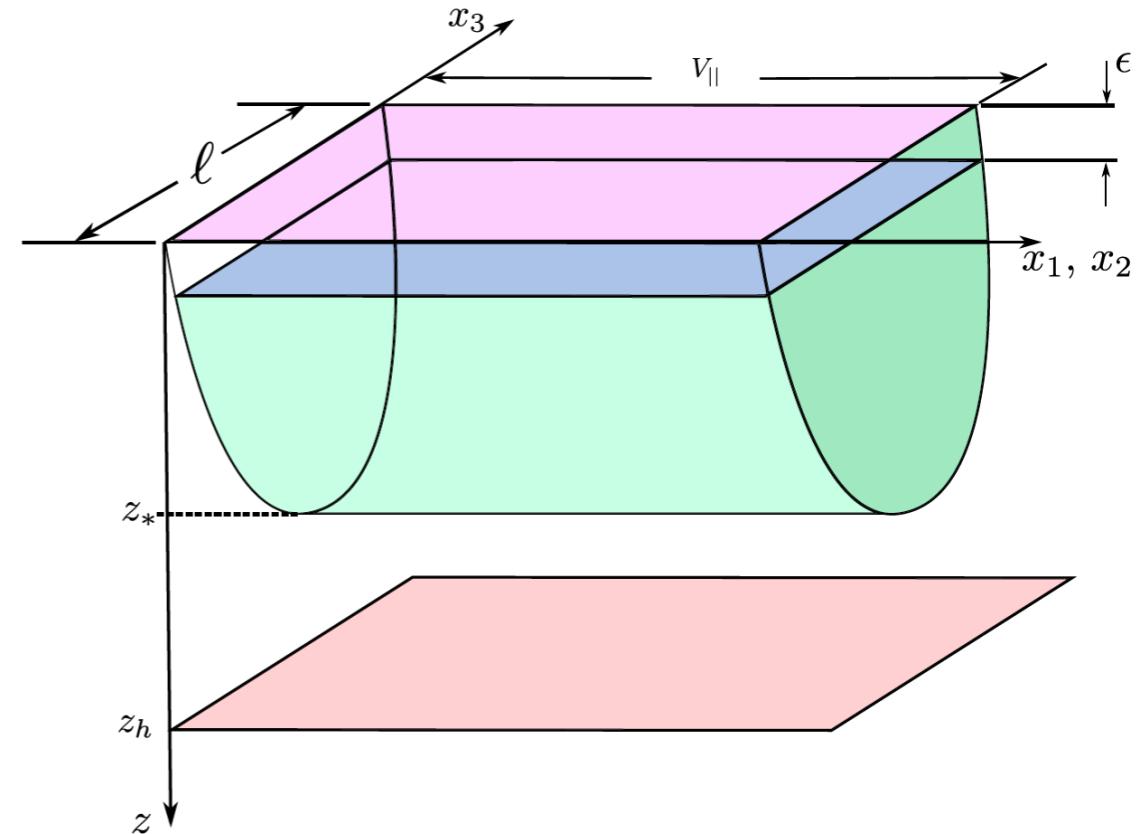


→ calculate a *geodesic* in conformally deformed AdS metric

	Transverse	Longitudinal
Embedding Coordinates	$\chi^\mu = (z(\sigma), t(\sigma), x_1(\sigma), x_2, x_3)$	$\chi^\mu = (z(\sigma), t(\sigma), x_1, x_2, x_3(\sigma))$
Surface Coordinates	$\sigma^i = (\sigma, x_2, x_3)$	$\sigma^i = (\sigma, x_1, x_2)$

Recall:

$$S_a = \frac{1}{4G_5} \mathcal{A}$$



Entanglement entropy

$$S_a = \frac{1}{4G_5} V_{\parallel} \int d\sigma \sqrt{\frac{v(z(\sigma))^4 \left(w(z(\sigma))^2 x'_3(\sigma)^2 + \frac{z'(\sigma)^2}{U(z(\sigma))} \right)}{z(\sigma)^6}}$$

$$V_{\parallel} = \int_a^b \int_{-a}^b dx_1 dx_3$$

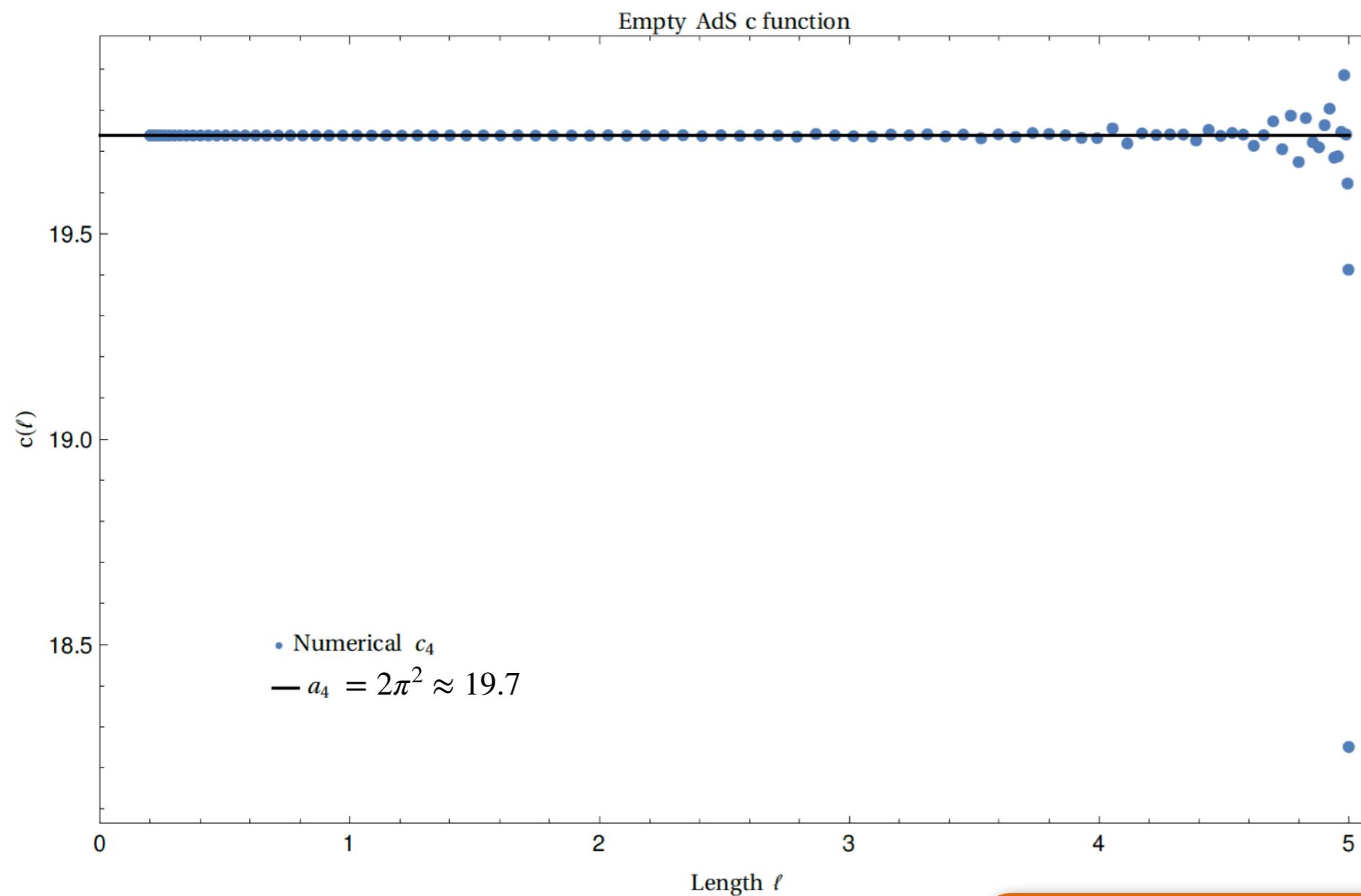
minimal surface area \mathcal{A}

Reminder: metric is

$$ds^2 = \frac{1}{z^2} \left(\frac{dz^2}{U(z)} - U(z)dt^2 + v(z)^2 (dx_1^2 + dx_2^2) + w(z)^2 (dx_3^2 + c(z)dt)^2 \right)$$

3.1 Entropic c-function *in N=4 SYM vacuum state*

[Cartwright, Kaminski; arXiv: 2107.12409]

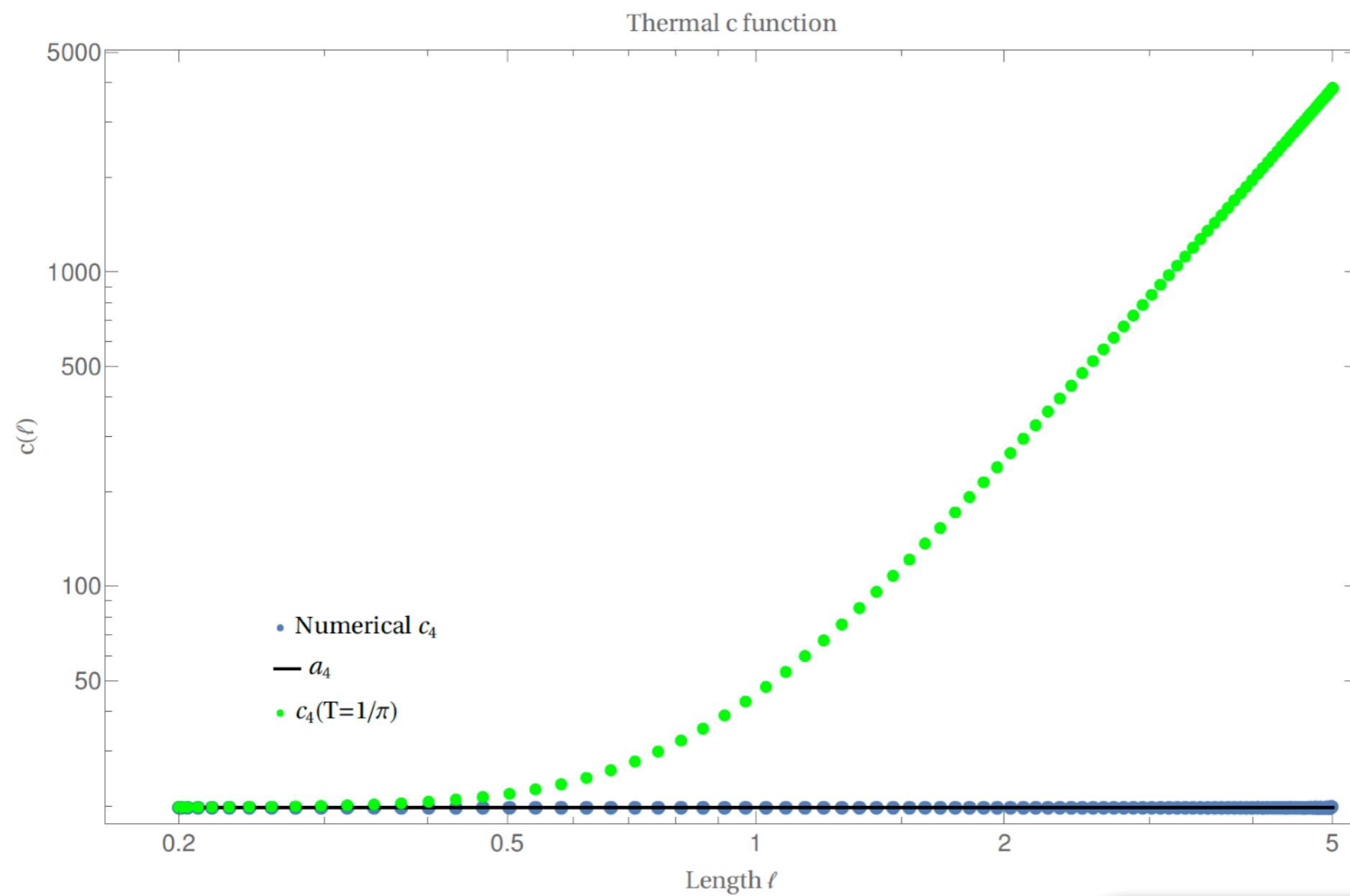


**zero temperature,
no magnetic field,
vanishing charge**

→ c-function at all scales
equal to central charge of
N=4 SYM, which is a CFT

3.2 Entropic c-Function *increases* in *thermal* state

[Cartwright, Kaminski; arXiv: 2107.12409]

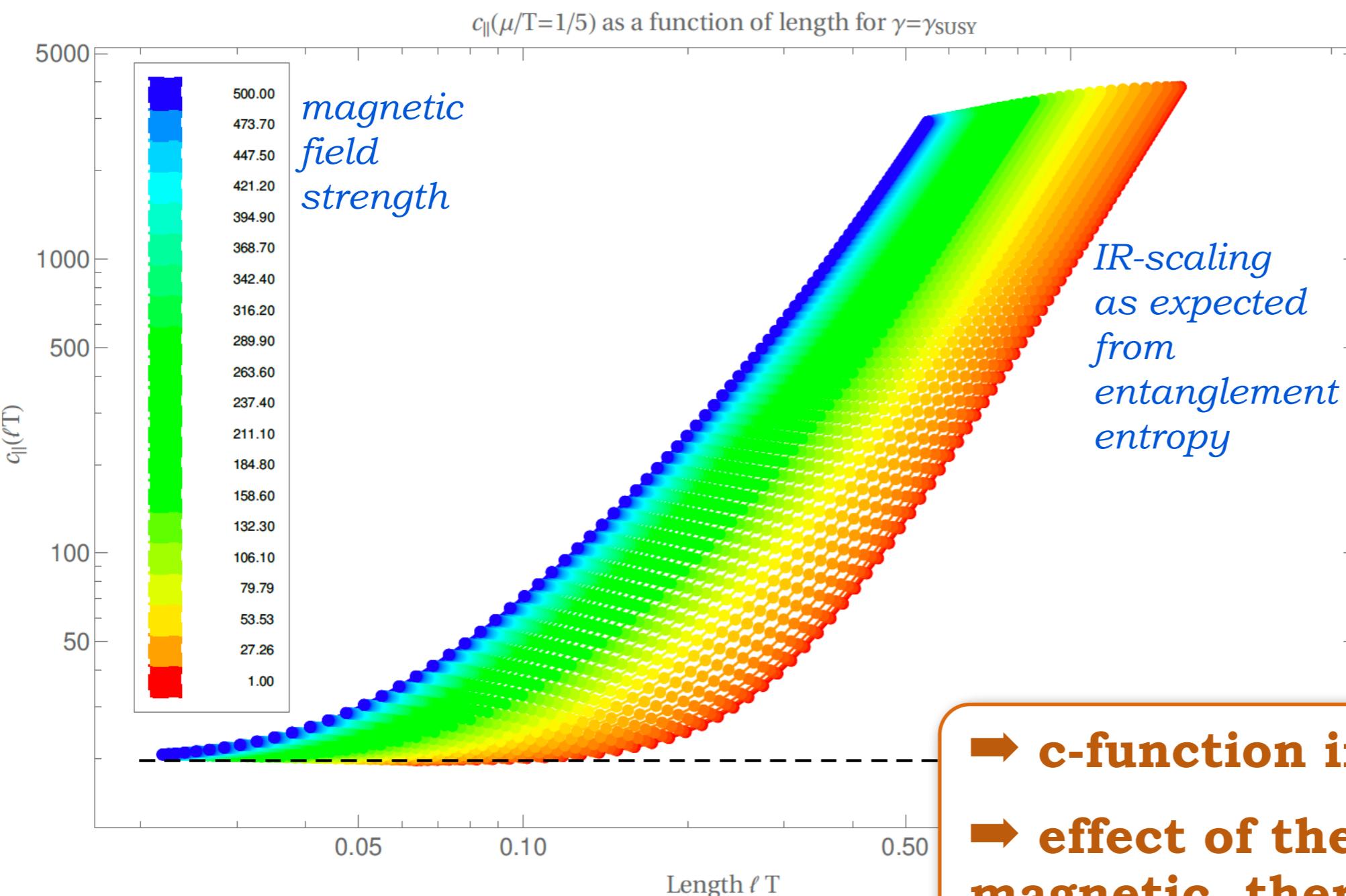


**nonzero temperature,
no magnetic field,
vanishing charge**

- ➡ c-function increases
- ➡ effect of the thermal state

3.3 Entropic c-Function *increases* in *thermal* state

[Cartwright, Kaminski; arXiv: 2107.12409]

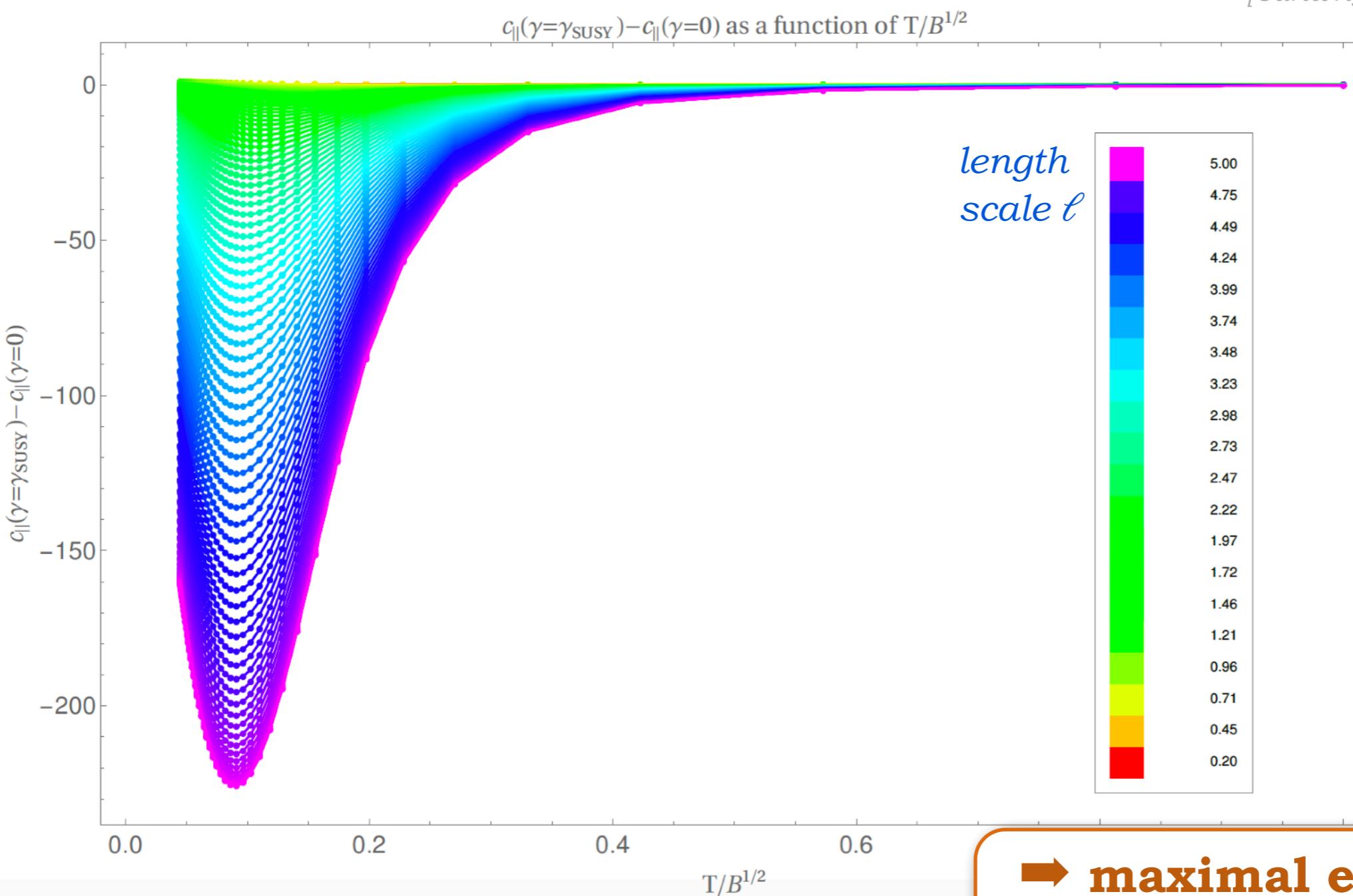


now with temperature, magnetic field, charge, chiral anomaly

- ➡ c-function increases
- ➡ effect of the charged, magnetic, thermal state
- ➡ IR limit: thermal entropy
- ➡ proposal: measure of occupation number

3.4 Effect of the chiral anomaly

[Cartwright, Kaminski; arXiv: 2107.12409]

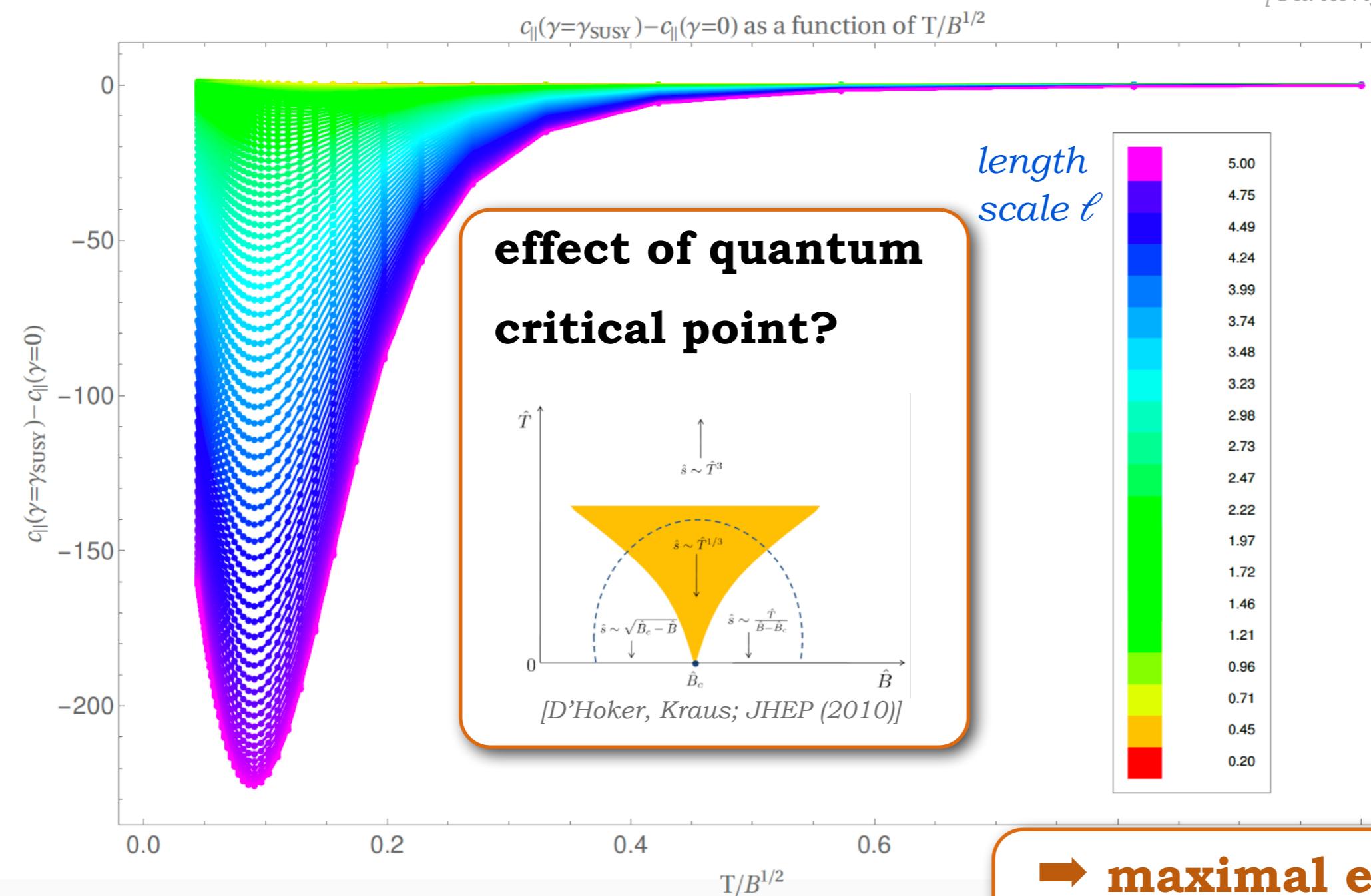


**now with
temperature,
magnetic field,
charge,
chiral anomaly**

→ maximal effect at 0.1
(thermal entropy has no
maximum)

3.4 Effect of the chiral anomaly

[Cartwright, Kaminski; arXiv: 2107.12409]

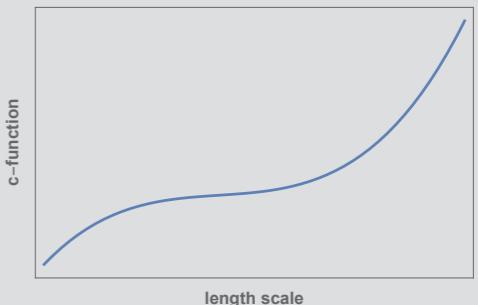


→ maximal effect at 0.1
(thermal entropy has no maximum)

4. Discussion

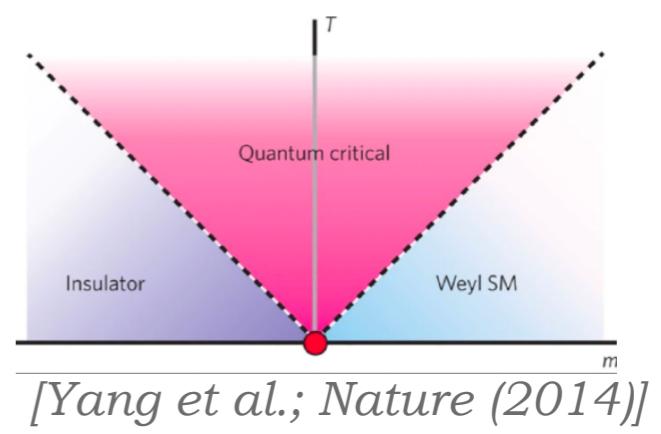
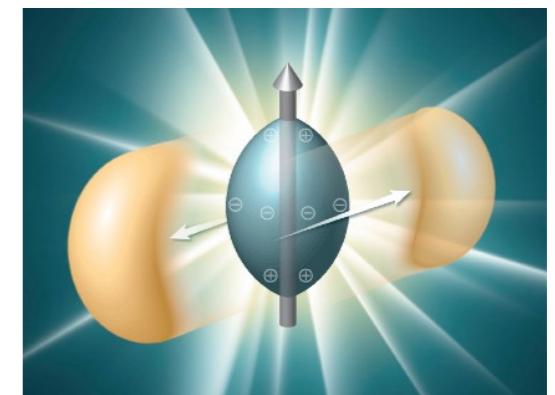
c-function

- **entropic** c-function defined from entanglement entropy
- entropic c-function in thermal states **increases** towards low energies (IR)
- proportional to **thermal** entropy in IR-limit (analytic result)
- **peaked** at intermediate scale of $B/T^2 \approx 0.1$



Applications

- heavy-ion-collisions: **measure for thermalization** via distribution of states
- quantum critical points (QCP): **detect QCP** via change in scaling behavior



[Yang et al.; Nature (2014)]

Collaborator on this project



Thank you for listening!

**University of
Alabama,
Tuscaloosa, USA**



Casey Cartwright

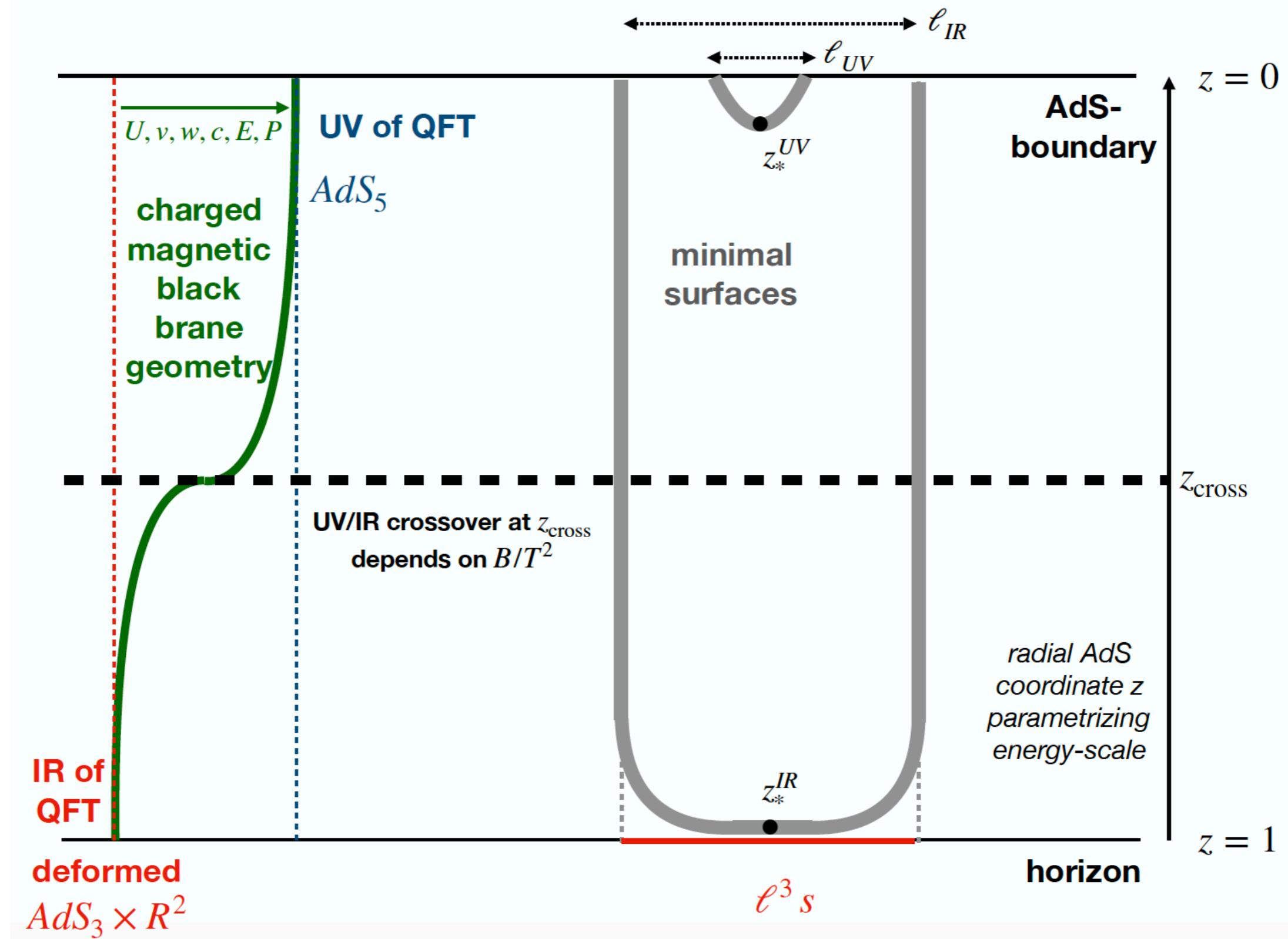
(PhD expected in May 2022)

Publications

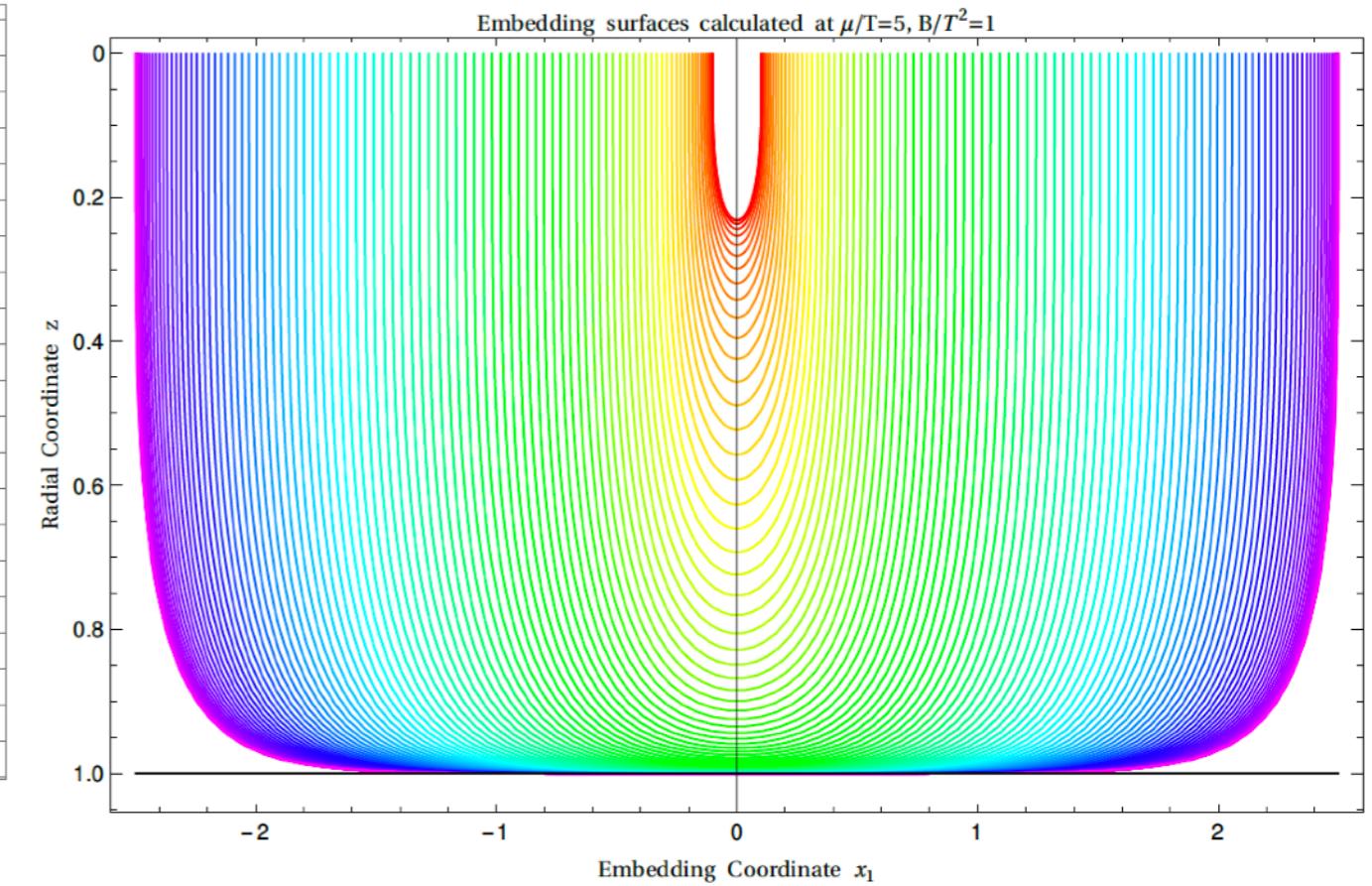
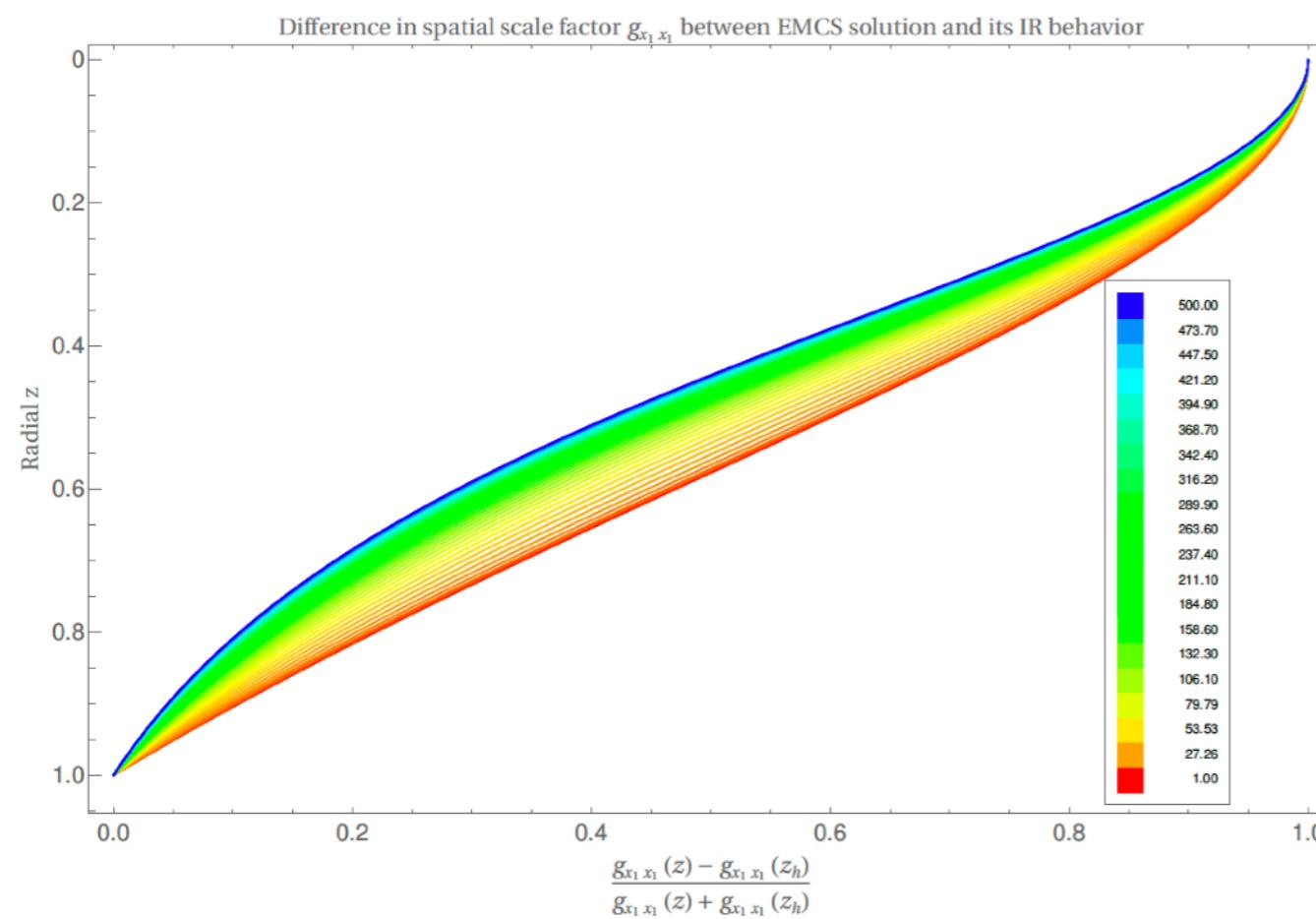
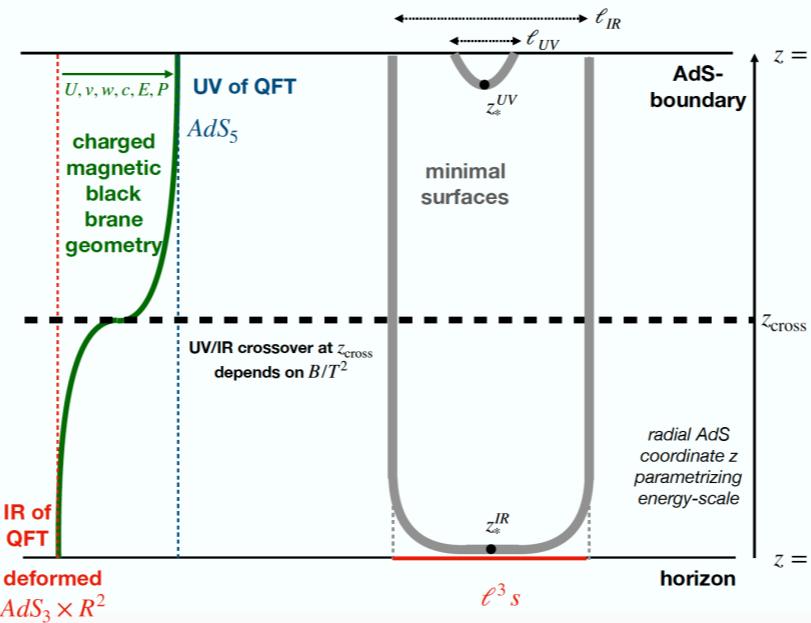
Homepage

APPENDIX

Schematic picture: probing energy scales

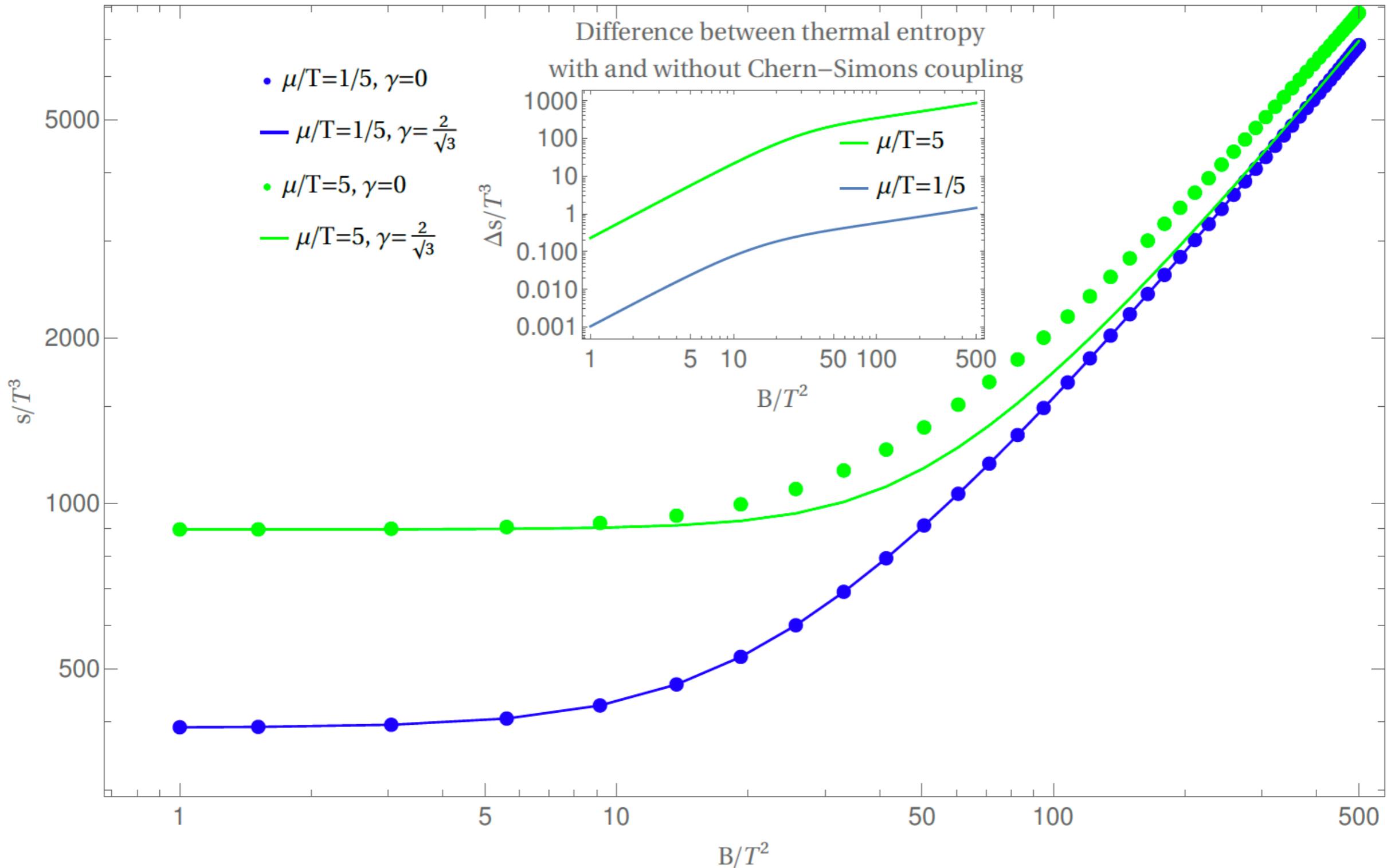


Numerical data confirming schematic picture



Thermal entropy

Thermal entropy density in the Einstein–Maxwell and Einstein–Maxwell–Chern–Simons theory



c-function proportional to thermal entropy in IR

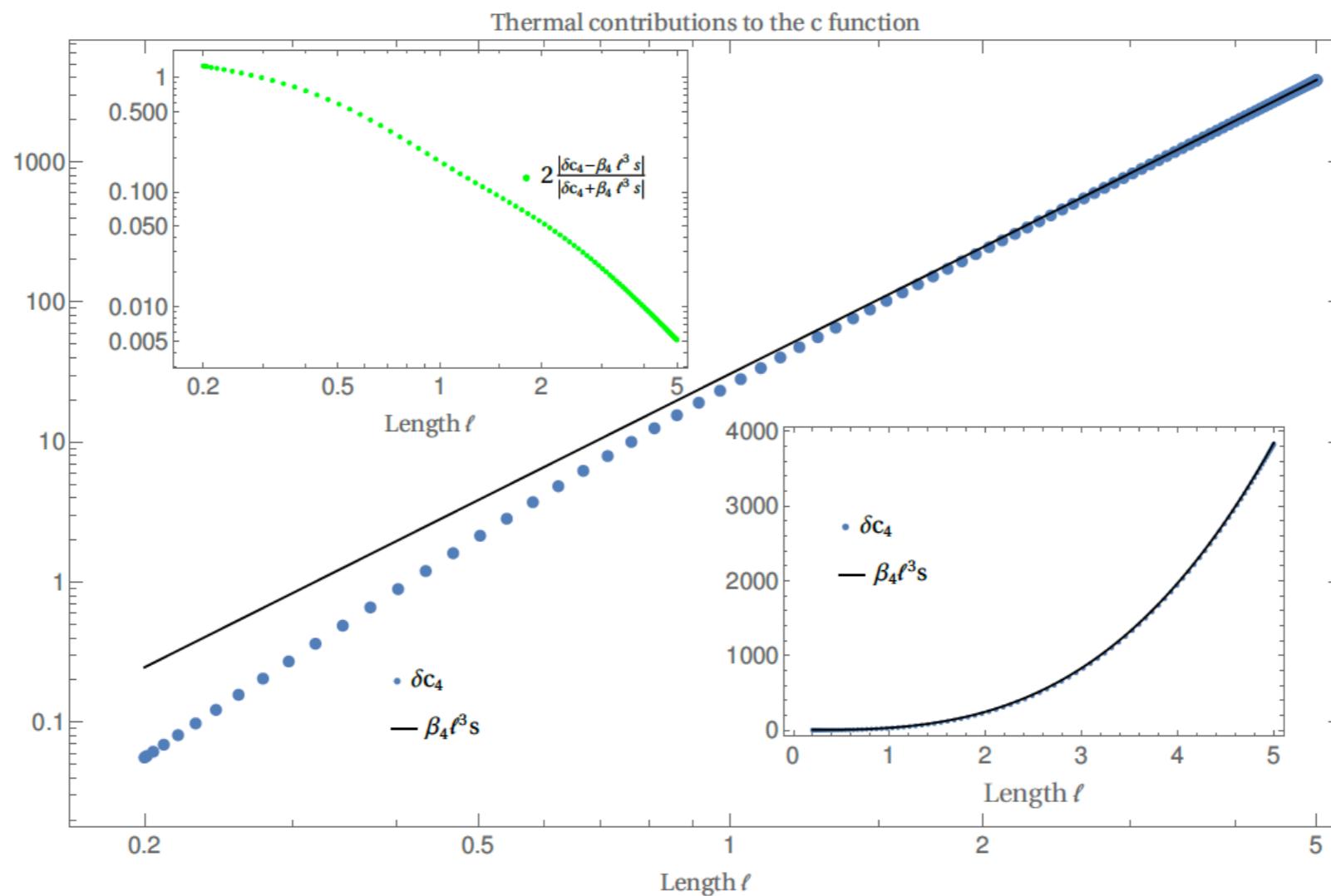


Figure 13: The deviation δc_4 of the thermal c-function, evaluated in the thermal state dual to the *AdS* Schwarzschild black brane, from the central charge a_4 , displayed as the blue points. The thermal subtraction $\beta_4 \ell^3 s$ is displayed as a black line. Its behavior precisely matches δc_4 in the IR. The bottom right inset graphic displays the same information only not in a log-log scale, the top left inset displays the difference as green points on a log-log scale.

Effect of chiral anomaly: entanglement entropy

