Useful results from the AdS/CFT correspondence?

- holographic quarks, mesons and the Super-Yang-Mills phase diagram

EMMI workshop ‘Quarks, Hadrons and the Phase Diagram of QCD’, St Goar, 31st August 2009

Matthias Kaminski
IFT-UAM/CSIC Madrid
Outline

I. Invitation: AdS/QGP Correspondence

II. Quarks

III. Mesons

IV. SYM phase diagrams

V. Holographic hydrodynamics

VI. Summary
I. Invitation: AdS/QGP Correspondence

-What can we do with gauge/gravity?

- Compute observables in strongly coupled QFTs
- Meson spectra/melting  
  \[ \text{Review: [Erdmenger, Evans, Kirsch, Threlfall 0711.4467]} \]
- Quark energy loss, Jets
- Thermodynamics/Phase diagrams
- Holographic hydrodynamics (beyond Muller-Israel-Stewart)
- Transport coefficients (e.g. ‘universal’ viscosity bound)
- Model QCD equation of state \((v_s, \xi/s\) match lattice-QCD)
- Deconfinement & Break: Chiral, Conformal, SUSY
- Condensed matter applications (strongly corr. electrons)

[AdS/QCD (bottom-up approach)]
I. Invitation: AdS/QGP Correspondence

- General features of gauge/gravity

- Large N, large $\lambda = g_{YM}^2 N$

\[ \begin{align*}
\text{Gauge} & \quad \text{duality} \quad \text{Gravity} \\
\text{strong (weak)} & \quad \text{weak (strong)} \\
d\text{-dimensional} & \quad (d+1)\text{-dimensional} \\
\text{symmetries} & \quad \text{symmetries}
\end{align*} \]
I. Invitation: AdS/QGP Correspondence

-General features of gauge/gravity

Gauge theory \(\text{duality}\) Gravity theory

strong (weak) \(\text{duality}\) weak (strong)

dimensional \(\text{holography}\) \((d+1)\)-dimensional

symmetries \(\text{symmetries}\)

Large \(N\), large \(\lambda = g_{YM}^2 N\)

QFT lives on AdS-boundary

\(\rho\)

QFT

\(\rho_{bdy}\)

AdS
I. Invitation: AdS/QGP Correspondence
- General features of gauge/gravity

\[ \text{Large } N, \text{ large } \lambda = g_{YM}^2 N \]

\begin{align*}
\text{Gauge theory} & \quad \text{duality} \quad \text{Gravity theory} \\
\text{strong (weak)} & \quad \leftrightarrow \quad \text{weak (strong)} \\
\text{d-dimensional} & \quad \leftrightarrow \quad (d+1)-\text{dimensional} \\
\text{symmetries} & \quad \leftrightarrow \quad \text{symmetries}
\end{align*}

\[ \begin{array}{c}
\text{Dictionary:} \\
\text{Operator} J_\mu \quad \leftrightarrow \quad \text{Field} A_\mu \\
\text{Correlator} G^{\text{ret}} \quad \leftrightarrow \quad \frac{\delta^2}{\delta A_{\text{bdy}} \delta A_{\text{bdy}}} S_{\text{Sugra}} \\
\text{QFT Feature} \quad \leftrightarrow \quad \text{Geometry} \\
(\text{energy scale}) \quad \leftrightarrow \quad (\text{radial coord.}) \\
(\text{phase transition}) \quad \leftrightarrow \quad (\text{geom. transition})
\end{array} \]
I. Invitation: AdS/QGP Correspondence

-General features of gauge/gravity

Gauge theory $\leftrightarrow$ Gravity theory

Strong (weak) $\leftrightarrow$ weak (strong)

d-dimensional $\leftrightarrow$ (d+1)-dimensional

Symmetries $\leftrightarrow$ Symmetries

**Dictionary:**

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$J \mu$, $J^\mu$, $A^\mu$, $A_{\mu}$

**Holography:**

- Geometric transition ($\delta^2 A_{b \mu} d\gamma \delta^2 A_{b \mu} d\gamma$)
- Phase transition ($\delta^2 \mu$)

**Boundary Asymptotics:**

$A = A^{(0)} + \frac{A^{(2)}}{\rho^2} + \ldots$

- Normalizable ($\langle V \rangle$)
- Non-normalizable (source)

$A^{(2)} = \langle J \rangle$

$\lambda = g_{YM} \sqrt{N}$
I. Invitation: AdS/QGP Correspondence

-Why does it work?

Stack of N D3-branes (coincident) in 10 dimensions

Two distinct ways to describe this stack:

4-dimensional worldvolume theory on the D3-branes (e.g. $\mathcal{N} = 4$ Super-Yang-Mills)

gauge side

AdS$_5 \times S^5$ near-horizon geometry (e.g. Supergravity)

gravity side
I. Invitation: AdS/QGP Correspondence

-Why does it work?

**Geometric setup:** Strings/Branes

Find solution configuration

**Field Theory result**

Stack of N **D3-branes** (coincident) in 10 dimensions

Two distinct ways to describe this stack:

4-dimensional worldvolume theory on the D3-branes (e.g. $\mathcal{N} = 4$ Super-Yang-Mills)  

**gauge side**

Near-horizon geometry

$AdS_5 \times S^5$

Add/change geometric objects on ‘gravity side’:

**Geometric setup:** Strings/Branes

Find solution configuration

**Field Theory result**

Example: Schwarzschild radius corresponds to temperature

-How does it work?
II. Quarks
- Geometric setup
- Gravity solution
- Results
- Discussion
II. Quarks - Geometric setup (general)

String (ends charged)

$q_{\text{fundamental}}$

$A_\mu$

? 

black hole horizon ($T = T_{\text{Hawking}}$)

(created by massive D3-branes)

AdS boundary ($R^{3,1}$)
II. Quarks - Geometric setup (general)

- AdS boundary ($R^{3,1}$)
- D7-brane (optional)
- String (ends charged)
- $A_\mu$
- $q_{fundamental}$
- black hole horizon ($T = T_{Hawking}$)
  (created by massive D3-branes)
II. Quarks - Gravity solutions & results (drag)

- left: trailing string solution
- right: unphysical analytic solution

quark E, p-loss rate:

\[
\frac{dp}{dt} = \frac{1}{v} \frac{dE}{dt} = -\frac{\pi}{2} \sqrt{\lambda T^2} \frac{v}{\sqrt{1 - v^2}}
\]

equilibration times:

\[
\frac{dp}{dt} = -\frac{p}{\tau_q}, \quad \tau_q = \frac{2m_q}{\pi T^2 \sqrt{\lambda}}
\]

\[\tau_{\text{charm}} \approx 2 \, \text{fm}\]
\[\tau_{\text{bottom}} \approx 6 \, \text{fm}\]
II. Quarks  -Discussion

- Drag on heavy quarks in thermal SYM

- Viscous drag has upper bound: $\mu \leq 2\pi T$ ($\frac{dp}{dt} = -\mu p$)

- Mechanism: Energy & Momentum flow along string
  - not scattering (string fluctuations)
  - not glueball emission (closed string ‘emission’)
  - rather like a wake of a boat [Gubser,Pufu,Yarom 0706.0213] [Chesler,Yaffe 0706.0368]

Jets: complete energy density and flux (heavy quark at v)
  - supersonic: Mach cone
  - near cone: most of energy flux flows orthogonal to front
  - diffusion wake behind quark
  - complete computation vs hydro: hydrodynamics valid! [Chesler]
III. Mesons

- Geometric setup
- Gravity solution
- Results
- Discussion
III. Mesons  - Geometric setup

Stationary gravity ‘background’ gives equilibrium thermodynamics
Gravity ‘fluctuations’ give field theory dynamics

\[ q_{\text{fund.}} \longrightarrow \bar{q}_{\text{anti}} \]

D7-brane

short string
(low-energy excitation)
\[ \tilde{A}_\mu \]

fluctuation of corresponding current
(fundamental-antifundamental)
\[ \tilde{J}_\mu \]
### III. Mesons - Geometric setup

Stationary gravity ‘background’ gives equilibrium thermodynamics

Gravity ‘fluctuations’ give field theory dynamics

D7-brane

short string
(low-energy excitation)

$\tilde{A}_\mu$

fluctuation of corresponding current
(fundamental-antifundamental)

$\tilde{j}_\mu$

Chemical potential:

$\hat{A}_\mu = \delta_{\mu 0}A_0 + \tilde{A}_\mu$

[Nakamura et al., hep-th/0611021]
[Myers et al., hep-th/0611099]
III. Mesons - Gravity solution

Effective action:

\[ S_{D7} = \int d^8x \sqrt{\det\{[g + F] + \tilde{F}\}} \], \quad F_{\mu\nu} = \partial_{[\mu} A_{\nu]} \]

[Erdmenger, M.K., Rust 0710.0334]
III. Mesons - Gravity solution

Effective action: 
\[ S_{D7} = \int d^8 x \sqrt{\det \left\{ [g + F] + \tilde{F} \right\}} \], \quad F_{\mu\nu} = \partial_{[\mu} A_{\nu]} \\

Equation of motion: 
\[ 0 = \dddot{A}'' + \frac{\partial_\rho \left[ \sqrt{\det G} |G^{22} G^{44}| \right]}{\sqrt{\det G} |G^{22} G^{44}|} \dot{A}' - \frac{G^{00}}{G^{44}} \varrho_H^2 \omega^2 \ddot{A} \]
III. Mesons  -Gravity solution

Effective action:

\[
S_{D7} = \int d^8 x \sqrt{\det \left\{\left[ g + F \right] + \tilde{F} \right\}} \quad , \quad F_{\mu\nu} = \partial_{[\mu} A_{\nu]}
\]

Equation of motion:

`Curved' Maxwell equations:

\[
\partial_{\mu} F^{\mu\nu} = 0
\]
\[
\partial_{\mu} \left( \sqrt{-G} G^{\mu\nu} G^{\rho\sigma} F_{\nu\sigma} \right) = 0
\]
\[
\partial_{\mu} \left( \sqrt{-G} G^{\mu\nu} G^{\rho\sigma} \partial_{[\nu} \tilde{A}_{\sigma]} \right) = 0
\]
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Effective action:

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Equation of motion:

\[ 0 = \dddot{A} + \frac{\partial_\rho \left[ \sqrt{\left| \det G \right| G^{22} G^{44} } \right]}{\sqrt{\left| \det G \right| G^{22} G^{44} } } \dot{A} - \frac{G_{00}}{G_{44}} \rho_H^2 \omega^2 \ddot{A} \]
III. Mesons -Gravity solution

Effective action: \[ S_{D7} = \int d^8 x \sqrt{|\det[G + F] + \tilde{F}|} \ , \ F_{\mu\nu} = \partial_{[\mu} A_{\nu]} \]

Equation of motion: \[ 0 = \tilde{A}'' + \frac{\partial \rho}{\sqrt{\det G}} \frac{\sqrt{|\det G|^2 G^{22} G^{44}]}{\sqrt{\det G'} G^{22} G^{44}} \tilde{A}' - \frac{G^{00}}{G^{44} \rho_H^2} \omega^2 \tilde{A} \]

Boundary conditions: \[ \tilde{A} = (\rho - \rho_H)^{-i\omega} [1 + \frac{i\omega}{2} (\rho - \rho_H) + \ldots] \]

[Erdmenger, M.K., Rust 0710.0334]
III. Mesons - Gravity solution

[Erzinger, M.K., Rust 0710.0334]

Effective action:

\[ S_{D7} = \int d^8 x \sqrt{\left| \det \{ [g + F] + \tilde{F} \} \right|} , \quad F_{\mu\nu} = \partial_{[\mu} A_{\nu]} \]

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Translation to gauge theory by duality:

\[ A_{\mu}^{\text{AdS/CFT}} \leftrightarrow J_{\mu} \]

(source)
III. Mesons - Gravity solution

Effective action: \[ S_{D7} = \int d^8 x \sqrt{\det \left\{ [g + F] + \tilde{F} \right\}} \cdot G \]

Equation of motion: \[ 0 = \tilde{A}'' + \frac{\partial_\rho \left[ \sqrt{\det G} G^{22} G^{44} \right]}{\sqrt{\det G}} \tilde{A}' - \frac{G^{00}}{G^{44}} \varrho_H^2 \omega^2 \tilde{A} \]

Boundary conditions: \[ \tilde{A} = (\varrho - \varrho_H)^{-i\omega} \left[ 1 + \frac{i\omega}{2} (\varrho - \varrho_H) + \ldots \right] \]

Translation to gauge theory by duality: \[ A_\mu^{\text{AdS/CFT}} \leftrightarrow J^\mu \]

Gauge correlator: \[ G^{\text{ret}} = \frac{N_f N_c T^2}{8} \lim_{\rho \to \rho_{\text{bdy}}} \left( \rho^3 \frac{\partial_\rho \tilde{A}(\rho)}{\tilde{A}(\rho)} \right) \]
**III. Mesons**

- **Results**

Finite baryon density:

\[ \tilde{d} = 0.25 \]

\[ \chi_0 = 0.1 \]

\[ \chi_0 = 0.5 \]

\[ \chi_0 = 0.7 \]

\[ \chi_0 = 0.8 \]

**Thermal spectral function:**

\[ \Re(\omega, q) = -2 \text{Im} G^{\text{ret}}(\omega, q) \]

\[ L(\varrho) = \varrho \chi(\varrho) \]

\[ \chi_0 = \chi(\rho) \bigg|_{\rho \to \rho_H} \sim \frac{m_{\text{quark}}}{T} \]
III. Mesons - Results

Finite baryon density:

\[ L(\varrho) = \varrho \chi(\varrho) \]

Lower temperature

Thermal spectral function:

\[ \mathcal{R}(\omega, q) = -2 \text{Im} G^\text{ret}(\omega, q) \]

\[ L(\varrho) = \varrho \chi(\varrho) \]

\[ \chi_0 = \chi(\rho) \big|_{\rho \rightarrow \rho_H} \sim \frac{m_{\text{quark}}}{T} \]
### III. Mesons - Results

[Ernmenger, M.K., Rust 0710.0334]

Finite baryon density:

\[
L(\rho) = \rho \chi(\rho)
\]

\[
\chi_0 = \chi(\rho) \big|_{\rho \to \rho_H} \sim \frac{m_{\text{quark}}}{T}
\]

Thermal spectral function:

\[
\mathcal{R}(\omega, q) = -2 \text{Im} G_{\text{ret}}(\omega, q)
\]

\[
L(\varrho) = \varrho \chi(\varrho)
\]

\[
\tilde{d} = 0.25
\]

\[
\chi_0 = 0.8
\]

\[
\chi_0 = 0.94
\]

\[
\chi_0 = 0.962
\]

\[
R(\omega, \varrho) = -2 \text{Im} G_{\text{ret}}(\omega, \varrho)
\]
III. Mesons - Results

Finite baryon density:

\[ \tilde{d} = 0.25 \]
\[ \chi_0 = 0.25 \]
\[ n = 0 \]
\[ n = 1 \]
\[ n = 2 \]
\[ n = 3 \]

Thermal spectral function:

\[ \mathcal{R}(\omega, q) = -2 \text{Im} G^{\text{ret}}(\omega, q) \]

\[ L(\rho) = \rho \chi(\rho) \]

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III. Mesons - Results

Finite baryon density:

Thermal spectral function:

\[ \Im G_{\text{ret}}(\omega, q) = -2 \Im G_{\text{ret}}(\omega, q) \]

\[ L(\varrho) = \varrho \chi(\varrho) \]

\[ \chi_0 = \chi(\varrho)|_{\varrho \rightarrow \varrho_H} \sim \frac{m_{\text{quark}}}{T} \]

Finite isospin density:
III. Mesons - Results

Finite baryon density:

\[ \tilde{d} = 0.25 \]
\[ \chi_0 = 0.8 \]
\[ \chi_0 = 0.94 \]
\[ \chi_0 = 0.962 \]

Finite isospin density:

\[ \mathcal{R}(\omega, q) = -2 \text{Im}G_{\text{ret}}(\omega, q) \]
\[ L(\varphi) = \varphi \chi(\varphi) \]
\[ \chi_0 = \chi(\rho)|_{\rho \to \rho_H} \sim \frac{m_{\text{quark}}}{T} \]

Thermal spectral function:

Analytically: [PhD thesis '08]

[Erndemenger, M.K., Rust 0710.0334]
III. Mesons  -Discussion

- Quite stable quark bound states survive deconfinement
- Resonances: vector mesons (like QCD’s Rho-meson)
- Correlators encode transport coefficients (Kubo formulae)
- Poles of correlators in complex frequency plane are QNMs

Other hadron results:  
- Charmonium diffusion suppressed at strong coupling:
  \[ \frac{dp_i}{dt} = \xi_i(t) - \eta_D p_i \]  
  \[ \tau_{\text{relax}}^{\text{strong}} \approx 4\tau_{\text{relax}}^{\text{weak}} \]  
  [Dusling,Erdmenger, M.K., Rust,Teaney,Young 0808.0957]

- Baryons modeled by classical solutions  
  [Witten, hep-th/9805112]
- Problem: N quarks needed, N large  
  [Sfetsos,Siampos 0807.0236]
- Even worse: baryons with less than N quarks allowed
IV. Super-Yang-Mills Phase Diagrams
IV. Super-Yang-Mills phase diagram

- Gravity setup

- $D_3$-branes

[Karch, Katz; hep-th/0205236]
IV. Super-Yang-Mills phase diagram

-Gravity setup

D\textsubscript{7}-branes

‘Minkowski’ embedding

- D\textsubscript{3}-branes

[Karch, Katz; hep-th/0205236]
IV. Super-Yang-Mills phase diagram

-Gravity setup

D3-branes

D7-branes

‘Minkowski’ embedding

[Karch, Katz; hep-th/0205236]
IV. Super-Yang-Mills phase diagram

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D\textsubscript{3}-branes

\textbf{‘Black hole’ embedding}

D\textsubscript{7}-branes

[Karch, Katz; hep-th/0205236]
IV. Super-Yang-Mills phase diagram

-Gravity setup

D\textsubscript{3}-branes
D\textsubscript{7}-branes

‘Black hole’ embedding

Geometric transition

\[ L(\rho) \]

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\text{D}_3 & x & x & x & x & & & & & \\
\text{D}_7 & x & x & x & x & x & x & x & x & x \\
\end{array}
\]

D\textsubscript{3}-branes (black)

[Karch, Katz; hep-th/0205236]
IV. Super-Yang-Mills phase diagram - Results

Baryonic phase diagram:
[Erdmenger, M.K., Rust 0710.0334]
[Myers et al., hep-th/0611099]

Isospin phase diagram:
[Erdmenger, M.K., Kerner, Rust 0807.2663]

- Results

Meson melting transition
- Continuous range
- First order range
- No confinement

Meson melting
- Flavor fluct. instability
- Superconducting phase

[Erdmenger, M.K., Kerner 0903.1864]
IV. Super-Yang-Mills phase diagram

- How useful are these SYM results?

SYM-coupling not running

Finite T: SUSY broken, non-conformal, BUT: large N

Energy densities (free theories): $\epsilon_{\text{SYM}} = 39T^4 \gg \epsilon_{\text{QCD}} = 16T^4$

SYM vs QCD equation of state:

$\epsilon_{\text{SYM}} = 39T^4 \gg \epsilon_{\text{QCD}} = 16T^4$

QCD-equation of state modeled with gravity potential; speed of sound and bulk viscosity similar to lattice-QCD
IV. Super-Yang-Mills phase diagram

-Other phases at strong coupling

- Sakai-Sugimoto model (D4, D8 and anti-D8-branes)
- chiral symmetry breaking (CSB)
- deconfinement can be tuned to coincide with CSB

Short thermalization times: \( \tau_{\text{RHIC, therm}} = 0.6 \text{fm}/c \)
\[ \tau_{\text{therm}} = 0.4 \text{fm}/c \] \[ \tau_{H} = 0.3 \text{fm}/c \]

Black hole formation (far-from-eq. isotropization)

[Chesler,Yaffe 0812.2053]
V. Conformal hydrodynamics

-First order hydrodynamics

Conservation equations

\[ \partial_\mu T^{\mu\nu} = 0 \quad \partial_\mu j^\mu = 0 \]

Constitutive equations

\[ T^{\mu\nu} = \frac{\epsilon}{3}(4u^{\mu}u^{\nu} + g^{\mu\nu}) + \Pi^{\mu\nu} \]

\[ j^\mu = nu^\mu - \sigma T(g^{\mu\nu} + u^\mu u^\nu)\partial_\nu \left( \frac{\mu}{T} \right) + \xi \omega^\mu \]

\[ \omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho \]

[Erndmenger, Haack, M.K., Yarom 0809.2488]

New vorticity term arises!

(related to triangle anomaly)

\[ \partial_\mu j^\mu = -\frac{1}{8} C \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \]

\[ \xi = C \left( \mu^2 - \frac{2}{3} \frac{\mu^3 n}{\epsilon + P} \right) \]

[Son, Surowka 0906.5044]

(see also chiral magnetic effect)

[talk by H.J. Warringa]
V. Conformal hydrodynamics

- New coefficient at first order hydrodynamics (~viscosity)
- $\xi$ completely determined by C and equation of state
- 3 ways to compute $\xi$:
  - E, p conservation & Weyl symmetry (conf.rescaling)
  - positivity of entropy current (anomaly requires new coeff)
  - directly in specific holographic model (microscopic)
- Second order hydro: even more new terms (beyond MIS)
[see also talk by Rischke]

Relativistic hydrodynamics needs to be completed.
VI. Summary

- Perils: large N & ’t Hooft coupling (conformality, SUSY)
- Terrifying agreement with lattice & ‘QCD’
- Heavy quarks: jets & drag (viscosity bound)
- Vector mesons survive deconfinement
- Baryon/Isospin phase diagrams with meson melting
- Flavor superconducting phase at high isospin density
- Hydrodynamics: neglected terms at the order of viscosity

Answer: YES!
VI. Summary

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→ Useful results from the AdS/CFT correspondence?

Answer: YES!
APPENDIX: Geometric setup (detailed)

\[
\begin{align*}
SU(N_f) & \quad \hat{A}_\nu \\
L(\rho) & \\
N_fD_7\text{-branes} & \\
D_3 \times & \times & \times & \times & \\
D_7 \times & \times & \times & \times & \times & \times & \times
\end{align*}
\]

Chemical potential:

\[
\hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu
\]

[Refs: Karch, Katz; hep-th/0205236, Nakamura et al., hep-th/0611021, Myers et al., hep-th/0611099]
## APPENDIX: Extension of the correspondence

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<th>AdS Schwarzschild black hole (D3/D7)</th>
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<td>QCD</td>
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<td>thermal Yang-Mills</td>
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<td>?</td>
<td>Type II Sugra in AdS</td>
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- $\mathcal{N} = 4$ SuperYangMills
- Type II Sugra in AdS

| Relations | $T \leftrightarrow \text{horizon}$
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### Relations

\[
g_{YM}^2 = g_s \quad \frac{R^4}{(\alpha')^2} = 4\pi N_c g_s \equiv \lambda
\]

\[T \leftrightarrow \text{horizon} \quad \mu_B, \mu_I \leftrightarrow A_0(\rho)\]
## APPENDIX: Extension of the correspondence

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<td>Relations</td>
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<td>$T \leftrightarrow \text{horizon}$</td>
</tr>
<tr>
<td>Relations</td>
<td></td>
<td>$\mu_B, \mu_I \leftrightarrow A_0(\rho)$</td>
</tr>
</tbody>
</table>

\[
g_{YM}^2 = g_s
\]
\[
\frac{R^4}{(\alpha')^2} = 4\pi N_c g_s \equiv \lambda
\]
## APPENDIX: Extension of the correspondence

<table>
<thead>
<tr>
<th>Gauge theory symmetry</th>
<th>Original AdS/CFT correspondence</th>
<th>AdS Schwarzschild black hole (D3/D7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge theory symmetry</td>
<td>QCD $\mathcal{N} = 4$ SuperYangMills</td>
<td>thermal Yang-Mills</td>
</tr>
<tr>
<td>Gravity</td>
<td>?</td>
<td>Type II Sugra in AdS</td>
</tr>
<tr>
<td>Gravity</td>
<td>?</td>
<td>Type II Sugra in AdS</td>
</tr>
<tr>
<td>Gauge theory symmetry</td>
<td>non-conf.</td>
<td>✓</td>
</tr>
<tr>
<td>Gauge theory symmetry</td>
<td>non-SUSY</td>
<td>✓</td>
</tr>
</tbody>
</table>

- $N = 4$ SuperYangMills
- Type II Sugra in AdS

\[
g_{YM}^2 = g_s
\]

\[
\frac{R^4}{(\alpha')^2} = 4\pi N_c g_s \equiv \lambda
\]