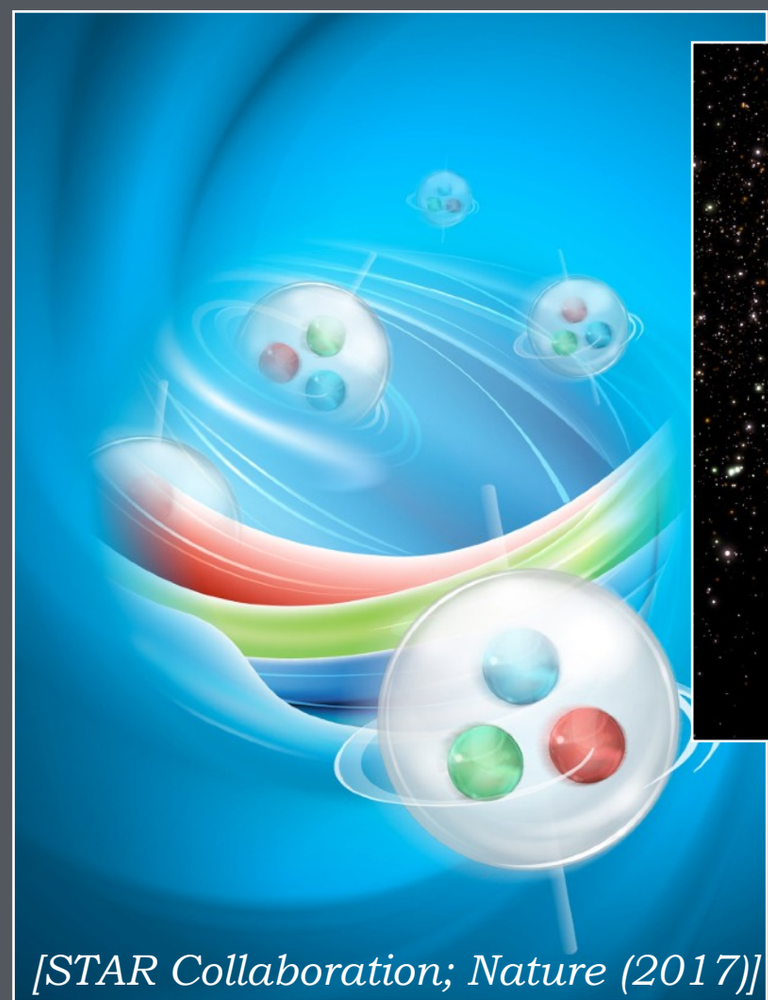
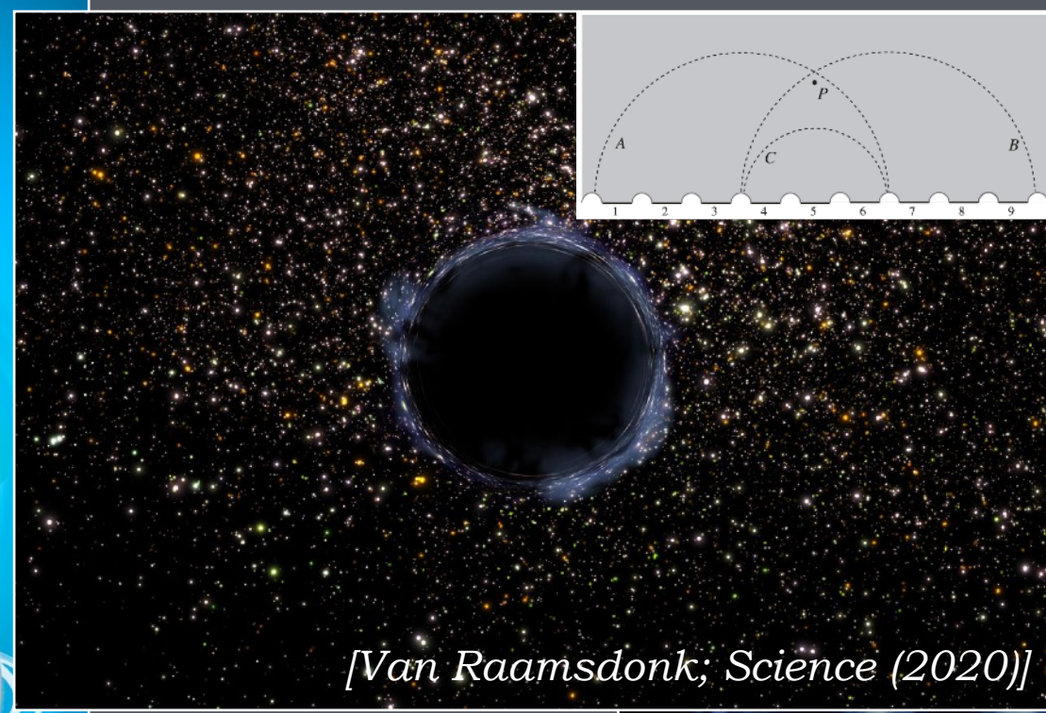
Hydrodynamics, Thermalization & Entanglement from Spacetime Geometry

CTQMAT22 - International Conference on Complexity and Topology in Quantum Matter

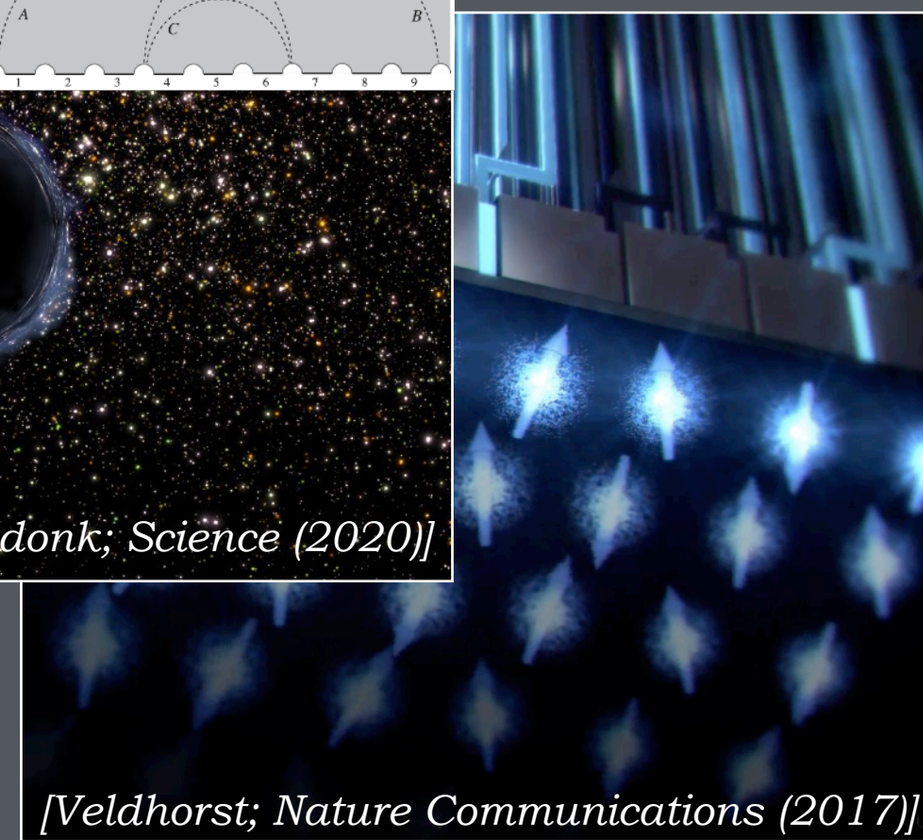
Würzburg, Germany, July 27th, 2022



[STAR Collaboration; Nature (2017)]



[Van Raamsdonk; Science (2020)]



[Veldhorst; Nature Communications (2017)]

Matthias Kaminski
University of Alabama

U.S. DEPARTMENT OF ENERGY

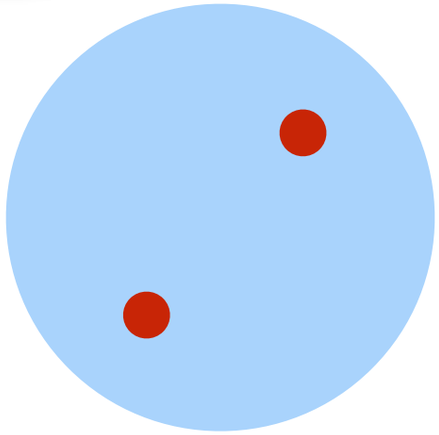
Thermodynamics: entropy increases

Statistical mechanics:

An **isolated system** evolves such that it maximizes its entropy.

Entropy = Non-Information

$S \propto$ # of configurations



time

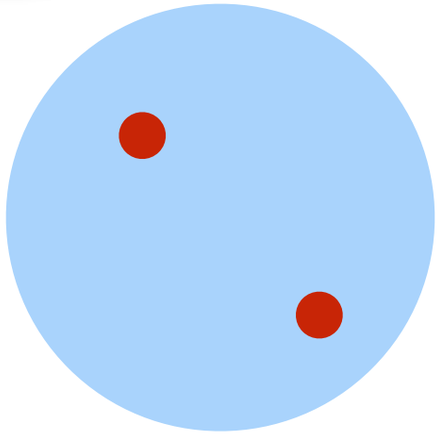
Thermodynamics: entropy increases

Statistical mechanics:

An **isolated system** evolves such that it maximizes its entropy.

Entropy = Non-Information

$S \propto$ # of configurations



time

Definition via density matrix

$$\rho = \sum_i p_i |n_i\rangle\langle n_i|$$

$$S = -\,\mathrm{tr}\left(\rho \log \rho\right)$$

Clash of two fundamental concepts

Statistical mechanics:

An **isolated system** evolves such that it maximizes its entropy.

Quantum mechanics:

A **pure state** (with zero entropy) remains pure with zero entropy.

[Kaufman et al.; Science (2016)] *"Quantum thermalization through entanglement in an isolated many-body system"*

Clash of two fundamental concepts

Statistical mechanics:

An **isolated system** evolves such that it maximizes its entropy.

Quantum mechanics:

A **pure state** (with zero entropy) remains pure with zero entropy.

[Kaufman et al.; Science (2016)] *"Quantum thermalization through entanglement in an isolated many-body system"*

Pure state density matrix

$$\rho = 1 \cdot |\text{eigenstate}\rangle\langle\text{eigenstate}|$$

with $\mathrm{tr}\left(\rho^2\right) = 1$.

Vanishing entropy $S = -\,\mathrm{tr}\left(\rho \log \rho\right) = 0$

Unitary time evolution $\rho \to U^{-1}\rho U$ keeps S=*0*.

Clash of two fundamental concepts

Statistical mechanics:

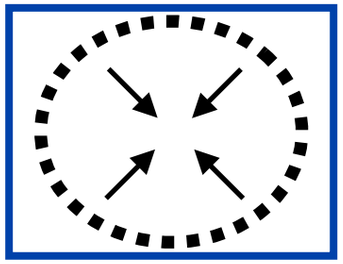
An **isolated system** evolves such that it maximizes its entropy.

Quantum mechanics:

A **pure state** (with zero entropy) remains pure with zero entropy.

Example 1:
Black hole evaporation

[Almheiri et al.; Rev.Mod.Phys. (2021)]



Pure state. **Zero entropy.**
(inward falling mass shell)

time

Clash of two fundamental concepts

Statistical mechanics:

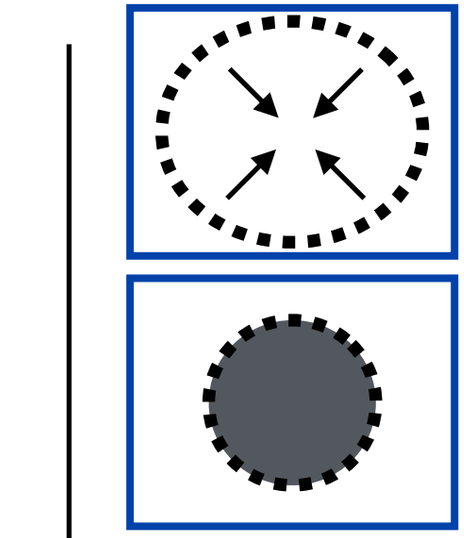
An **isolated system** evolves such that it maximizes its entropy.

Quantum mechanics:

A **pure state** (with zero entropy) remains pure with zero entropy.

Example 1:
Black hole evaporation

[Almheiri et al.; Rev.Mod.Phys. (2021)]



Pure state. **Zero entropy.**
(inward falling mass shell)

Black hole is formed.

Clash of two fundamental concepts

Statistical mechanics:

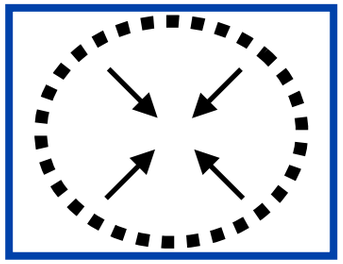
An **isolated system** evolves such that it maximizes its entropy.

Quantum mechanics:

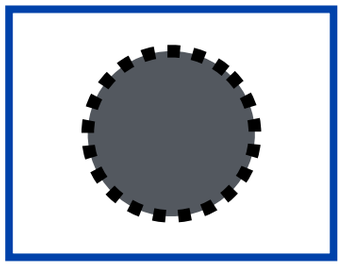
A **pure state** (with zero entropy) remains pure with zero entropy.

Example 1:
Black hole evaporation

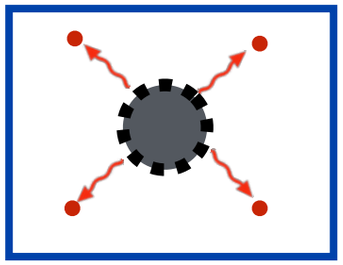
[Almheiri et al.; Rev.Mod.Phys. (2021)]



Pure state. **Zero entropy.** (inward falling mass shell)



Black hole is formed.



Hawking radiation, black hole evaporates.

time

Clash of two fundamental concepts

Statistical mechanics:

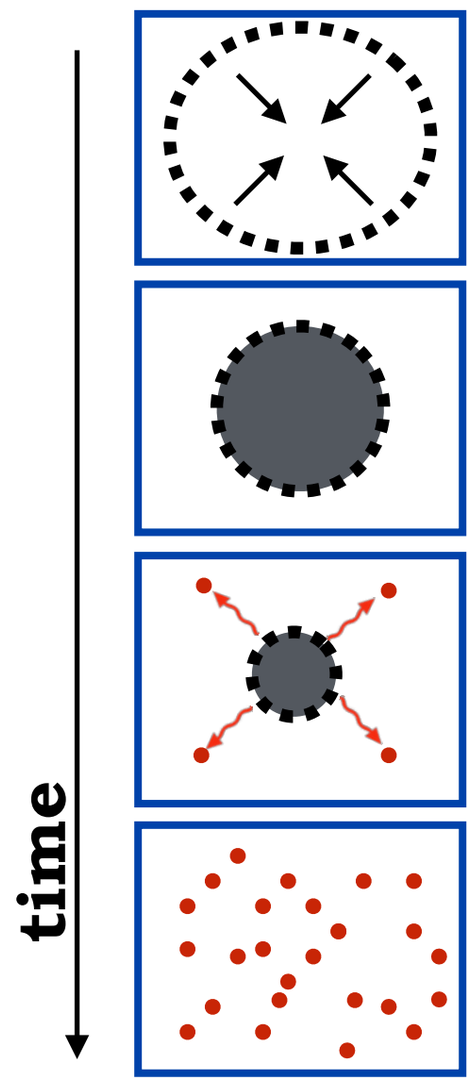
An **isolated system** evolves such that it maximizes its entropy.

Quantum mechanics:

A **pure state** (with zero entropy) remains pure with zero entropy.

Example 1:
Black hole evaporation

[Almheiri et al.; Rev.Mod.Phys. (2021)]



Pure state. **Zero entropy.**
(inward falling mass shell)

Black hole is formed.

Hawking radiation,
black hole evaporates.

Thermal Hawking radiation.
Nonzero entropy!?

Clash of two fundamental concepts

Statistical mechanics:

An **isolated system** evolves such that it maximizes its entropy.

Quantum mechanics:

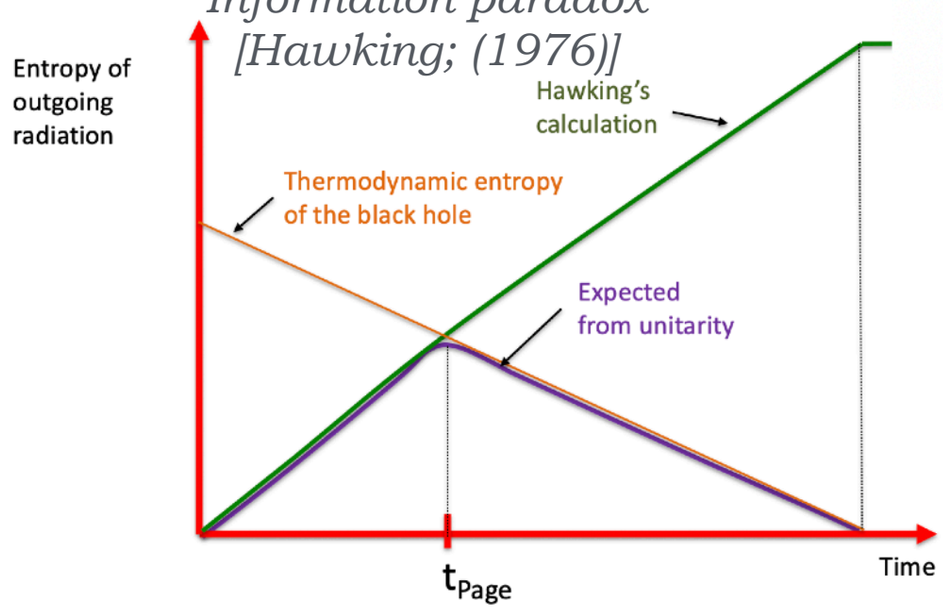
A **pure state** (with zero entropy) remains pure with zero entropy.

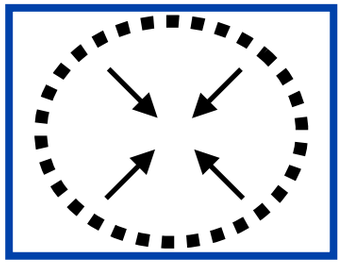
Example 1:
Black hole evaporation

[Almheiri et al.; Rev.Mod.Phys. (2021)]

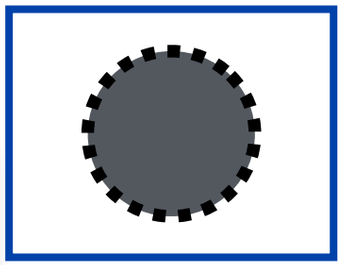
Page curve (fine-grained S)

Information paradox
[Hawking; (1976)]

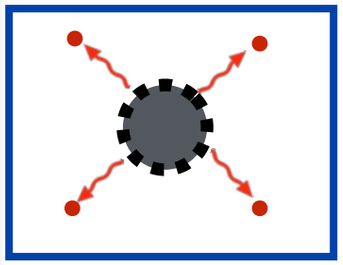




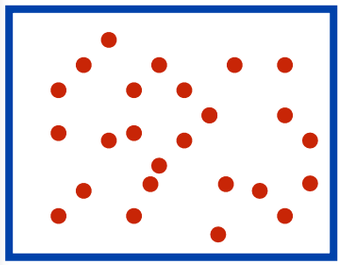
Pure state. **Zero entropy.** (inward falling mass shell)



Black hole is formed.



Hawking radiation, black hole evaporates.



~~Thermal~~ Hawking radiation.
~~**Nonzero entropy!?**~~

Clash of two fundamental concepts

Statistical mechanics:

An **isolated system** evolves such that it maximizes its entropy.

Quantum mechanics:

A **pure state** (with zero entropy) remains pure with zero entropy.

Example 1:
Black hole evaporation

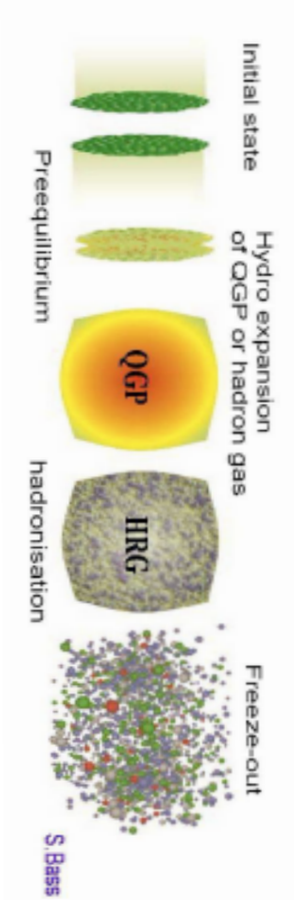
Example 2:
Heavy ion collisions (HIC)

Quantum information approach to HIC
[Kharzeev; Phil.Trans.A.Math.Phys.Eng.Sci. (2021)]

[Zhang et al.; PRD (2021)]

[Florio,Kharzeev; PRD (2021)]

time



Pure state. **Zero entropy.**
(two colliding ions)
[Müller,Schäfer; (2017)]

Particle distributions in detectors look thermal. **Nonzero entropy!?**

Clash of two fundamental concepts

Statistical mechanics:

An **isolated system** evolves such that it maximizes its entropy.

Quantum mechanics:

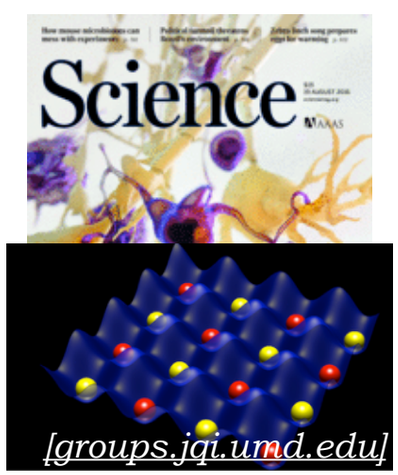
A **pure state** (with zero entropy) remains pure with zero entropy.

Example 1:
Black hole evaporation

Example 2:
Heavy ion collisions

Example 3:
Ultracold atoms

[Kaufman et al.; Science (2016)]
"Quantum thermalization through entanglement in an isolated many-body system"



- Rb atoms in optical lattice
- six-site Bose-Hubbard system
- quench and microscopy

time ↓

Pure state. **Zero entropy. Remains pure!**

BUT subsystems thermalize! (nonzero entropy)

Clash of two fundamental concepts

Statistical mechanics:

An **isolated system** evolves such that it maximizes its entropy.

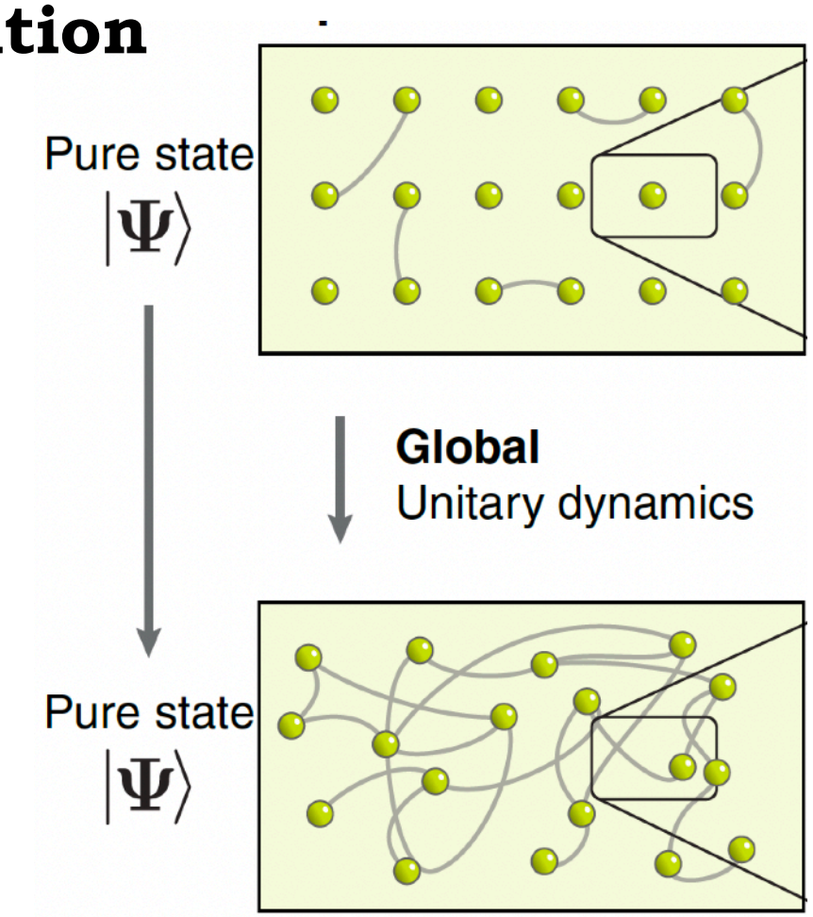
Quantum mechanics:

A **pure state** (with zero entropy) remains pure with zero entropy.

General proposed resolution

- Total system remains pure.
- Subsystems thermalize (nonzero entropy).
- Eigenstate thermalization hypothesis (ETH)? *[Srednicki; (1993)]*

. . . many open questions!



[Kaufman et al.; Science (2016)]

Clash of two fundamental concepts

Statistical mechanics:

An **isolated system** evolves such that it maximizes its entropy.

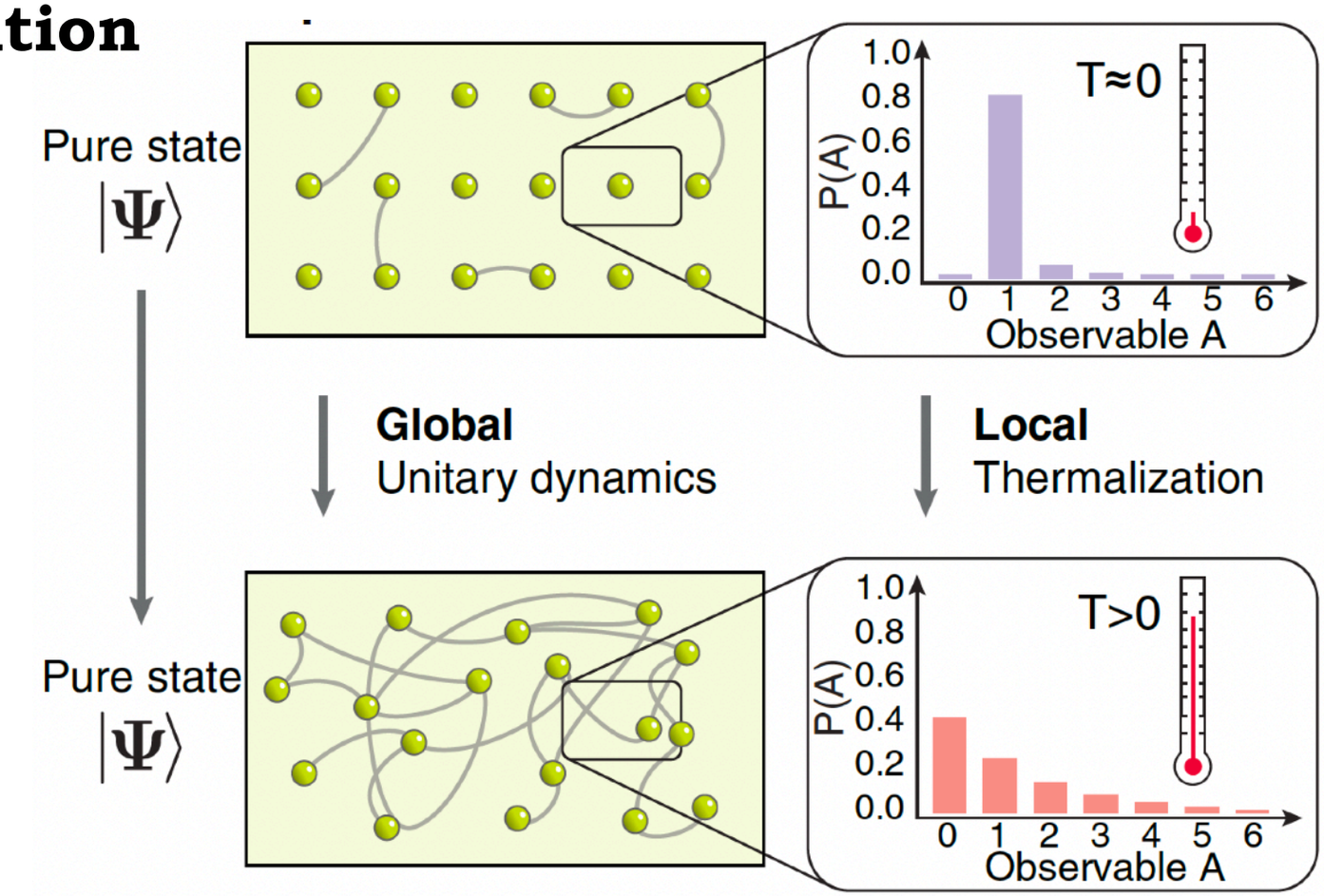
Quantum mechanics:

A **pure state** (with zero entropy) remains pure with zero entropy.

General proposed resolution

- Total system remains pure.
- Subsystems thermalize (nonzero entropy).
- Eigenstate thermalization hypothesis (ETH)? *[Srednicki; (1993)]*

. . . many open questions!



[Kaufman et al.; Science (2016)]

entanglement entropy!

Clash of two fundamental concepts

Statistical mechanics:

An **isolated system** evolves such that it maximizes its entropy.

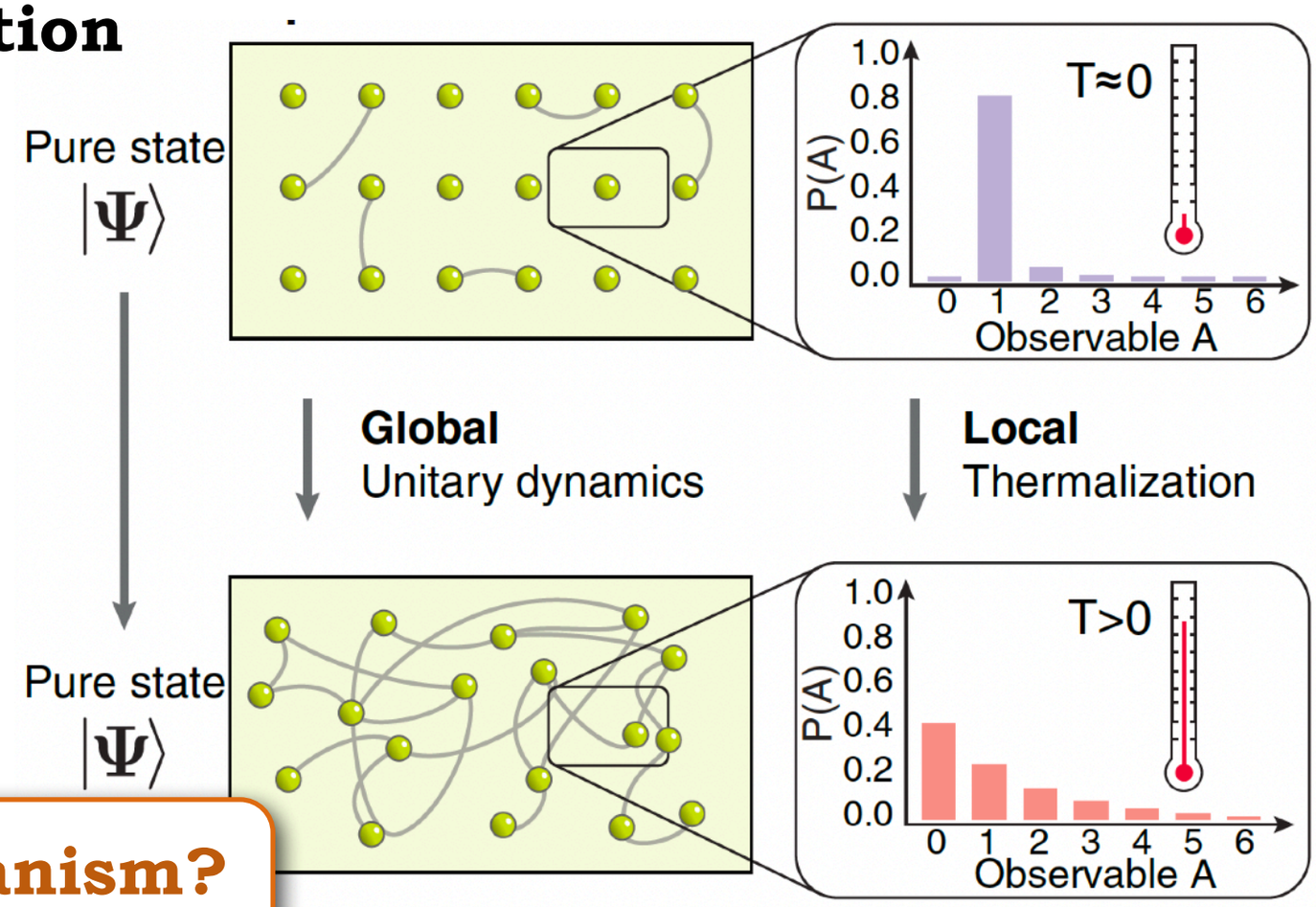
Quantum mechanics:

A **pure state** (with zero entropy) remains pure with zero entropy.

General proposed resolution

- Total system remains pure.
- Subsystems thermalize (nonzero entropy).
- Eigenstate thermalization hypothesis (ETH)?

[Srednicki; (1993)]

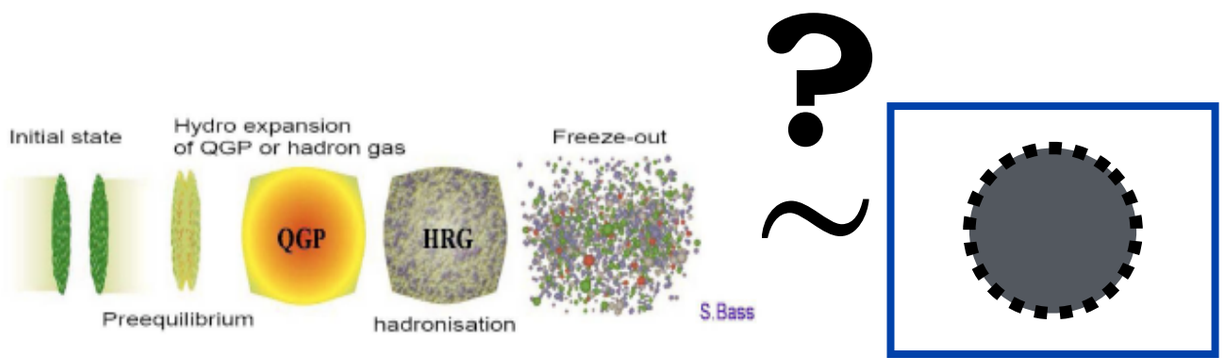


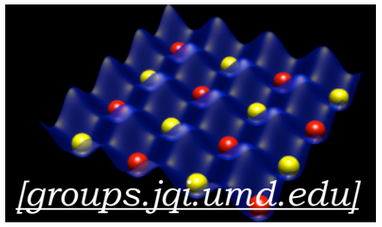
n et al.; Science (2016)]

entanglement entropy!

➡**thermalization mechanism?**
➡**entanglement/decoherence?**

Ion collision/Ultracold atoms ~ Black hole formation?



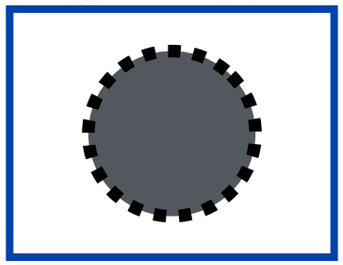


Exact correspondence: Holography (gauge/gravity)

see talk by Koenraad Schalm

N=4 Super-Yang-Mills in **3+1 dimensions** (gauge, QFT) $\Leftrightarrow$ Typ II B Supergravity in **(4+1)-dimensional** Anti de Sitter space (gravity)

$\Leftrightarrow$



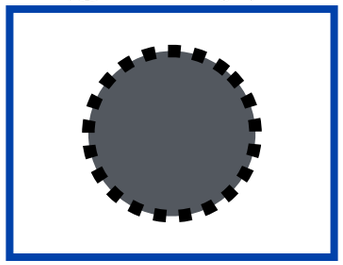
['t Hooft (1993)]
[Susskind; J.Math.Phys. (1995)]
[Maldacena; Adv.Theor.Math.Phys. (1997)]

Exact correspondence: Holography (gauge/gravity)

see talk by Koenraad Schalm

N=4 Super-Yang-Mills in **3+1 dimensions** (gauge, QFT) $\Leftrightarrow$ Typ II B Supergravity in **(4+1)-dimensional** Anti de Sitter space (gravity)

$\Leftrightarrow$



[*'t Hooft (1993)*]
[*Susskind; J.Math.Phys. (1995)*]
[*Maldacena; Adv.Theor.Math.Phys. (1997)*]

Black hole entropy grows as its **surface area** (not as its volume).

[*Bekenstein*]
[*Hawking*]

$$S \propto A_h$$

➡**Exact thermalization results from holographic models**
➡**Compare to hydrodynamic predictions**

[*Cartwright,Kaminski,Knipfer (2022)*]
[*Cartwright,Kaminski,Schenke; PRC (2022)*]
[*Cartwright,Kaminski; JHEP (2019)*]

Hydrodynamics and holography in parallel

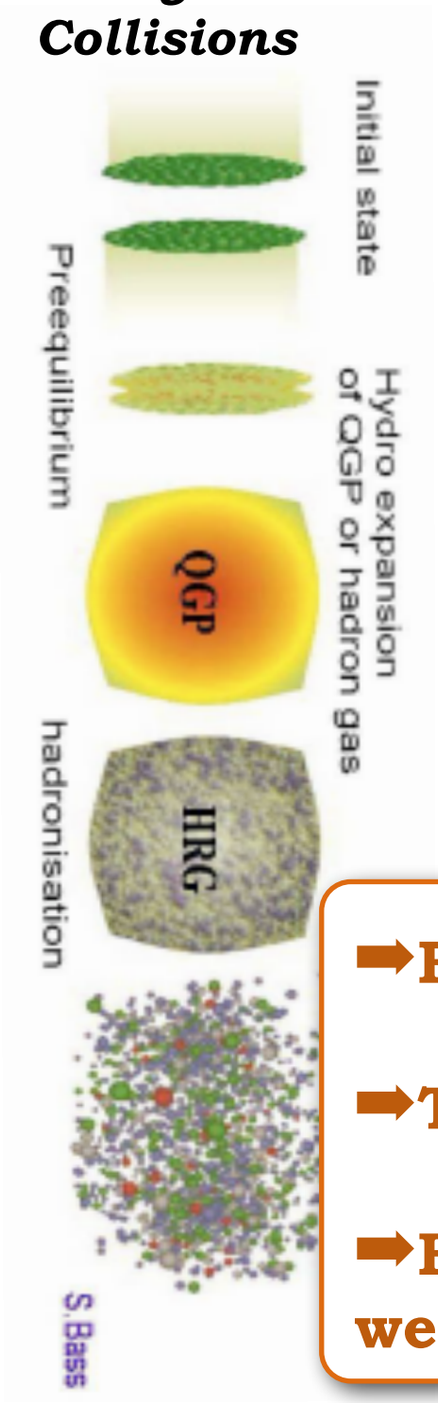
HYDRODYNAMICS & THERMODYNAMICS (quantum field theory methods)

- fluids/plasmas
- strong magnetic fields
- transport

[Jensen,Kaminski,Kovtun,Meyer et al.; PRL (2012)]

cf. hydrodynamics in graphene:
[Sachdev et al]
[Lucas et al]

Example: heavy ion Collisions



time

holographic idea: [Janik, Peschanski; PRD (2006)]

HOLOGRAPHY

- far from equilibrium
- strong coupling
- strong magnetic fields
- transport

➡Robust theoretical results

➡Theoretical discoveries (e.g. CVE)
[Erdmenger,Haack,Kaminski,Yarom; JHEP (2008)]

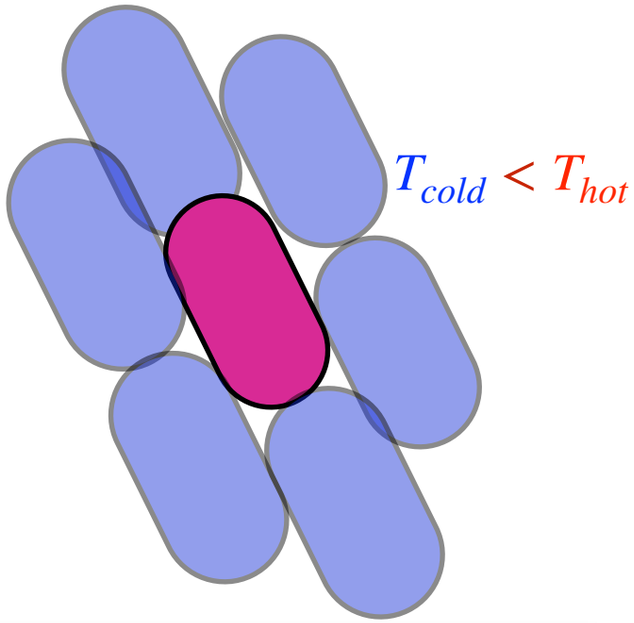
➡Heavy-ion collision data extremely well described by *hydrodynamics*

Hydrodynamics as "far-from-equilibrium quantum dynamics"?

[Romatschke, PRL (2017)]

False myths about hydrodynamic requirements:

- ~~isotropy~~
- ~~small gradients ($\omega, k \ll T$)~~
- ~~local thermal equilibrium~~



Heavy-ion collision data *[Romatschke, textbook (2017)]*
and numerical holographic model data *[Chesler, Yaffe; PRL (2009)]*
show hydrodynamics is a valid approximation
even far away from equilibrium.

[Cartwright,Kaminski,Knipfer; under review with JHEP (2022)]

➡**due to hydrodynamics being asymptotic expansion (resurgence, re-summation, attractors)** *[Heller, Spalinski; PRL (2015)]*
[Wondrak,Kaminski,Bleicher; Phys.Lett.B (2020)]

➡**quantum chaos data encoded in hydrodynamics**

[Blake et al.; (2018)] *[Grozdanov,Schalm et al.; (2015)]* *[Blake,Cartwright,Kaminski,Thompson; to appear]*

Example 1: Enhanced convergence radius of hydrodynamics

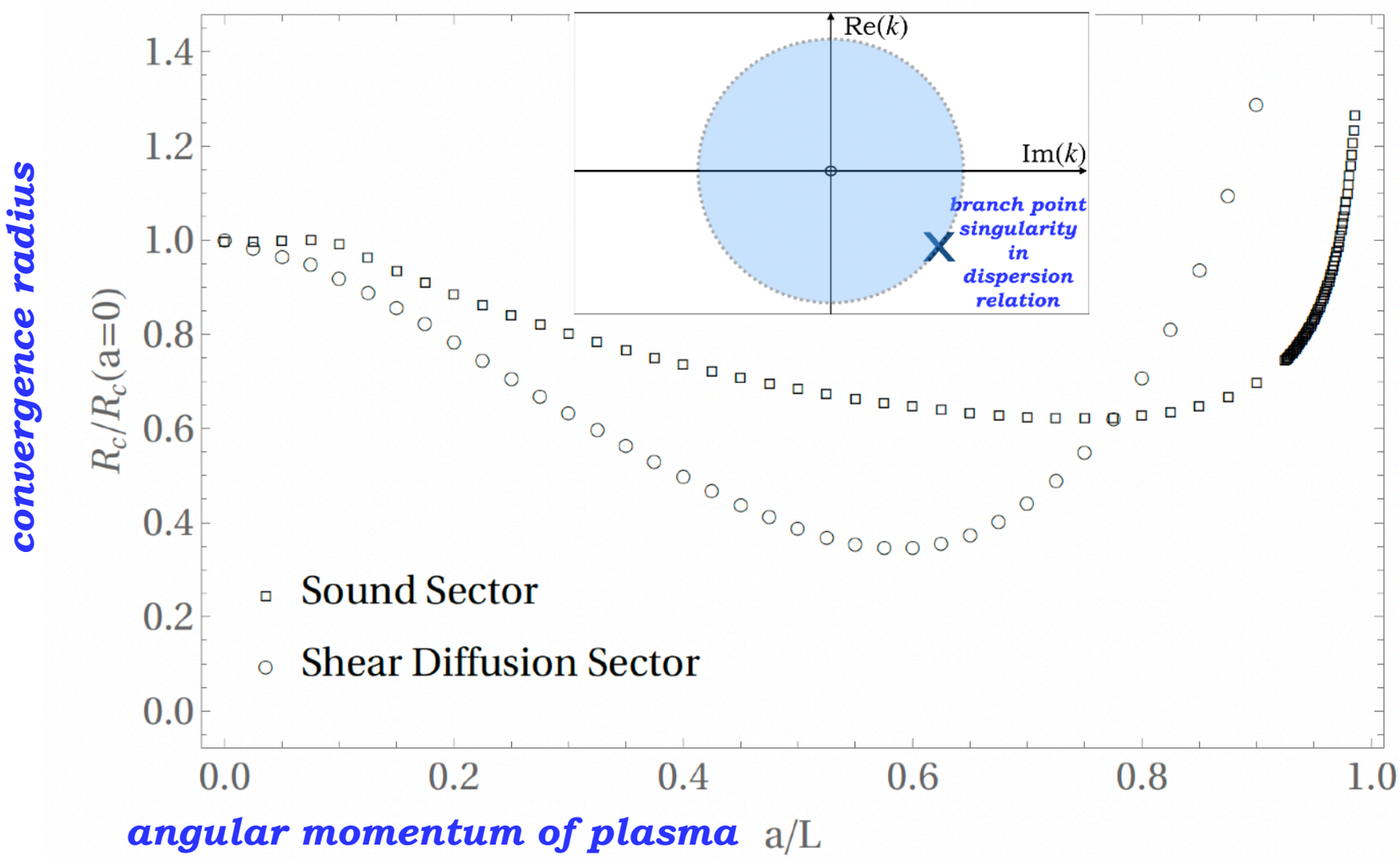
(of N=4 Super-Yang-Mills theory in a rotating highly vortical thermal state)

[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; under review with PRL (2022)]

Calculation: branch point singularities in dispersion relations (holography)

[Grozdanov,Kovtun,Starinets,Tadic; PRL (2019)]

[Abbasi,Kaminski; JHEP (2020)] *[Heller,Serantes,Spalinski,Svensson,Withers; PRD (2020)]*



Casey Cartwright (Fall 2022: Utrecht U.)

➡ **Hydrodynamics converges in highly vortical fluids.**

➡ **Can use hydrodynamic simulations for data analysis.**

Example 2: Discovery of novel transport effects

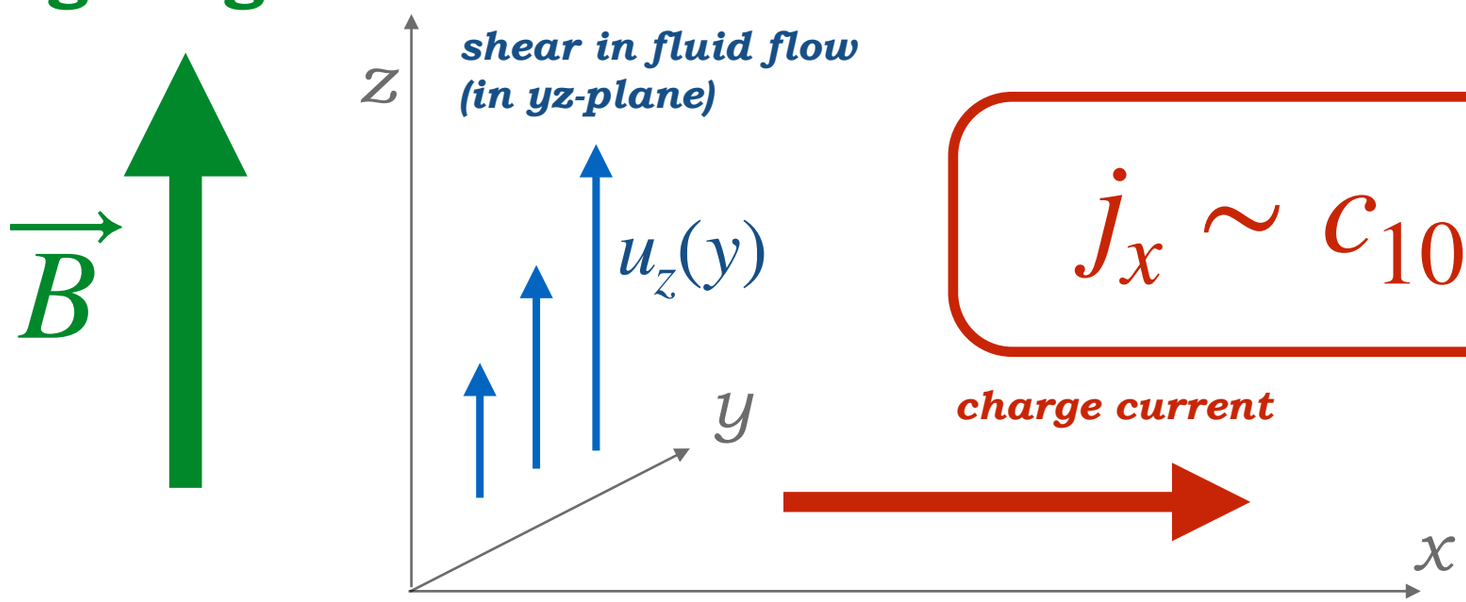
Calculation: field theoretic construction of hydrodynamics.

phenomenological hydrodynamics → **formal hydrodynamics (quantum field theory methods)**

[Jensen,Kaminski,Kovtun,Meyer et al.; PRL (2012)]

Shear-induced Hall conductivity c_{10} in strong magnetic field

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]



$$j_x \sim c_{10} \cdot \partial_y u_z$$

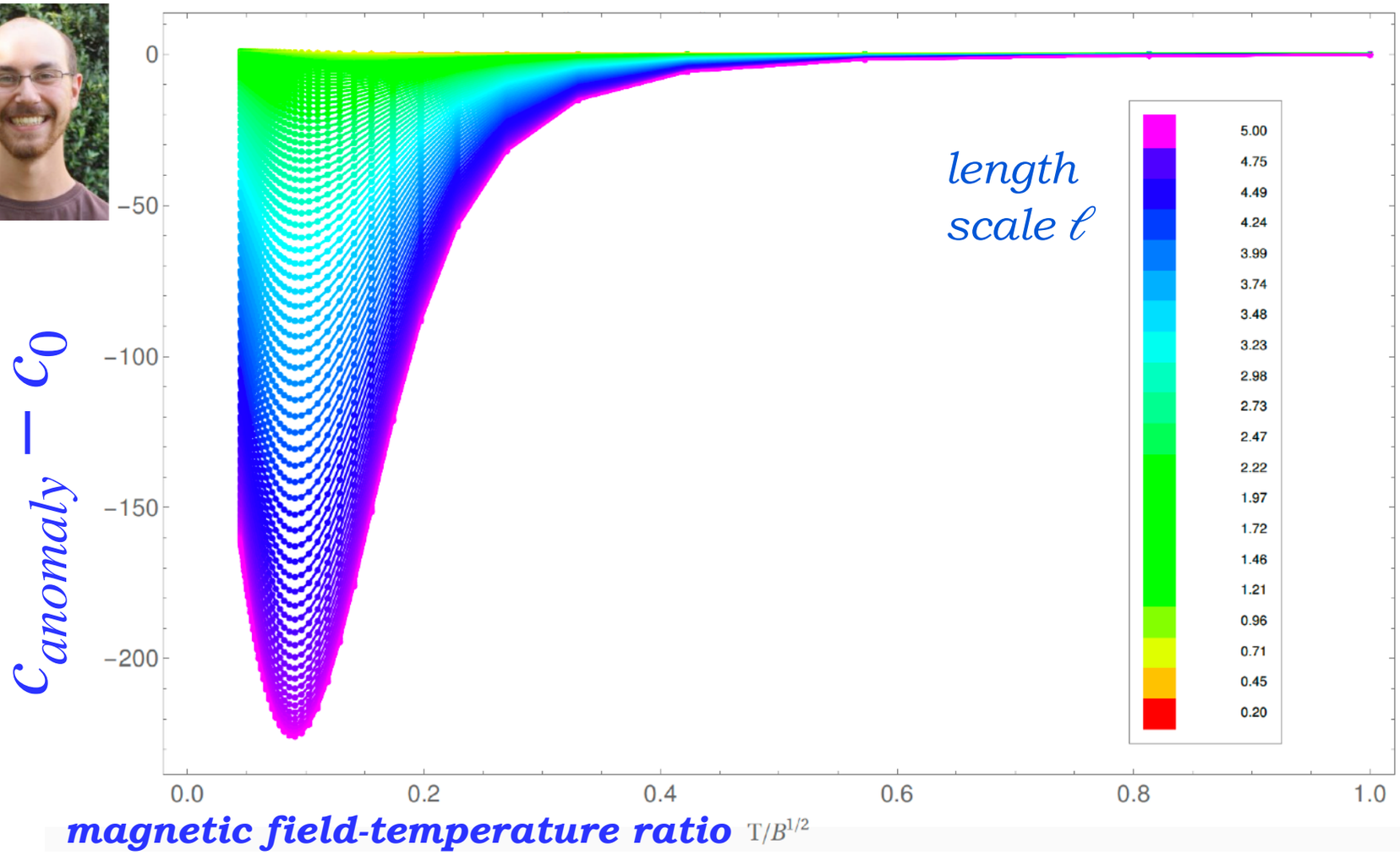
- ➡ **novel Hall response (one among 4 novel effects)**
- ➡ **non-dissipative**
- ➡ **propose cond-mat experiment with B.Trauzettel**

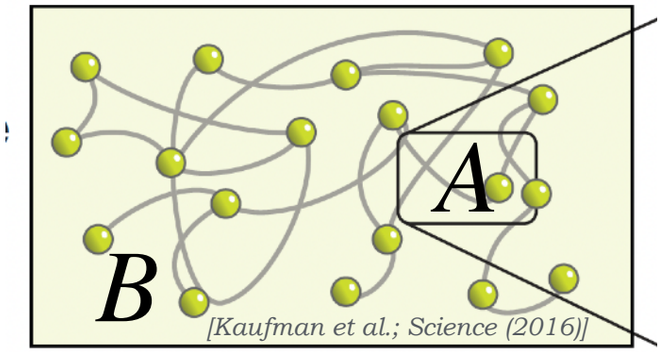
Example 3: effect of quantum anomaly on entanglement

Calculation: strongly coupled *N*=4 Super-Yang-Mills theory; compute minimal surfaces *[Ryu,Takayanagi; JHEP (2006)]*

Entropic c-function (with anomaly minus without anomaly)

[Cartwright,Kaminski; JHEP (2021)]





Reduced density matrix:

$$\rho_A = \mathrm{tr}_B\, \rho$$

Entanglement entropy:

$$S_A = -\,\mathrm{tr}\left(\rho_A \log \rho_A\right)$$

Entropic c-function:

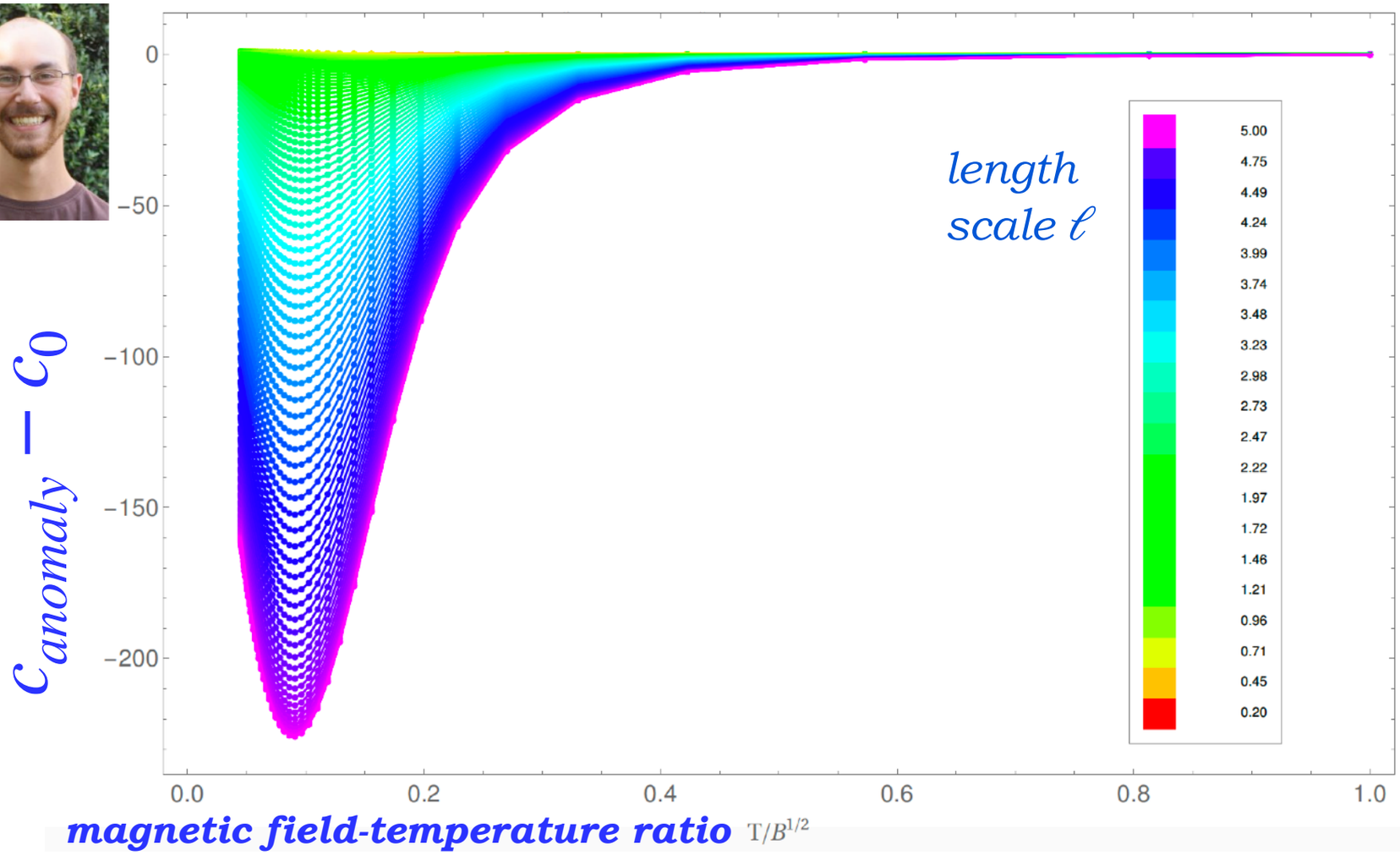
$$c \propto \frac{\partial S_A}{\partial \ell}$$

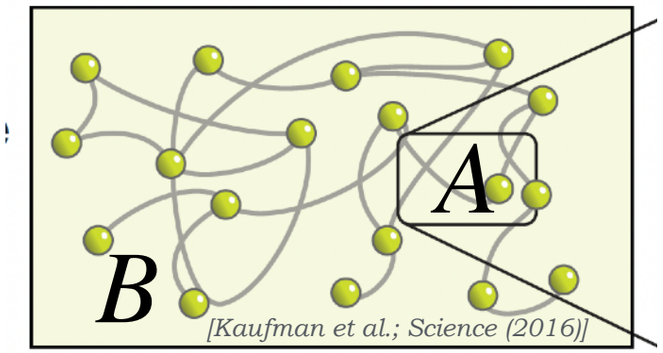
Example 3: effect of quantum anomaly on entanglement

Calculation: strongly coupled *N*=4 Super-Yang-Mills theory; compute minimal surfaces *[Ryu,Takayanagi; JHEP (2006)]*

Entropic c-function (with anomaly minus without anomaly)

[Cartwright,Kaminski; JHEP (2021)]





Reduced density matrix:

$$\rho_A = \mathrm{tr}_B\, \rho$$

Entanglement entropy:

$$S_A = -\,\mathrm{tr}\left(\rho_A \log \rho_A\right)$$

Entropic c-function:

$$c \propto \frac{\partial S_A}{\partial \ell}$$

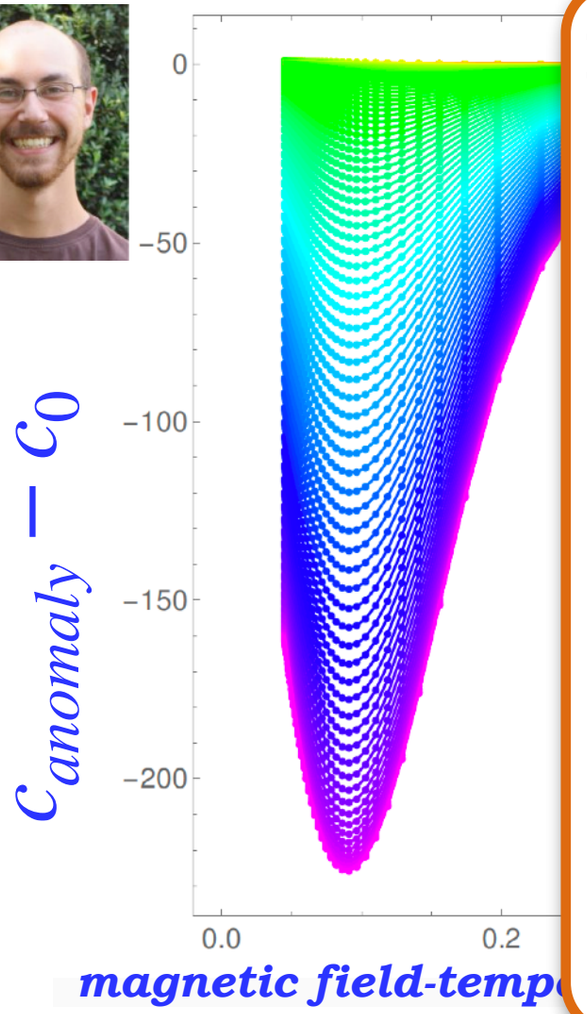
➡anomaly effect peaked at 0.1 (scale?)

Example 3: effect of quantum anomaly on entanglement

Calculation: strongly coupled *N*=4 Super-Yang-Mills theory; compute minimal surfaces *[Ryu,Takayanagi; JHEP (2006)]*

Entropic c-function (with anomaly minus without anomaly)

[Cartwright,Kaminski; JHEP (2021)]

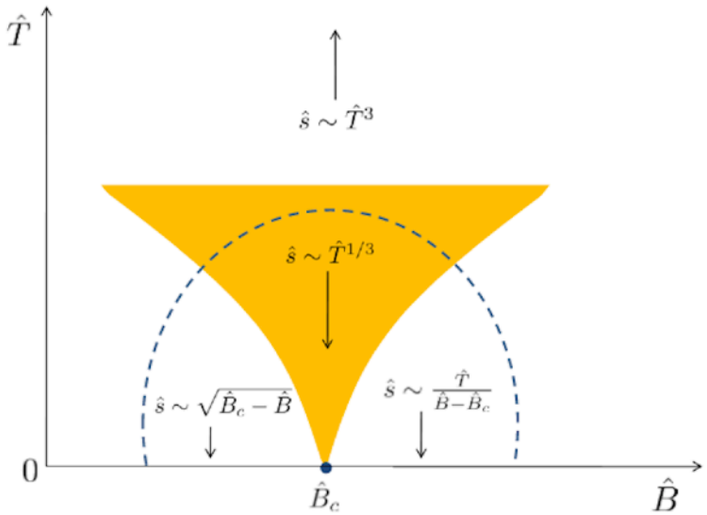


Effect of quantum critical point? (cf. strange metals)

see talks by Subir Sachdev and Koenraad Schalm

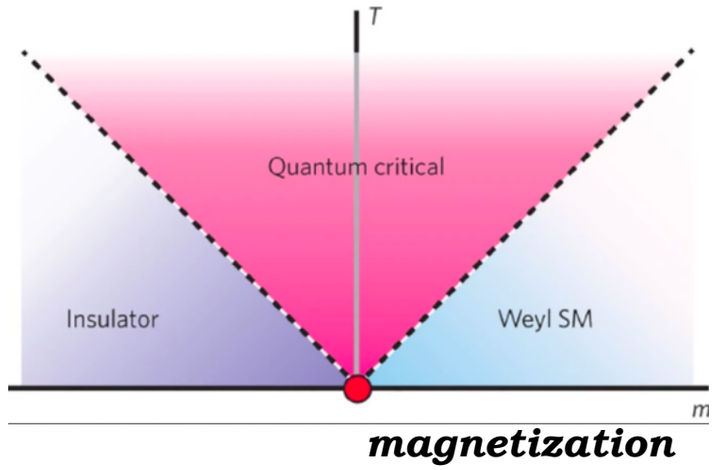
cf. [Baggioli,Giataganas; PRD (2021)]

This model:



[D'Hoker, Kraus; JHEP (2010)]

Weyl semimetal:

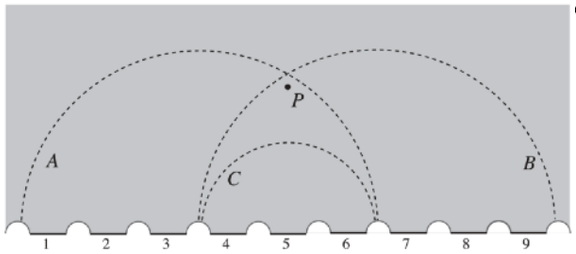


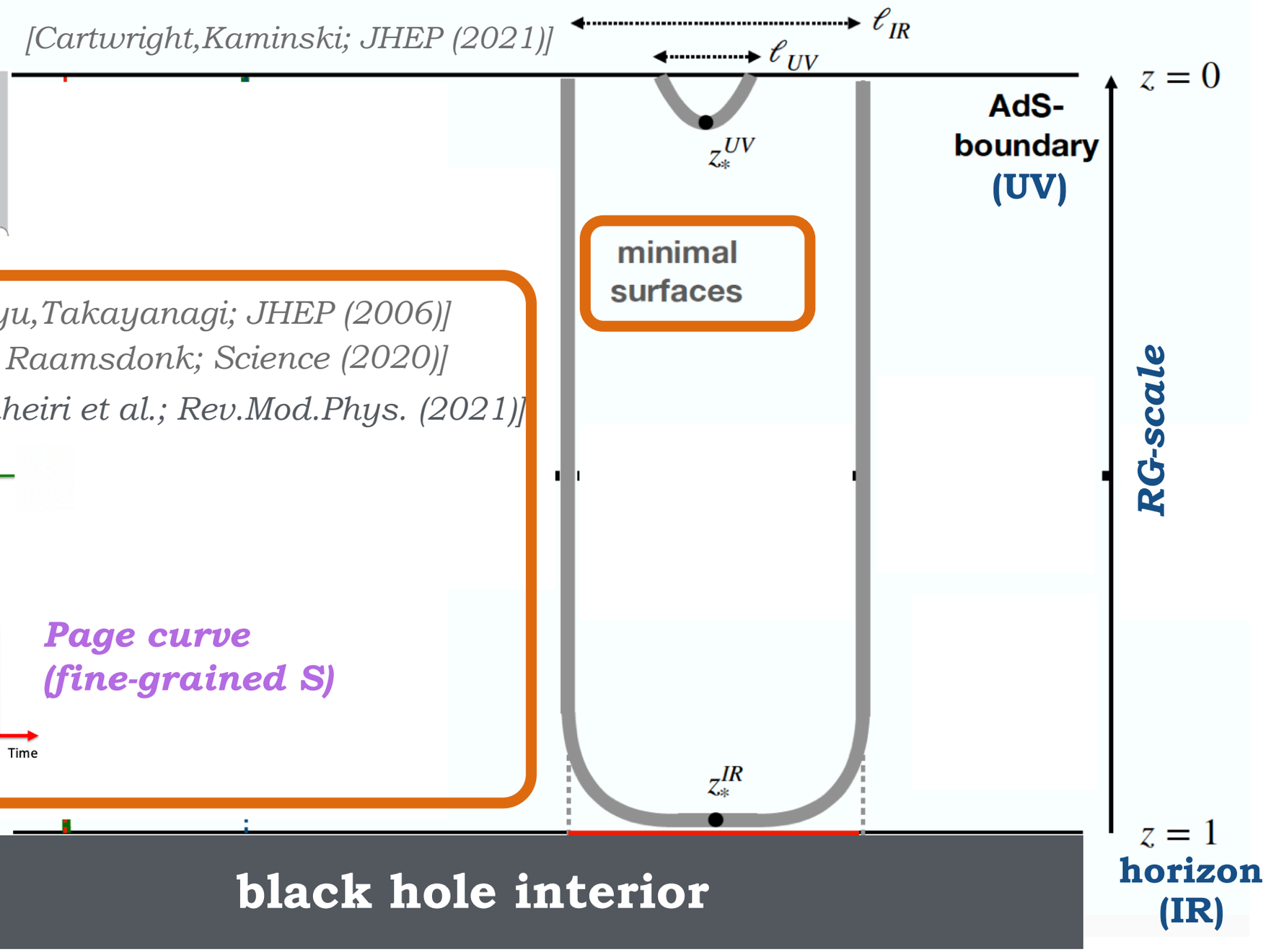
[Yang et al.; Nature (2014)]
[Wang et al.; PRB (2019)]

➡anomaly effect peaked at 0.1 (scale?)

Geometric picture: three faces of *minimal surfaces*

[Cartwright,Kaminski; JHEP (2021)]

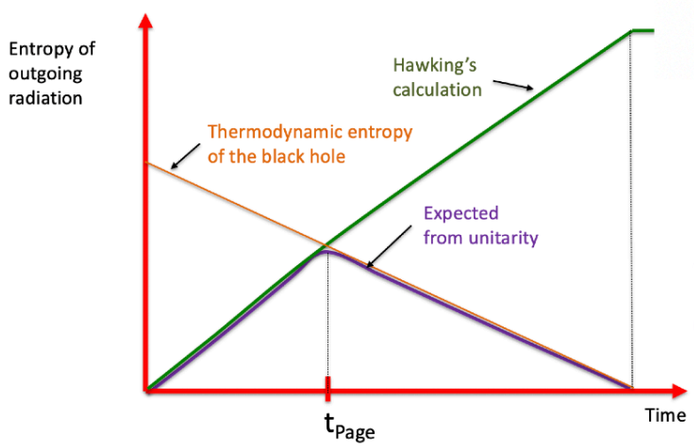




1) Entanglement entropy: *[Ryu,Takayanagi; JHEP (2006)]*

2) Emergent spacetime: *[Van Raamsdonk; Science (2020)]*

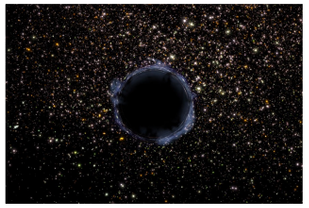
3) Fine-grained entropy: *[Almheiri et al.; Rev.Mod.Phys. (2021)]*



Page curve (fine-grained S)

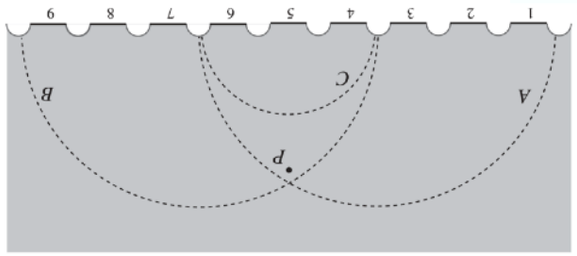
➡ Entanglement = Spacetime

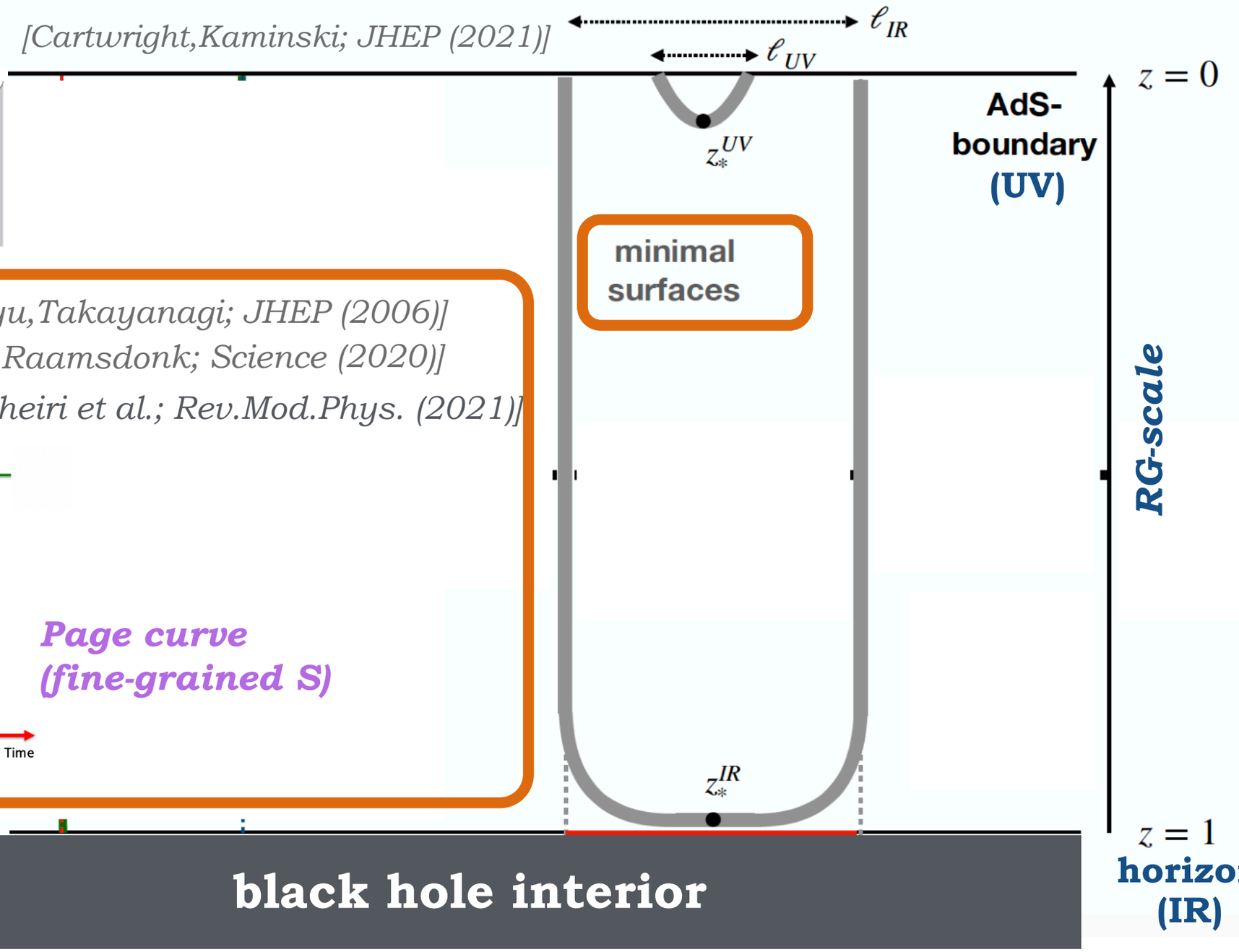
[Van Raamsdonk; Gen.Rel.Grav. (2010)]



Geometric picture: three faces of *minimal surfaces*

[Cartwright,Kaminski; JHEP (2021)]

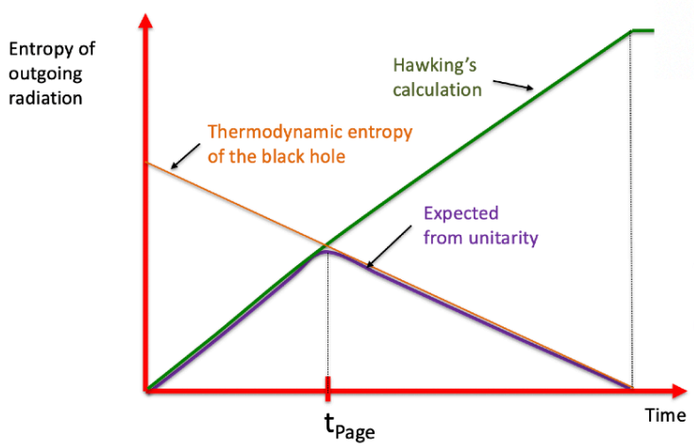




1) Entanglement entropy: *[Ryu,Takayanagi; JHEP (2006)]*

2) Emergent spacetime: *[Van Raamsdonk; Science (2020)]*

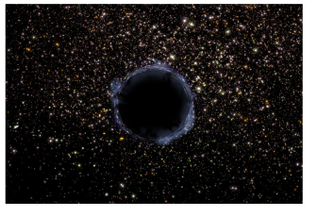
3) Fine-grained entropy: *[Almheiri et al.; Rev.Mod.Phys. (2021)]*



Page curve (fine-grained S)

➡ Entanglement = Spacetime

[Van Raamsdonk; Gen.Rel.Grav. (2010)]



Example 4: Construction of spin hydrodynamics

[Hongo,Huang,Kaminski,Stephanov,Yee; JHEP (2021)]
[Hongo,Huang,Kaminski,Stephanov,Yee; JHEP (2022)]

➡**Kubo formula for novel *rotational viscosity***

$$\eta_s = - \lim_{(\Gamma_s \ll \omega \ll \Gamma)} \lim_{k \to 0} \langle \underbrace{\theta^x_{(a)y}}_{\text{anti-symmetric stress tensor}} \underbrace{\Sigma^{\hat{0}x}{}_{\hat{y}}}_{\text{spin current}} \rangle (\omega, k)$$

➡**applications in spintronics / graphene?**

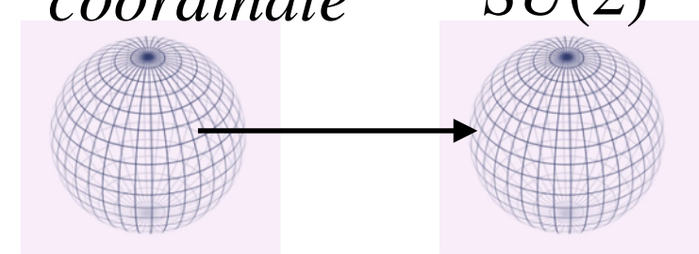
see talks by Christoph Tegenkamp and Niels Schroeter

Example 5: Topological confinement in Skyrmion holography

[Cartwright,Harms,Kaminski,Thomale; Class.Quant.Grav (2022]
[Cartwright,Harms,Kaminski; JHEP (2021)]

see talk by Christian Pfleiderer

$S^3_{coordinate}$ $S^3_{SU(2)}$



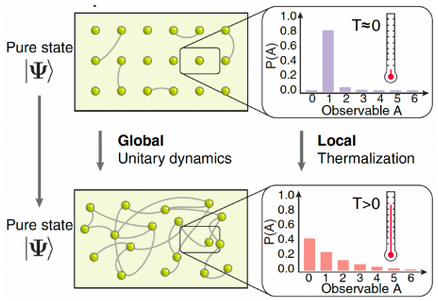
➡**topology alters *confinement transition* (cf. quarks)**

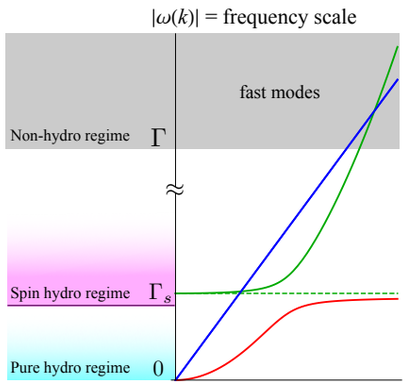
➡**quantum spin chains, spin liquids, FQH**

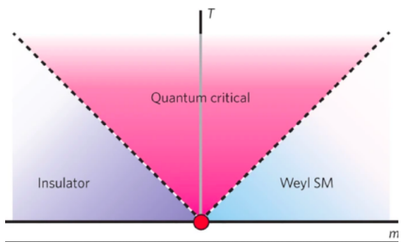
SUMMARY

- **isolated quantum systems thermalize in similar ways**
- **robust quantum results from combination of holography+hydrodynamics**
- **hydrodynamics is unreasonably powerful (resurgence, attractors, chaos)**
- **entanglement = spacetime**

1. hydrodynamic convergence radius enhanced by rotation
2. novel shear-induced Hall charge current
3. entanglement detecting topological order?
4. constructed **spin hydrodynamics** (Kubo formula for rotational viscosity)
5. **topological confinement**

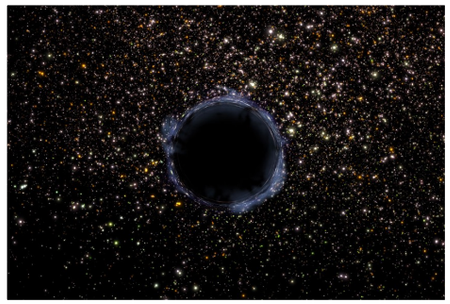






OUTLOOK

- **measurements** of hydrodynamic effects (spin, strong magnetic field, far-from equilibrium)
- entanglement & thermalization in **holography** and **far-from-equilibrium fluid dynamics**



Thanks to my collaborators (since 2012) and Thank You!

University of Victoria, Canada

Prof. Dr. Pavel Kovt

Prof. Dr. Adam Ritz

Prof. Dr. Kristan Jensen

University of Washington, Seattle, USA

Prof. Dr. Laurence Yaffe

BNL, USA

Dr. Björn Schenke

Heriot-Watt University, United Kingdom

Prof. Dr. Richard Davison

Bristol University, United Kingdom

Prof. Dr. Mike Blake

Niigata University, Japan

Prof. Dr. Masaru Hongo

Regensburg University, Germany

Prof. Dr. Andreas Schaefer

Lanzhou University, China

Prof. Dr. Navid Abbasi

University of Alabama, Tuscaloosa, USA

Prof. Dr. Benjamin Harms

Dr. Markus Garbiso Amano (now Henan U., China)

Dr. Casey Cartwright (Fall 2022: Utrecht U.)

Marco Knipfer

UT Austin, USA

Prof. Dr. Andreas Karch

Lubljana University, Slovenia

Prof. Dr. Saso Grozdanov

Fudan University, Shanghai

Prof. Dr. Xu-Guang Huang

Frankfurt University, Germany

Prof. Dr. Marcus Bleicher

Prof. Dr. Dirk Rischke

Prof. Dr. Jürgen Schaffner-Bielich

Technion, Israel

Prof. Dr. Amos Yarom

Stony Brook University, USA

Prof. Dr. Dmitri Kharzeev

Jena University, Germany

Prof. Dr. Martin Ammon

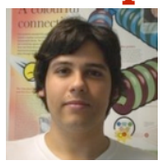
University of Illinois, USA

Chicago

Prof. Dr. Misha Stephanov

Prof. Dr. Ho-Ung Yee

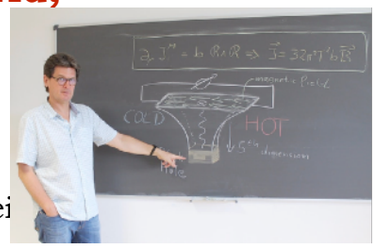
Urbana-Champaign


Prof. Dr. Jorge Noronha

Dr. Enrico Speranza

IFT Madrid, Spain

Prof. Dr. Karl Landstei



Bielefeld University, Germany

Dr. Gergely Endrődi

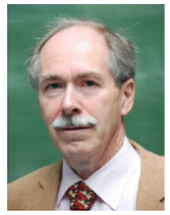
Würzburg University, Germany

Prof. Dr. Ronny Thomale

Prof. Dr. Björn Trauzettel

Prof. Dr. Johanna Erdmenger

Utrecht University, Netherlands


Prof. Dr. Gerard 't Hooft (Nobel Laureate)

APPENDIX

Ad example 4: Green-Kubo formula for rotational viscosity

Calculation: construct **spin hydrodynamics**, then linear mode analysis.

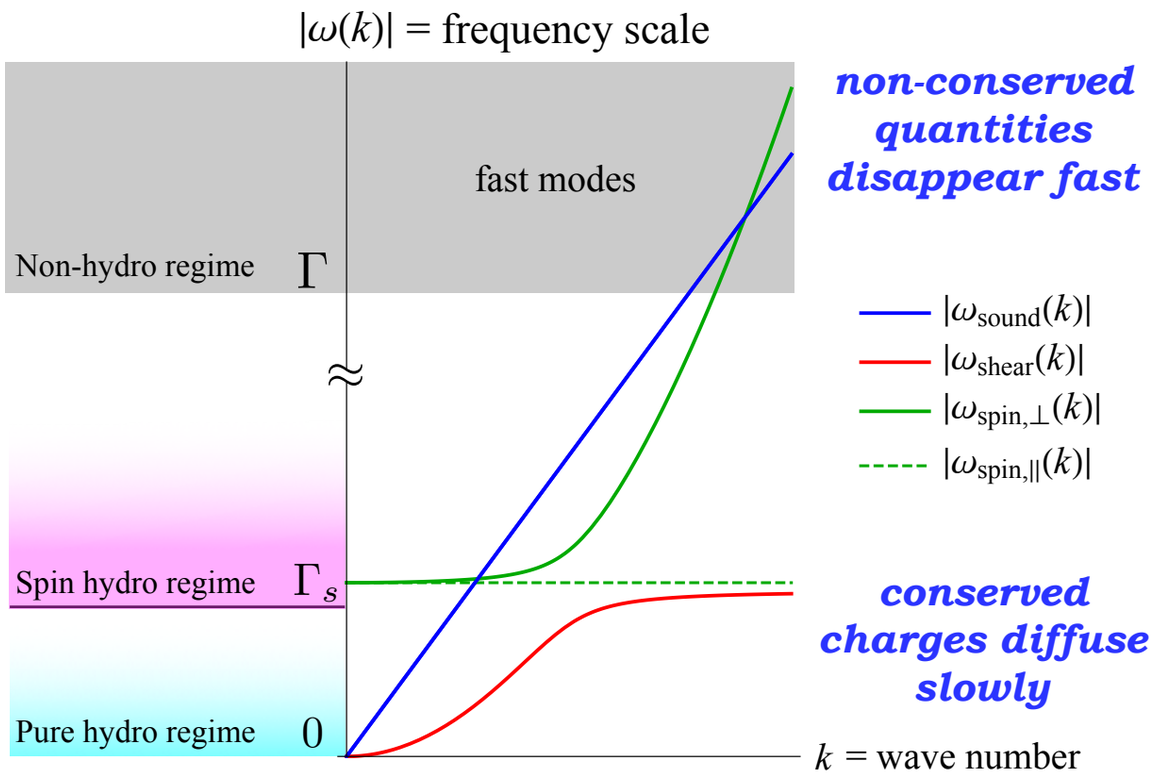
[Hongo,Huang,Kaminski,Stephanov,Yee; JHEP (2021)]

$$\eta_s = -\lim_{\Gamma_s \ll \omega \ll \Gamma} \lim_{k \to 0} \langle \theta^x{}_{y,\text{anti-symm.}} \Sigma^{\hat{0}x}{}_{\hat{y}} \rangle (\omega, k)$$

anti-symmetric stress tensor ***spin current***

➡**computed in quantum chromodynamics**

[Hongo,Huang,Kaminski,Stephanov,Yee; JHEP (2022)]



➡**included spin in hydrodynamic description.**

➡**applications in spintronics / graphene?**

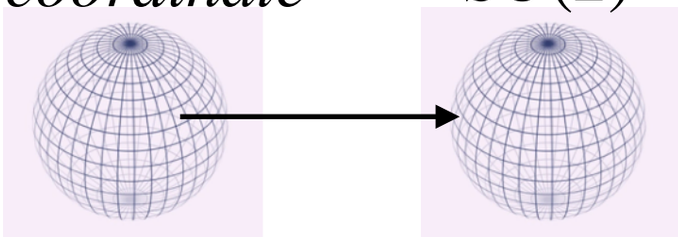
see talk by Christoph Tegenkamp

Ad example 5: Topological confinement

Calculation: Einstein gravity + SU(2) field; topologically distinct solutions

Gauge fields map $A_j^a \tau_a(x)$:

$S^3_{coordinate}$ $S^3_{SU(2)}$



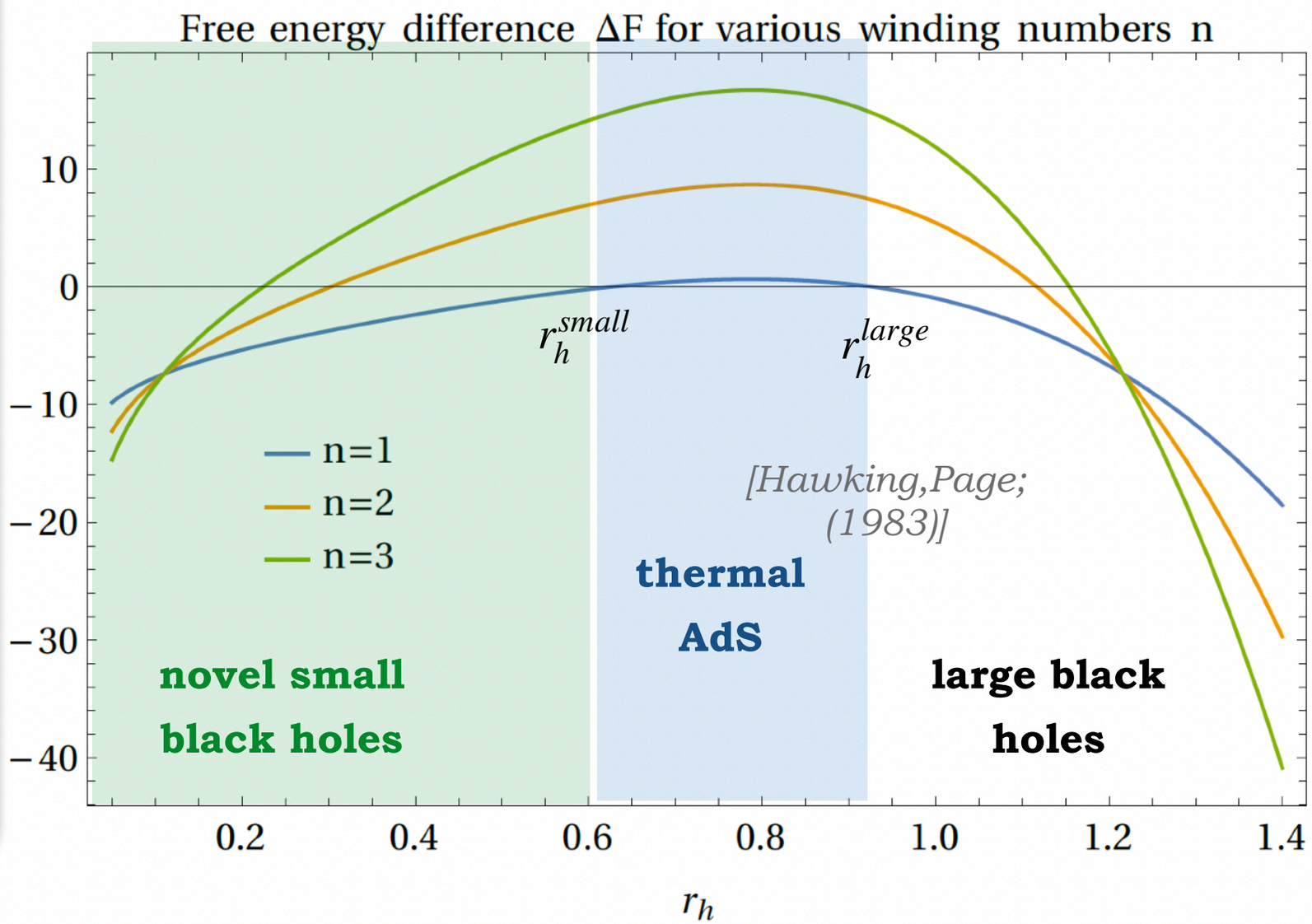
➡**gauge field configuration has non-trivial topology with Chern number *n***

➡**novel black hole solutions**

➡**topology alters confinement transition**

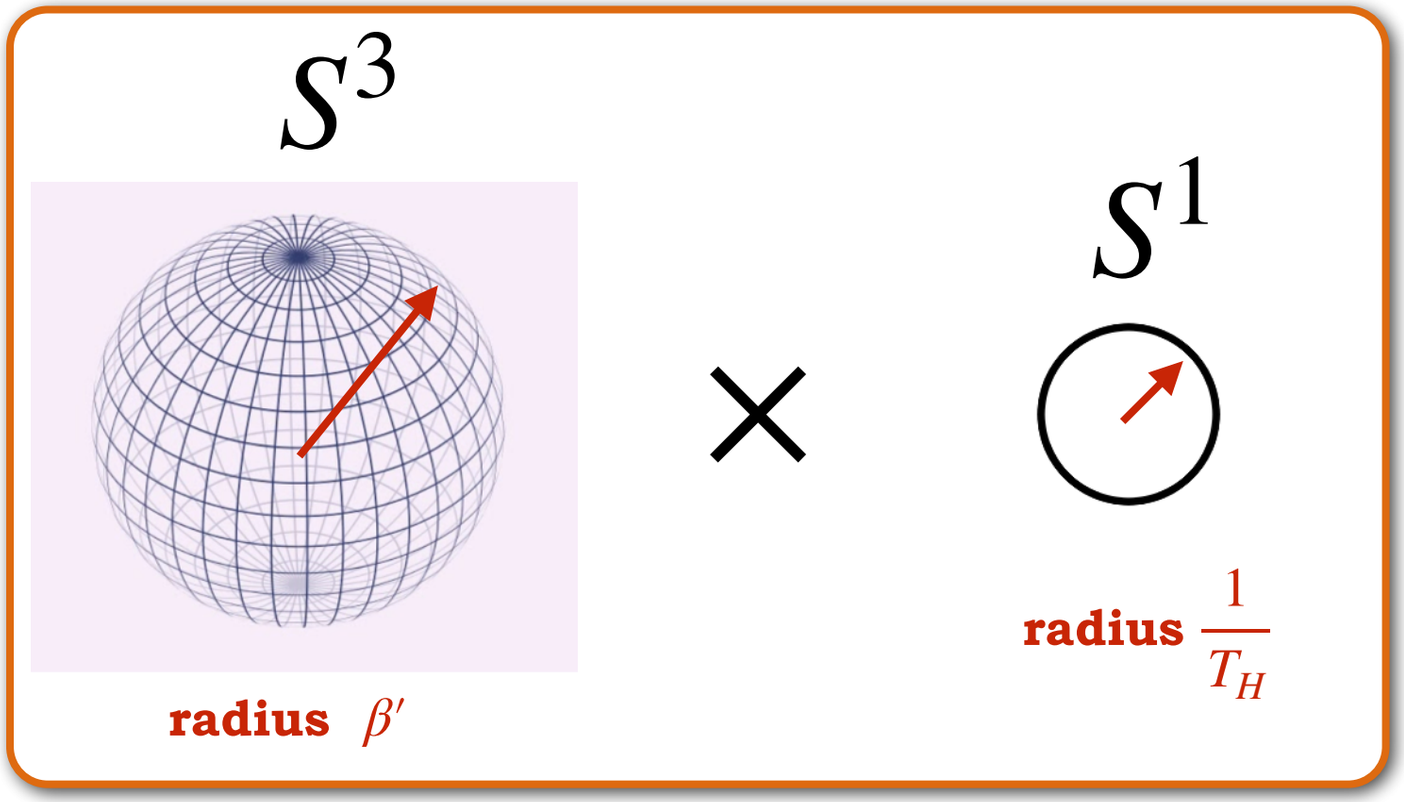
➡**quantum spin chains, spin liquids, fractional quantum Hall**

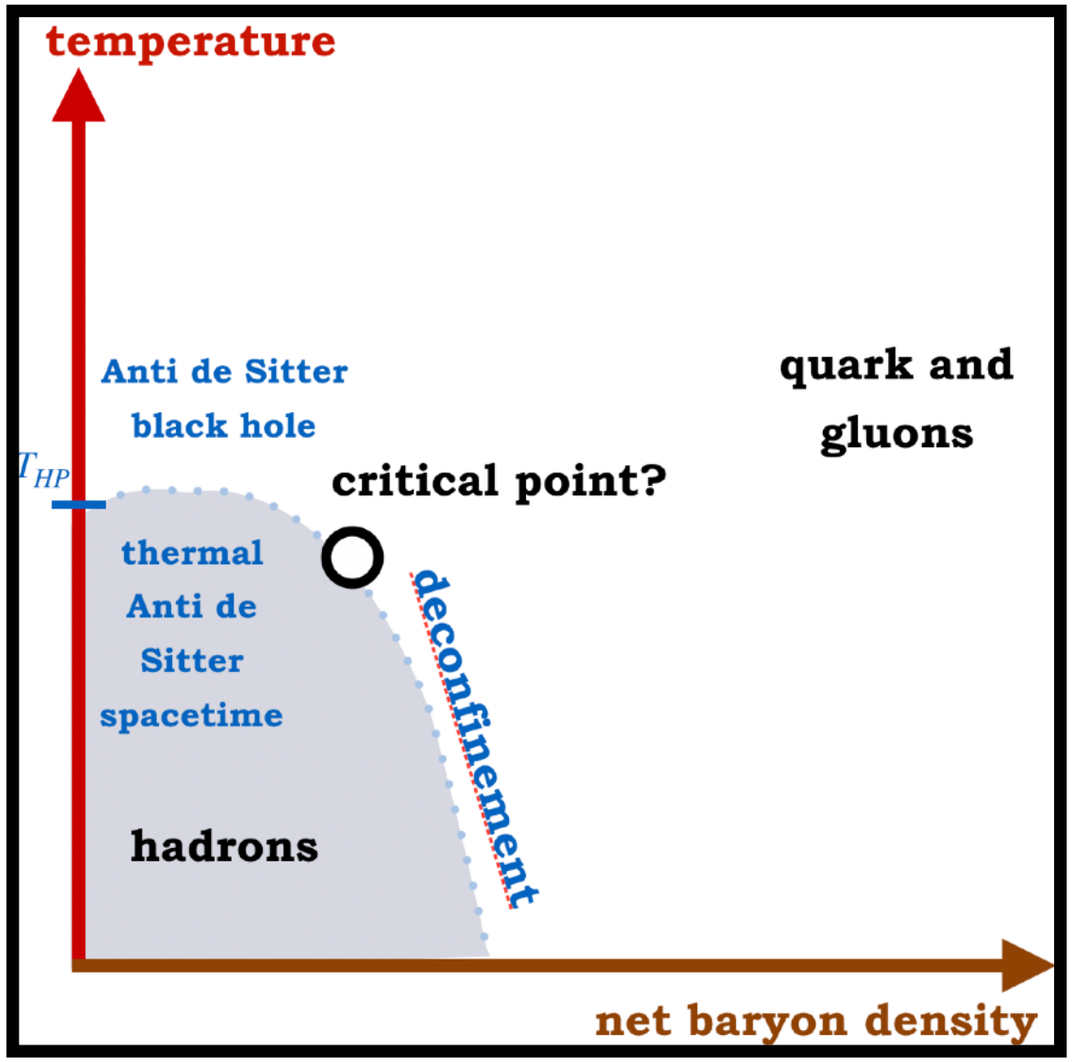
[Cartwright,Harms,Kaminski,Thomale; Class.Quant.Grav (2022]
[Cartwright,Harms,Kaminski; JHEP (2021)]



Ad ex 5: Holographic Deconfinement: Witten's Interpretation

- **strongly coupled** thermal **Conformal** Field Theory (CFT) at **large number of colors** N_c *have* ***phase transition*** ***dual to HP transition***
  [Witten; (1998)]
- these are ***deconfinement*** transitions (Wilson loops calculated) *[Witten; (1998)]*
- specific example: $\mathcal{N}$=4 Super-Yang-Mills (SYM) is holographically dual to Einstein-Hilbert action with negative cosmological constant (AdS5) *[Maldacena; (1997)]*





holography concepts mentioned
see talks by David Mateos, Matti Jarvinen, Maximilian Attems

Ad ex 5: Novel Topological Skyrme AdS5 Black Hole

[Cartwright,Harms,Kaminski; JHEP (2021)]

Coordinates: $\psi \in (0,\pi),\ \theta \in (0,\pi),\ \phi \in (0,2\pi)$

Pauli matrices: $\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Gauge configuration:

$$U = e^{in\chi v^i \tau^i}$$

$$v = (\cos(\theta), \sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi))$$

$$K_\mu = U^{-1}\partial_\mu U$$

Metric:

$$ds^2 = \frac{1}{A(r)}\mathrm{d}r^2 - A(r)\mathrm{d}t^2 + r^2\left(h_1(\psi)\mathrm{d}\psi^2 + h_2(\psi)(\mathrm{d}\theta^2 + \sin(\theta)^2\mathrm{d}\phi^2)\right)$$

with $h_1(\psi) = n^2, \quad h_2(\psi) = \sin(n\psi), \quad A(r) = \underbrace{\frac{1}{e^2r^2} + \frac{\log(r)}{e^2r^2}}_{\textbf{Skyrmion contributions}} + \frac{r^2}{L^2} - \underbrace{\frac{m_t}{r^2}}_{\textbf{mass parameter}} + 1$

Ad ex 5: Interpretation within Dual Field Theory

[Cartwright,Harms,Kaminski; JHEP (2021)]

SU(2)-current vanishes in field theory:

$$\langle J_i \rangle = \lim_{r\to\infty} r^2 A_i = \frac{-1}{2} A_i^{(2),a} \tau_a \,, \quad = 0$$

$$A_i^{ext} = \lim_{r\to\infty} A_i = \frac{-1}{2} A_i^{(0),a} \tau_a \,.$$

Two trace contributions:

$$\bar{\kappa} \left\langle T^i_{\ i} \right\rangle = \frac{3L^2}{4e^2} - \frac{3L^4}{4}$$

external field (explicit breaking)

conformal anomaly of N=4 SYM on $\mathbb{R} \times S^3$

Ad ex 5: Interpretation within Dual Field Theory

[Cartwright,Harms,Kaminski; JHEP (2021)]

SU(2)-current vanishes in field theory:

$$\langle J_i \rangle = \lim_{r\to\infty} r^2 A_i = \frac{-1}{2} A_i^{(2),a} \tau_a \,, \quad = 0$$

$$A_i^{ext} = \lim_{r\to\infty} A_i = \frac{-1}{2} A_i^{(0),a} \tau_a \,.$$

Two trace contributions:

$$\bar{\kappa} \left\langle T^i_{\ i} \right\rangle = \frac{3L^2}{4e^2} - \frac{3L^4}{4}$$

external field (explicit breaking)

conformal anomaly of N=4 SYM on $\mathbb{R} \times S^3$

Explicit field configuration of SU(2)-gauge field:

[Manton,Sutcliffe (2004)]

$$A_\psi^{(0),1} = -n\cos(\theta)\,,$$
$$A_\psi^{(0),2} = -n\sin(\theta)\cos(\phi)\,,$$
$$A_\psi^{(0),3} = -n\sin(\theta)\sin(\phi)\,.$$

$$A_\phi^{(0),1} = \sin^2(\theta)\sin^2(n\psi)\,,$$
$$A_\phi^{(0),2} = \sin(\theta)\sin(n\psi)\left(\sin(\phi)\cos(n\psi) - \cos(\theta)\cos(\phi)\sin(n\psi)\right)\,,$$
$$A_\phi^{(0),3} = -\sin(\theta)\sin(n\psi)\left(\cos(\theta)\sin(\phi)\sin(n\psi) + \cos(\phi)\cos(n\psi)\right)$$

$$A_\theta^{(0),1} = \sin(n\psi)\sin(\theta)\cos(n\psi)\,,$$
$$A_\theta^{(0),2} = \sin(n\psi)\left(\cos(\theta)\cos(\phi)\cos(n\psi) + \sin(\phi)\sin(n\psi)\right)\,,$$
$$A_\theta^{(0),3} = \sin(n\psi)\left(\cos(\phi)\sin(n\psi) - \cos(\theta)\sin(\phi)\cos(n\psi)\right)\,,$$

Gauge fields map $A_j^a \tau_a(x) : S^3 \to S^3$

$S^3_{coordinate}$ $\qquad$ $S^3_{SU(2)}$

➡**gauge field configuration has non-trivial topology with Chern number *n, counting how often coordinate sphere wraps SU(2) sphere***

Example 0: Discovery of new transport effect

Calculation: solve derivative-expanded Einstein equations (fluid/gravity).

Chiral Vortical Effect

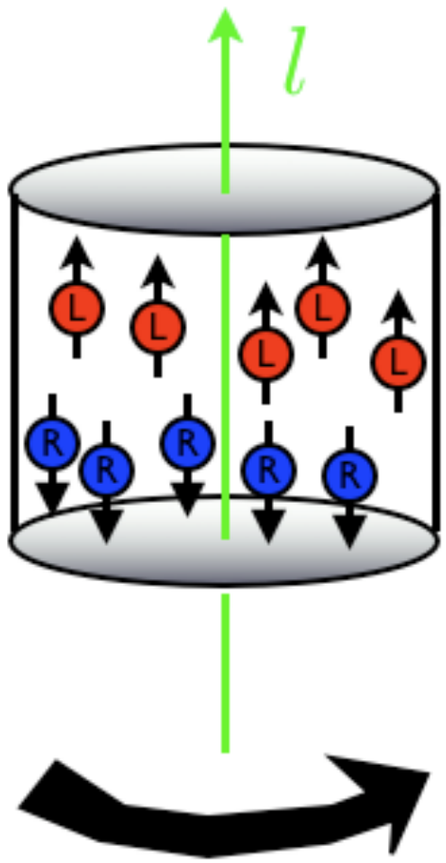
[Erdmenger,Haack,Kaminski,Yarom; JHEP (2008)]

[Banerjee et al.; JHEP (2011)]

[Son,Surowka; PRL (2009)]

$$J^\mu_A = \xi\, \Omega^\mu$$

vorticity



Chiral vortical conductivity: $\xi = C\,\mu_A{}^2 + c_g T^2$

Quantum transport effect originating from chiral anomaly (topologically protected, non-perturbative)

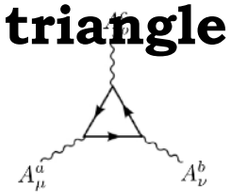
Measurements:

[Ball, Nature (2017)]

[Gooth, Landsteiner et al, Nature (2017)]

Chiral anomaly from triangle diagrams:

[Adler; Bell; Jackiw; (1969)]



Ad ex 1: Hydrodynamic modes live the longest ($k \approx 0$)

Interacting many-body systems at large temperature T have collective excitations, damped eigenmodes, with specific dispersion relations :

(assuming rotation invariance: $k \equiv |\vec{k}|$)

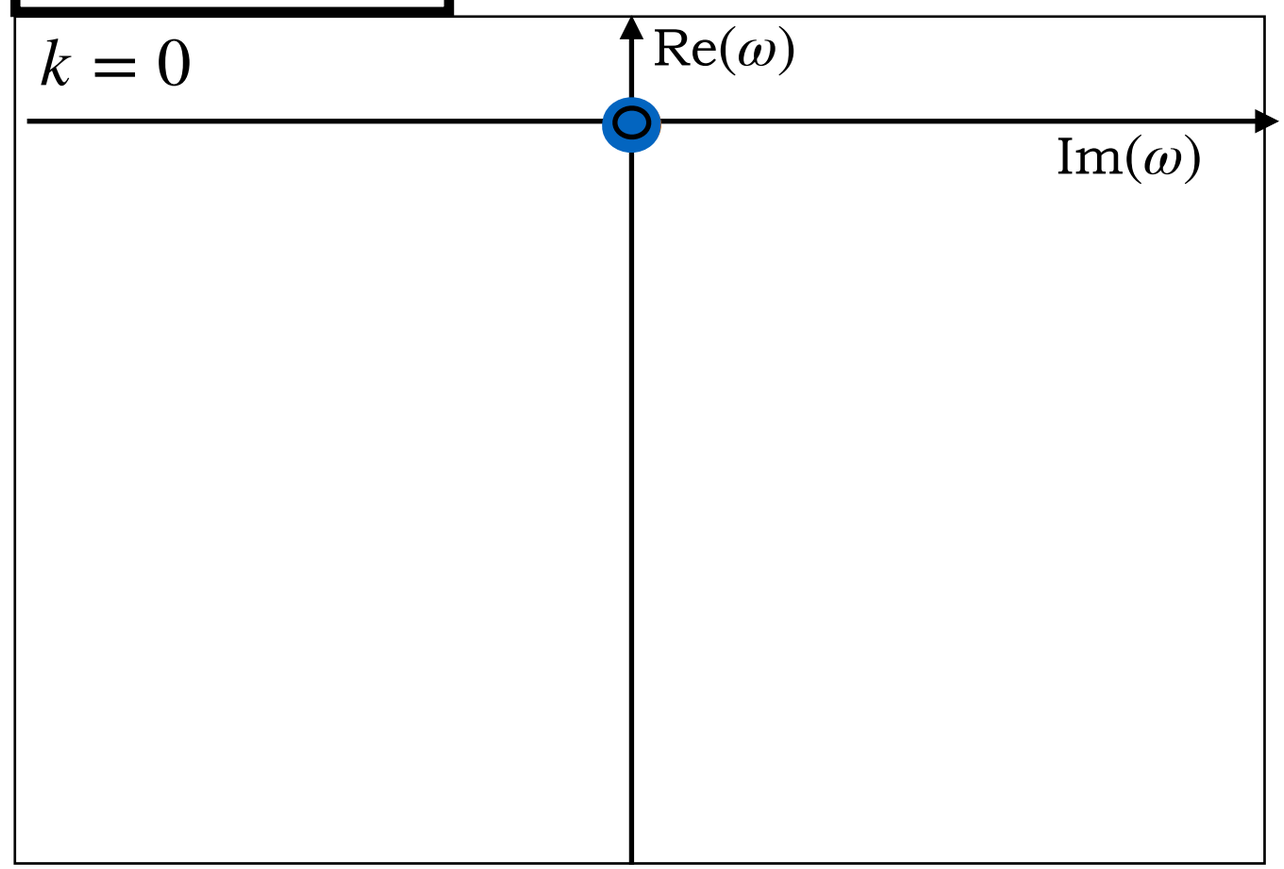
Sound modes (gapless)

$$\omega(k) = \pm v_s k - i\Gamma k^2 + \mathcal{O}(3)$$

Shear modes (gapless)

$$\omega(k) = -iDk^2 + \mathcal{O}(3)$$

$$G^R(\omega', k) \propto \frac{1}{\omega' - \omega(k)}$$



Complex frequency plane

Ad ex 1: Hydrodynamic modes live the longest ($k \approx 0$)

Interacting many-body systems at large temperature T have collective excitations, damped eigenmodes, with specific dispersion relations :

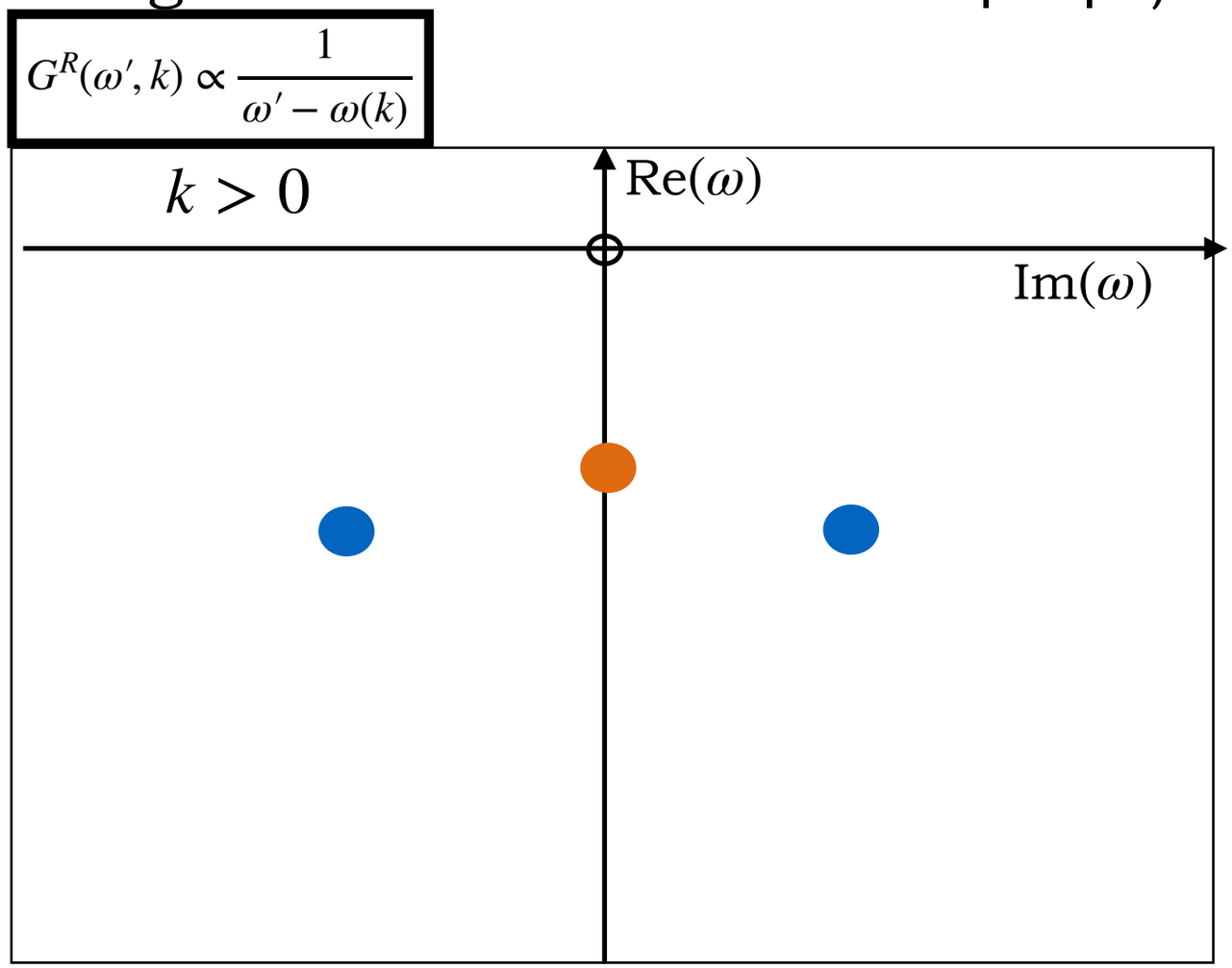
(assuming rotation invariance: $k \equiv |\vec{k}|$)

Sound modes (gapless)

$$\omega(k) = \pm v_s k - i\Gamma k^2 + \mathcal{O}(3)$$

Shear modes (gapless)

$$\omega(k) = -iDk^2 + \mathcal{O}(3)$$



Complex frequency plane

Ad ex 1: Hydrodynamic modes live the longest ($k \approx 0$)

Interacting many-body systems at large temperature T have collective excitations, damped eigenmodes, with specific dispersion relations :

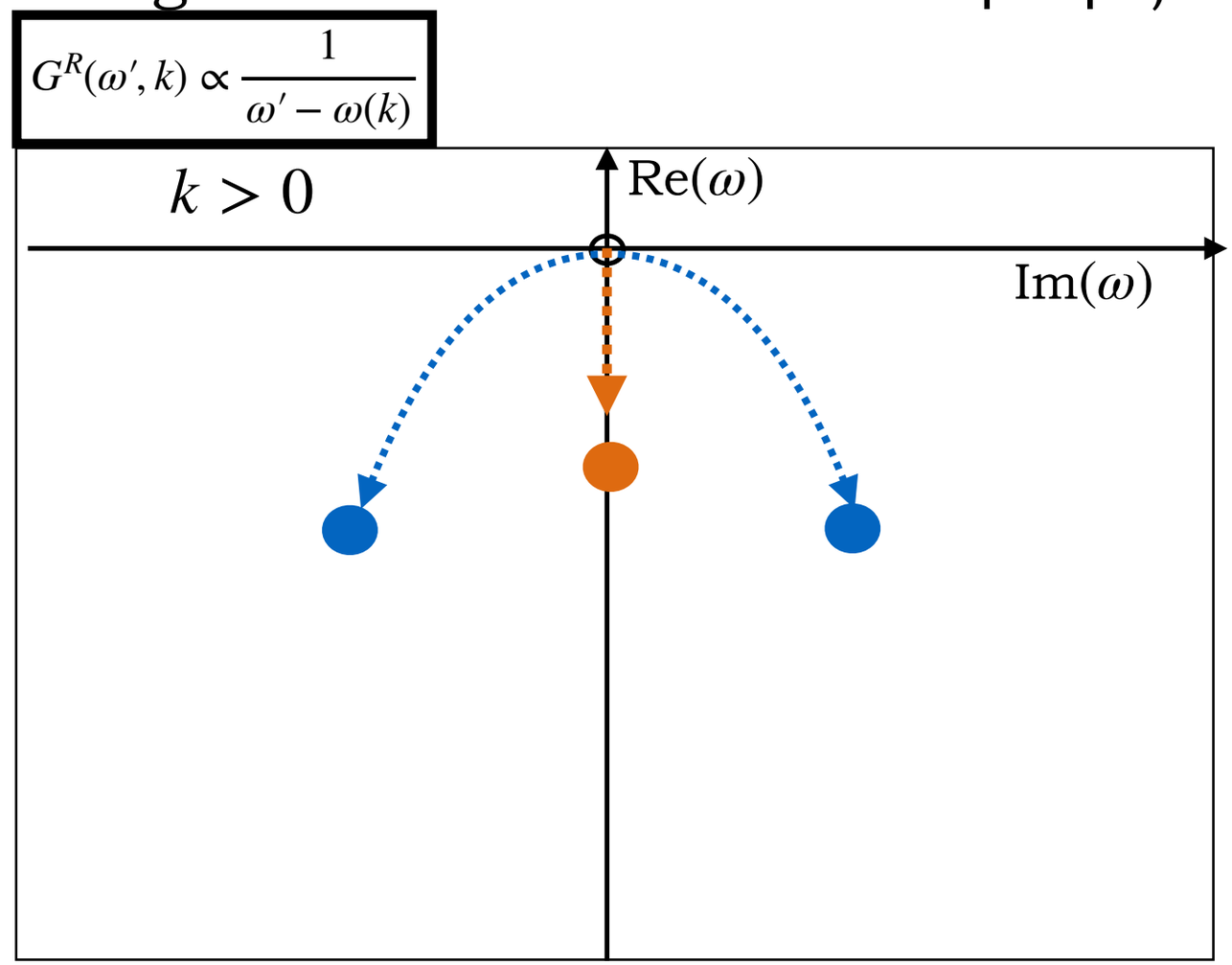
(assuming rotation invariance: $k \equiv |\vec{k}|$)

Sound modes (gapless)

$$\omega(k) = \pm v_s k - i\Gamma k^2 + \mathcal{O}(3)$$

Shear modes (gapless)

$$\omega(k) = -iDk^2 + \mathcal{O}(3)$$



Complex frequency plane

Ad ex 1: Non-hydrodynamic modes live the longest ($k \approx T$)

Interacting many-body systems at large temperature T have collective excitations, damped eigenmodes, with specific dispersion relations :

(assuming rotation invariance: $k \equiv |\vec{k}|$)

Sound modes (gapless)

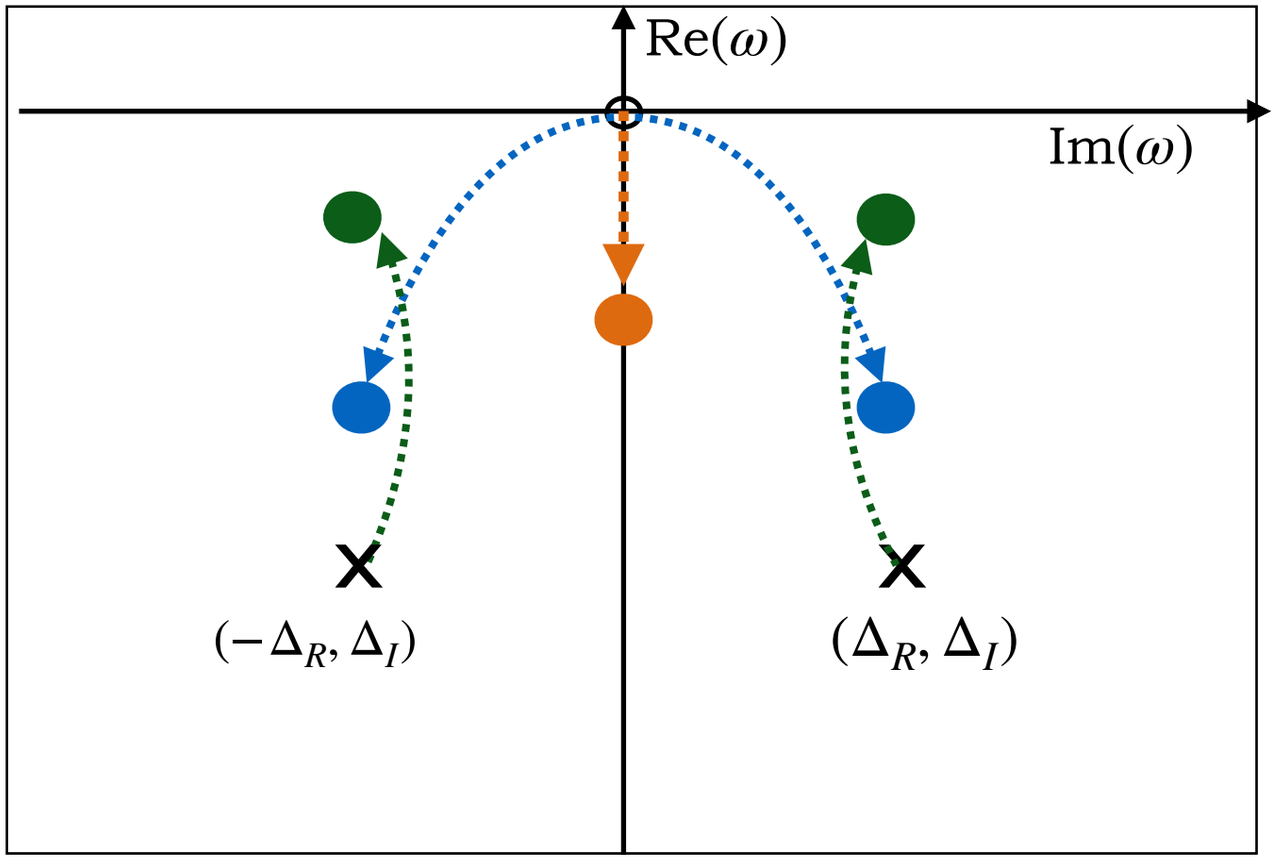
$$\omega(k) = \pm v_s k - i\Gamma k^2 + \mathcal{O}(3)$$

Shear modes (gapless)

$$\omega(k) = -iDk^2 + \mathcal{O}(3)$$

Non-hydrodynamic modes

$$\omega(k) = \underbrace{\pm\Delta_{\mathrm{R}} - i\Delta_{\mathrm{I}}}_{\text{gap}} + v_0 k - i\Gamma_0 k^2 + \mathcal{O}(3)$$



Complex frequency plane

Ad ex 1: Non-hydrodynamic modes live the longest ($k \approx T$)

Interacting many-body systems at large temperature T have collective excitations, damped eigenmodes, with specific dispersion relations :

(assuming rotation invariance: $k \equiv |\vec{k}|$)

Sound modes (gapless)

$$\omega(k) = \pm v_s k - i\Gamma k^2 + \mathcal{O}(3)$$

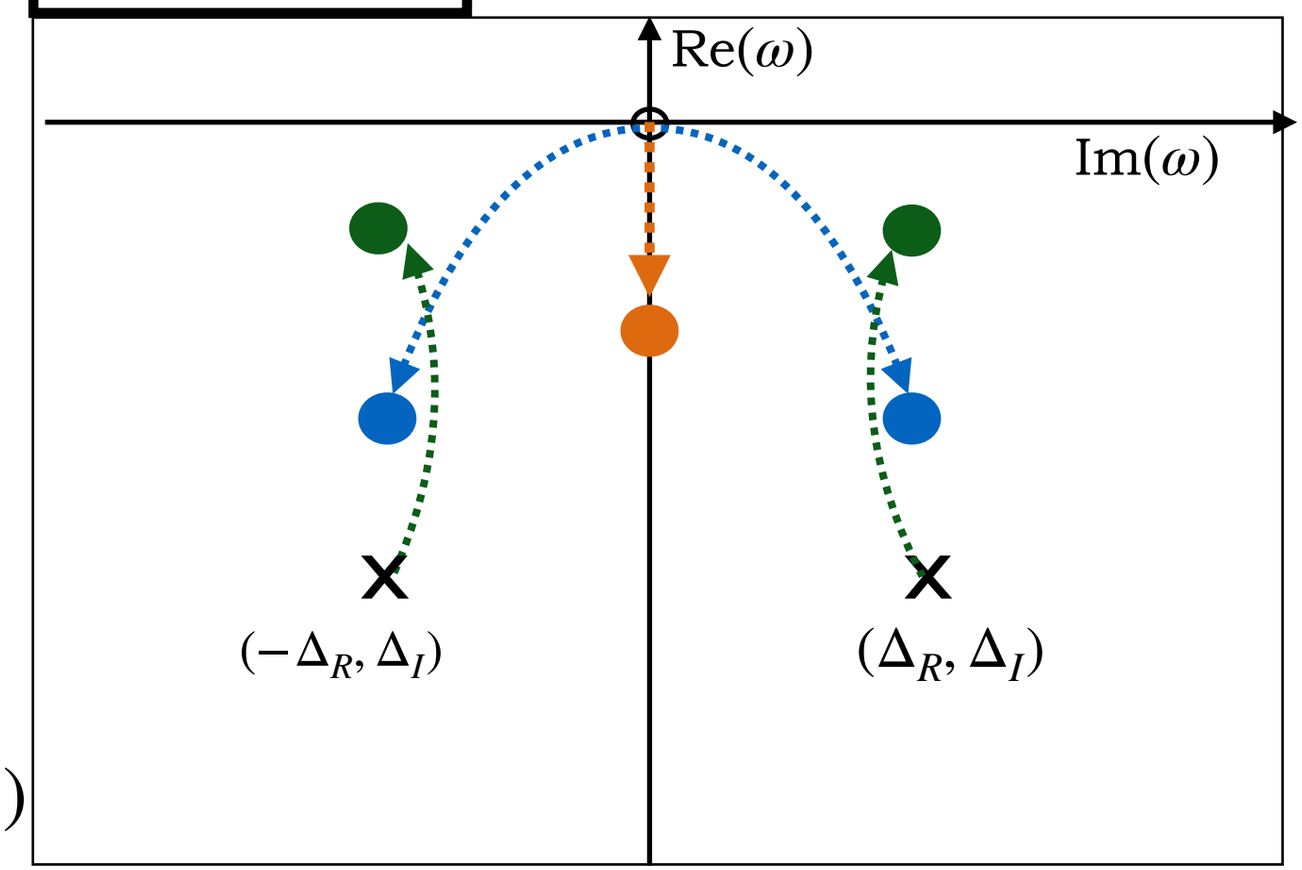
Shear modes (gapless)

$$\omega(k) = -iDk^2 + \mathcal{O}(3)$$

Non-hydrodynamic modes

$$\omega(k) = \underbrace{\pm\Delta_{\mathrm{R}} - i\Delta_{\mathrm{I}}}_{\text{gap}} + v_0 k - i\Gamma_0 k^2 + \mathcal{O}(3)$$

$$G^R(\omega', k) \propto \frac{1}{\omega' - \omega(k)}$$



Complex frequency plane

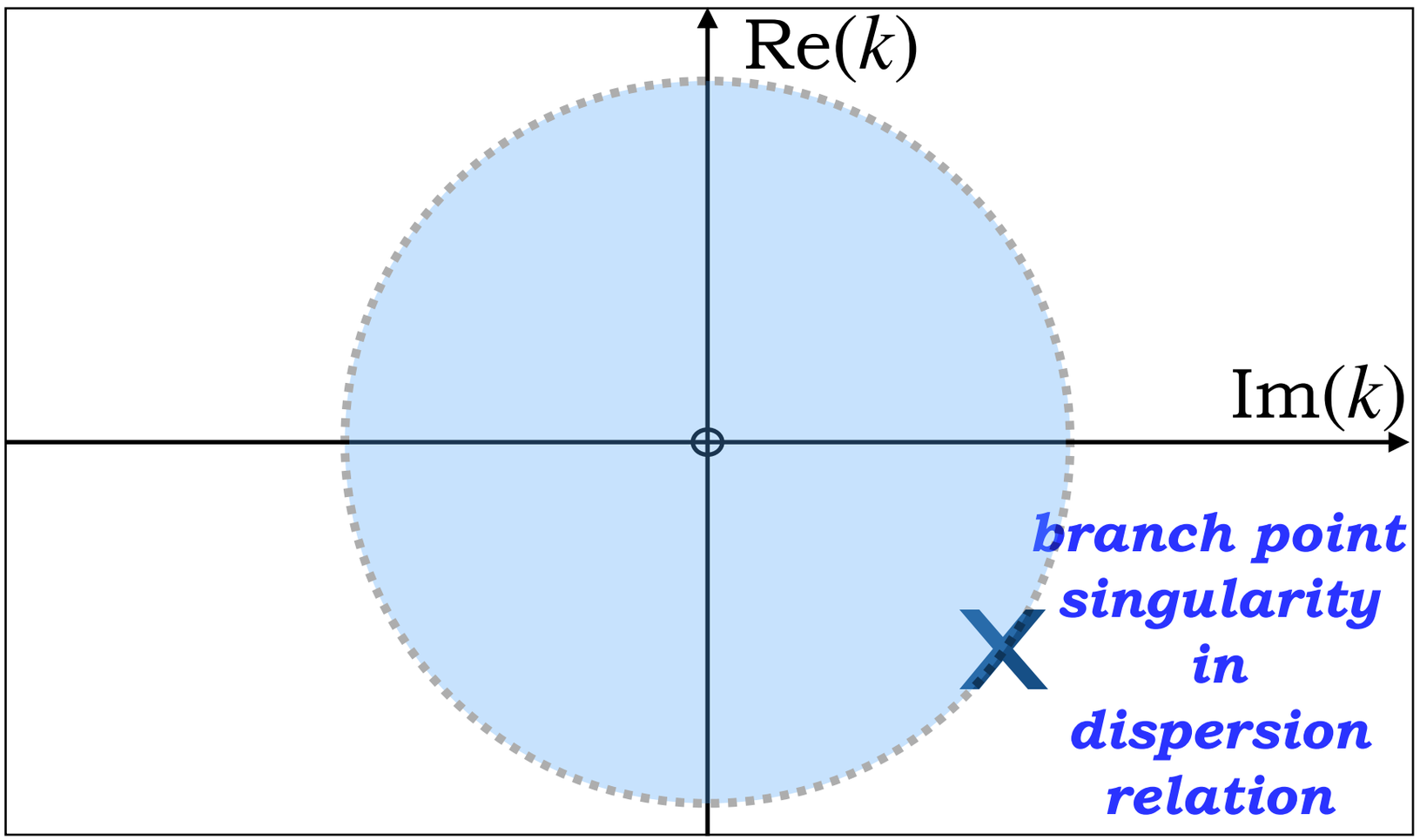
➡ When non-hydrodynamic modes are closer to origin, they live longer than hydrodynamic modes, hydrodynamics breaks down!

Ad ex 1: Branch point singularities in dispersion relations

Branch point singularities (critical points) in dispersion relations limit convergence radius of hydrodynamics

$$G^R(\omega', k) \propto \frac{1}{\omega' - \omega(k)}$$

$\omega(k), k \in \mathbb{C}$



Complex ***momentum*** *plane*

[Grozdanov,Kovtun,Starinets,Tadic; PRL (2019)]
[Heller,Serantes,Spalinski,Svensson,Withers; PRD (2020)]

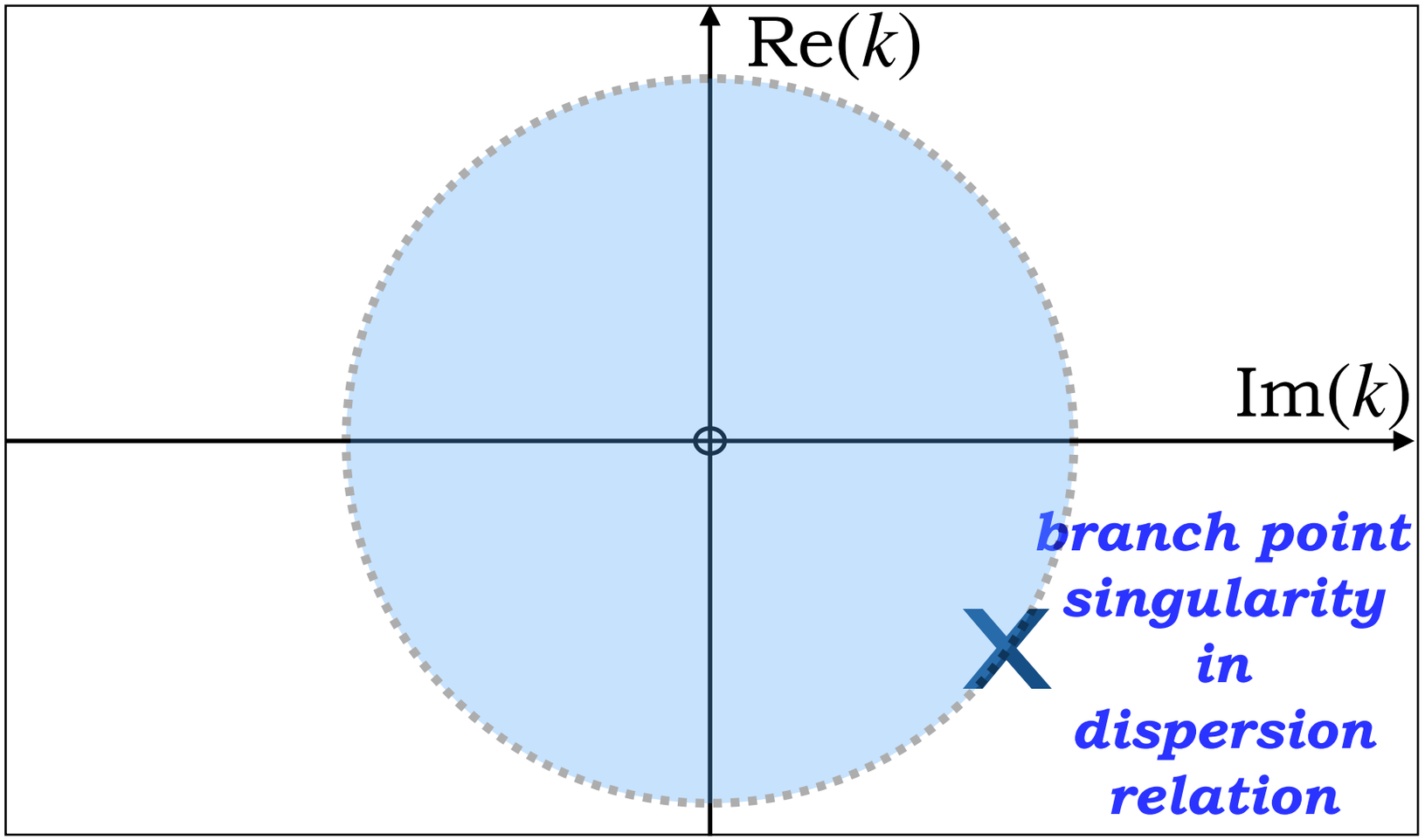
Ad ex 1: Branch point singularities in dispersion relations

Branch point singularities (critical points) in dispersion relations limit convergence radius of hydrodynamics

$$G^R(\omega', k) \propto \frac{1}{\omega' - \omega(k)}$$

[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza; under review with PRL]

$$\omega(k), k \in \mathbb{C}$$

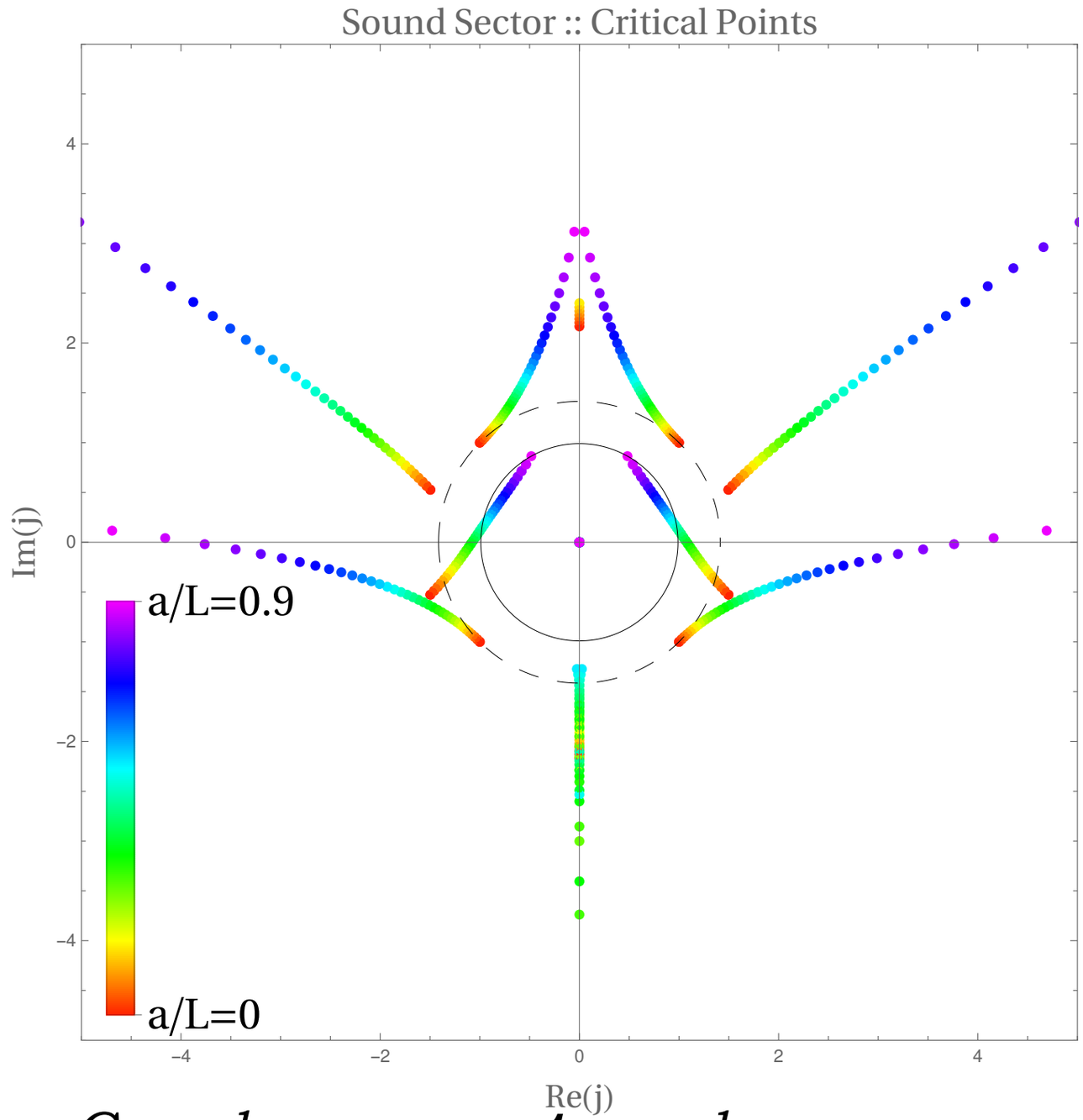


Complex ***momentum*** *plane*

[Grozdanov,Kovtun,Starinets,Tadic; PRL (2019)]
[Heller,Serantes,Spalinski,Svensson,Withers; PRD (2020)]

Ad ex 1: Singularities in the dispersion relations

Branch singularities (critical points) in dispersion relations of N=4 Super-Yang-Mills theory in a rotating state with angular momentum a/L.



Complex ***momentum*** *plane*

[Cartwright,Garbiso-Amano;Kaminski,Noronha,Speranza;arXiv:2112.10781]

Ad ex 2: Transport coefficients & Kubo formulae for plasma in strong magnetic field

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

➡ magnetic field + anomaly leads to anisotropy and breaks parity ($\mathcal{P}$) and time-reversal ($\mathcal{T}$)
➡ gold mine of novel Physics

coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
Thermodynamic $\left(\lim_{\mathbf{k}\to 0}\lim_{\omega\to 0}\right)$, non-dissipative					
helicity 1					
M_2	perp. magnetic vorticity susceptibility	$T^{xz}T^{yz}$ (2.30)	+	-	+
M_5	magneto-vortical susceptibility	$T^{tx}T^{yz}$ (2.30,2.31)	+	-	+
ξ	chiral vortical conductivity	J_xT_{ty} (2.38,2.39)	+	+	+
ξ_B	chiral magnetic conductivity	J^xJ^y (2.38,2.39)	+	-	+
ξ_T	chiral vortical heat conductivity	$T^{tx}T^{ty}$ (2.38,2.39)	+	-	+
helicity 0					
M_1	magneto-thermal susceptibility	J^tT^{xx} (2.32)	+	+	-
M_3	magneto-acceleration susceptibility	J^tT^{tt} (2.32)	+	+	-
M_4	magneto-electric susceptibility	J^tJ^t (2.32)	+	-	-

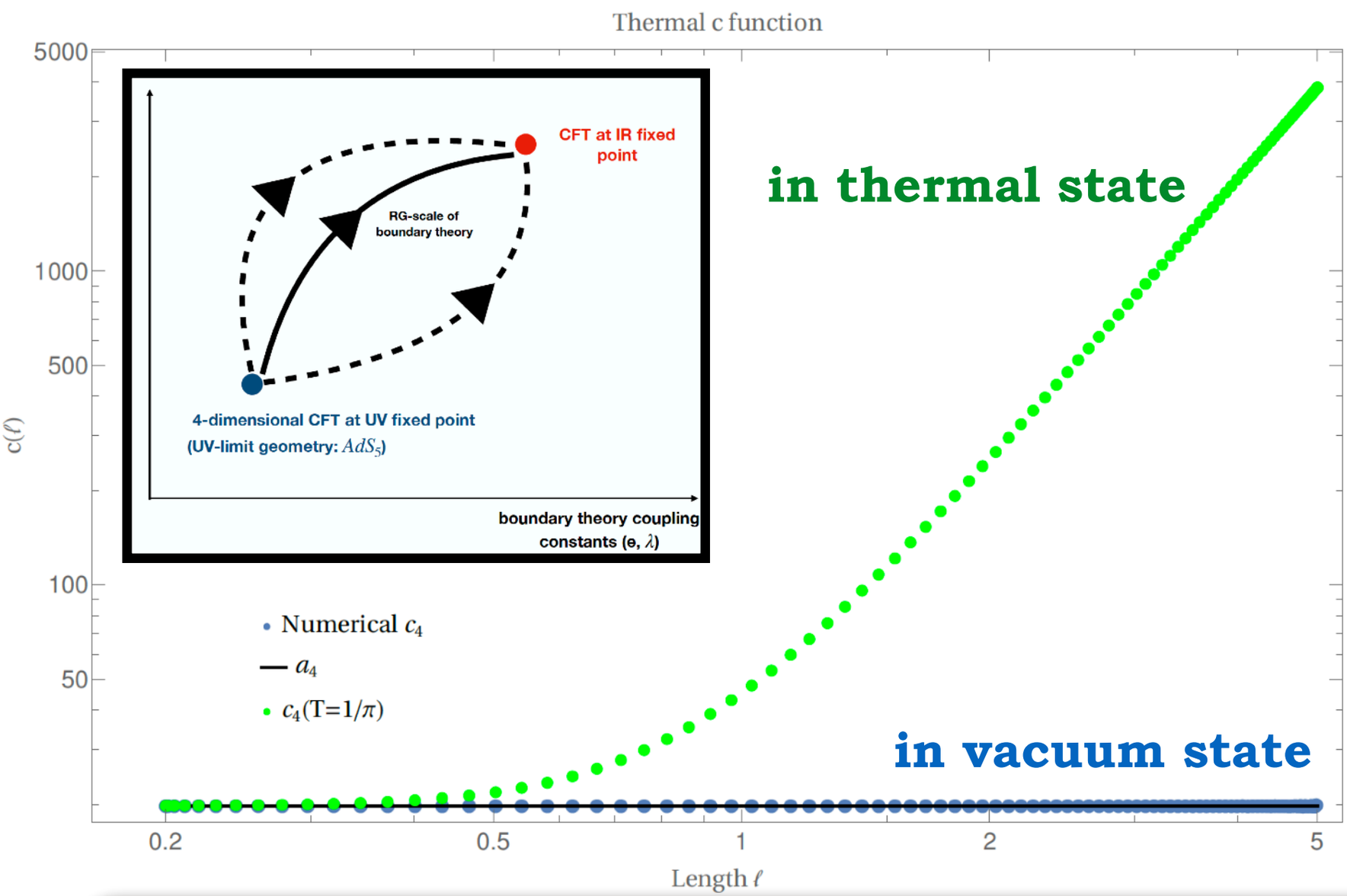
dissipative, hydrodynamic $\left(\lim_{\omega\to 0}\lim_{\mathbf{k}\to 0}\right)$					
coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
helicity 2					
$\eta_\perp$	perp. shear viscosity	$T_{xy}T_{xy}$ (2.55)	+	+	-
helicity 1					
$\eta_{\parallel}$	parallel shear viscosity	$T^{xz}T^{xz}$ (2.59a)	+	+	-
$\tilde{\eta}_{\parallel}$	parallel Hall viscosity	$T_{yz}T_{xz}$ (2.59b)	+	-	+
$\boxed{c_8} \propto c_{15}$	shear-induced conductivity	$T_{tx}T_{xz}$, $T_{tx}T_{yz}$ (2.57)	+	+	+
$\rho_\perp$	perp. resistivity	J^xJ^x (2.54)	+	+	-
$\tilde{\rho}_\perp$	Hall resistivity	J^xJ^y (2.55e)	+	+	-
$\sigma_{\parallel}$	long. conductivity	J^zJ^z (2.53a)	+	+	-
$\sigma_\perp$	perp. conductivity	$\rho_{ab} \equiv (\sigma^{-1})_{ab} = \rho_\perp\delta_{ab} + \tilde{\rho}_\perp\epsilon_{ab}$	+	+	-
helicity 0					
η_1	bulk viscosity	$\mathcal{O}_1\mathcal{O}_1$ (2.55c)	+	+	-
η_2	bulk viscosity	$\mathcal{O}_2\mathcal{O}_2$ (2.55d)	+	+	-
ζ_1	bulk viscosity	$T^{ij}(T^{xx}+T^{yy})$(2.55a)	+	+	-
ζ_2	bulk viscosity	$3\zeta_2 - 6\eta_1 = 2\eta_2$	+	+	-
$\boxed{c_4}$	expan.-induced long. cond.	J_xT_{xx} (2.57)	+	-	-
$\boxed{c_5}$	expan.-induced long. cond.	J_zT_{zz} (2.57)	+	-	-
c_3		$c_5 = -3(c_3 + c_4)$	+	-	-

Non-dissipative Hydrodynamic $\left(\lim_{\omega\to 0}\lim_{\mathbf{k}\to 0}\right)$					
coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
helicity 2					
$\tilde{\eta}_\perp$	transverse Hall viscosity	$T_{xy}(T_{xx} - T_{yy})$(2.55f)	+	-	+
helicity 1					
$\boxed{c_{10}} \propto c_{17}$	shear-induced Hall cond.	$T^{tx}T^{xz}, T^{tx}T^{yz}$ (2.60,2.62a,2.62b)	+	+	+
$\tilde{\sigma}_\perp$	Hall conductivity	J^xJ^x, J^xJ^y (2.54,2.53b,2.53c)	+	-	+

$$B \sim \mathcal{O}(1)$$
$$B \ll T^2$$

Ad ex. 3: Entropic c-function *increases* in *thermal* state

[Cartwright,Kaminski; JHEP (2021)]



$$a_4 = \beta_4 \frac{\ell^3}{H^2} \frac{\partial S_a}{\partial \ell}$$

[Casini,Huerta; Phys.Lett.B (2004)]
[Nishioka,Takayanagi; JHEP (2007)]
[Myers,Sinha; JHEP (2011)]

H: IR-regulator
β_4: known constant

- ➡ **c-function increases in thermal state (violates c-theorem?)**
- ➡ **constant in vacuum of CFT (c-theorem valid)**
- ➡ **IR limit: thermal entropy**

[Zamolodchikov; JETP Lett.(1986)]
[Komargodski,Schwimmer; (2011)]
[Osborn; Phys.Lett.B(1988)]
[Cardy; Phys.Lett.B(1988)]

Hydrodynamics and holography in parallel

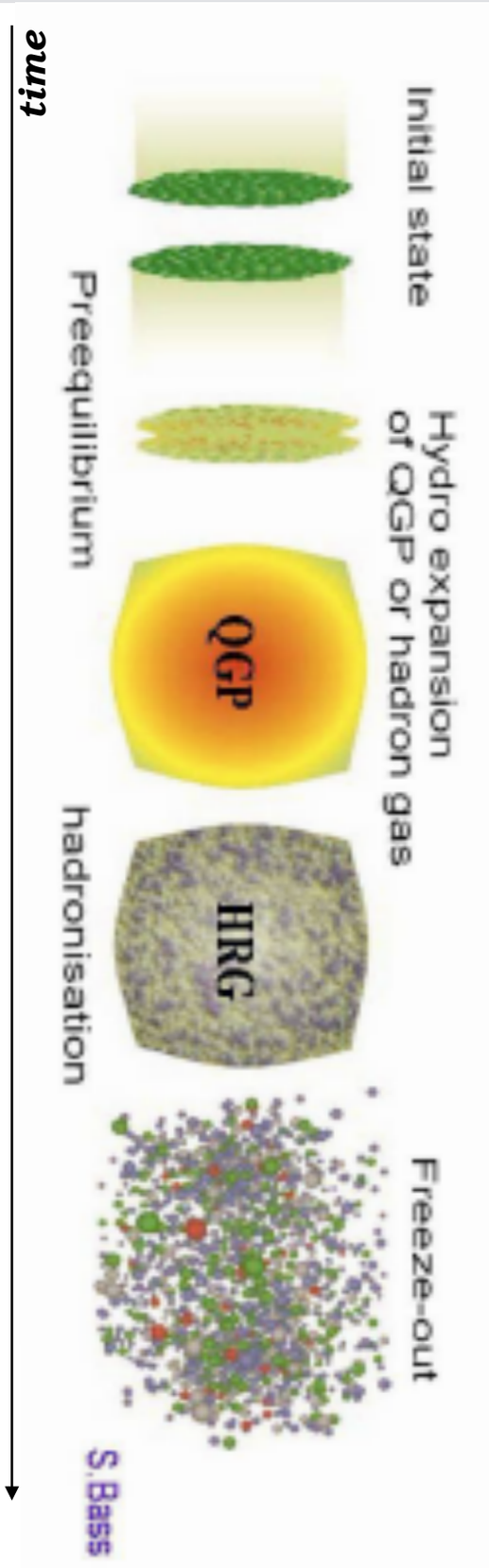
HYDRODYNAMICS & THERMODYNAMICS

Chiral Magnetic Effect (CME) from chiral anomaly

[Kharzeev; PRC (2004)]
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]

$$J_A^\mu = \xi_B B$$

$$\xi_B \sim C\mu_A \qquad \nabla_\mu J_A^\mu = C\, E \cdot B$$



HOLOGRAPHY

Hydrodynamics and holography in parallel

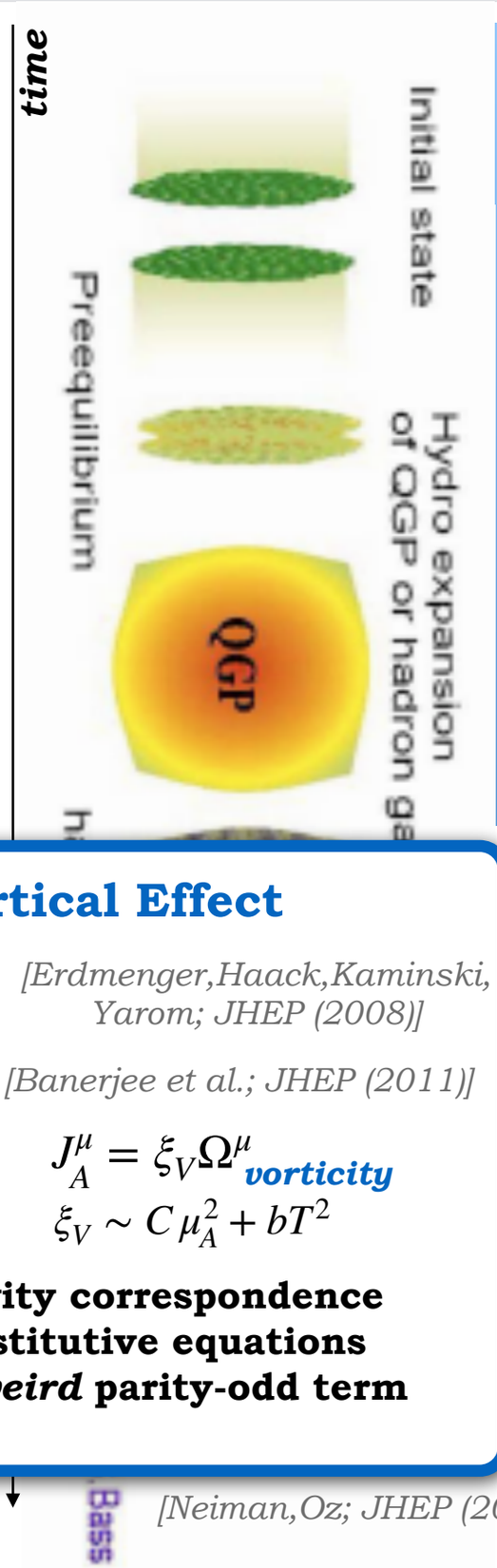
HYDRODYNAMICS & THERMODYNAMICS

Chiral Magnetic Effect (CME) from chiral anomaly

[Kharzeev; PRC (2004)]
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]

$$J_A^\mu = \xi_B B$$

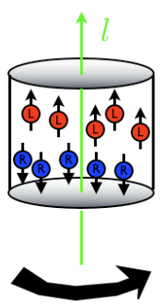
$\xi_B \sim C\mu_A$ $\qquad$ $\nabla_\mu J_A^\mu = C\, E \cdot B$



HOLOGRAPHY

hydro and holo in parallel

Chiral Vortical Effect



[Erdmenger,Haack,Kaminski, Yarom; JHEP (2008)]

[Banerjee et al.; JHEP (2011)]

$$J_A^\mu = \xi_V \Omega^\mu$$ ***vorticity***

$$\xi_V \sim C\mu_A^2 + bT^2$$

- **fluid/gravity correspondence**
- **gives constitutive equations**
- **contain *weird* parity-odd term**

[Neiman,Oz; JHEP (2010)]

Hydrodynamics and holography in parallel

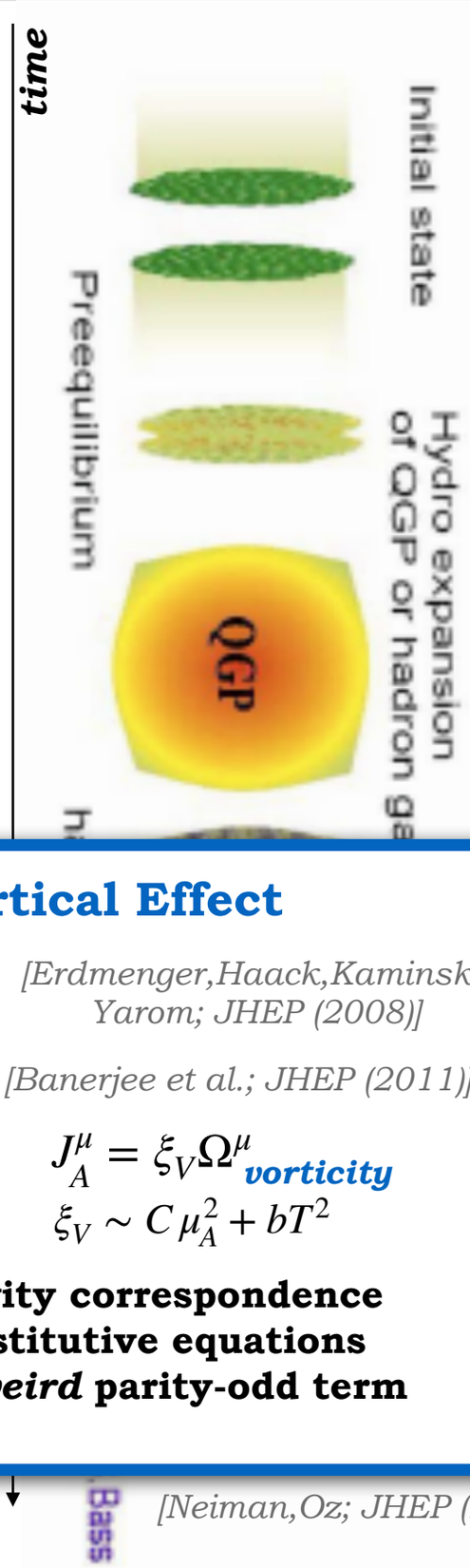
HYDRODYNAMICS & THERMODYNAMICS

Chiral Magnetic Effect (CME) from chiral anomaly

[Kharzeev; PRC (2004)]
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]

$$J_A^\mu = \xi_B B$$

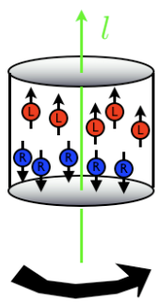
$$\xi_B \sim C\mu_A \qquad \nabla_\mu J_A^\mu = C\,E\cdot B$$



[Neiman,Oz; JHEP (2010)]

hydro and holo in parallel

Chiral Vortical Effect



[Erdmenger,Haack,Kaminski, Yarom; JHEP (2008)]

[Banerjee et al.; JHEP (2011)]

$$J_A^\mu = \xi_V \Omega^\mu \quad \textit{vorticity}$$

$$\xi_V \sim C\mu_A^2 + bT^2$$

- **fluid/gravity correspondence**
- **gives constitutive equations**
- **contain *weird* parity-odd term**

HOLOGRAPHY

CME far from equilibrium, strong *B*

- **non-expanding plasma**
  [Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]
- **expanding plasma**
  [Cartwright,Kaminski,Schenke; PRC (2022)]

Frequency dependence of CME

[Amado,Landsteiner,Pena_Benitez; JHEP (2011)]
[Li,Yee; PRD (2018)]
[Koirala; PhD thesis (2020)]

CME near equilibrium (+hydro)

- **weak magnetic field *B***
  [Son,Surowka; PRL (2009)]
  [Kharzeev,Yee; PRD (2011)]
  [Ammon, Kaminski et al.; JHEP (2017)]
- **strong *B*** *[Ammon,Leiber,Macedo; JHEP (2016)]*
  [Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

Selected own results strong fields & far from equilibrium

1. Strong field physics (and high density & temperature)

*entanglement entropy in strong B (effect of quantum anomalies)

*(fluid) quantum dynamics in strong B

[Cartwright,Kaminski; JHEP (2021)]

[Cartwright,Kaminski,Schenke; PRC (2022)]

[Koirala; PhD thesis (2020)]

[Ammon, Grieninger, Hernandez, Kaminski et al.; JHEP (2021)]

[Endrodi, Kaminski et al.; JHEP (2018)]

[Ammon, Kaminski et al.; JHEP (2017)]

['t Hooft et al.; Springer Proc.Phys. (2018)]

2. Far-from-equilibrium quantum dynamics

[Cartwright,Kaminski,Knipfer (2022)]

[Hongo,Huang,Kaminski et al.; JHEP (2022)]

[Hongo,Huang,Kaminski et al.; JHEP (2021)]

[Abbasi,Kaminski; PRD (2022)]

[Abbasi,Kaminski; JHEP (2020)]

[Garbiso,Kaminski; JHEP (2020)]

[Wondrak,Kaminski,Bleicher; Phys.Lett.B (2020)]

[Jensen,Kaminski et al.; Phys.Rev.Lett. (2012)]

[Erdmenger,Haack,Kaminski,Yarom; JHEP (2008)]

1.+2. Quantum dynamics far from equilibrium in strong fields

[Cartwright,Kaminski,Schenke; PRC (2022)]

[Cartwright,Kaminski; JHEP (2019)]