## Invitation

### Properties of Gauge/Gravity

<table>
<thead>
<tr>
<th>Negative</th>
<th>Positive</th>
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<td>• only toy models</td>
<td>• strong coupling effects</td>
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Properties of Gauge/Gravity

**Negative**
- only toy models
- no model of QCD or SM
- no quantitative results (mass)
- QCD in this universality class?

**Positive**
- strong coupling effects
- models thermalization, etc
- exact solutions exist
- qualitative results (scaling)
- some universal results

The Ridge Phenomenon
- strong coupling effects?
- pre-thermalization?
- needs qualitative explanation
- some “universal” results?
Invitation

Properties of Gauge/Gravity

Negative

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Positive

• strong coupling effects
• models thermalization, etc
• exact solutions exist
• qualitative results (scaling)
• some universal results

The Ridge Phenomenon

• strong coupling effects?
• pre-thermalization?
• needs qualitative explanation
• some “universal” results?

Gauge/Gravity seems like an appropriate tool.
Invitation

Gauge/Gravity Dictionary

Gauge Theory

“Medium” after collision

Gravity Theory

Background geometry (metric, gauge fields, ...)

Temperature

Hawking

$T \sim \text{horizon radius}$

\[ g_{\mu \nu}(r) \]

\[ g_{\mu \nu}(r; r_{\text{Horizon}}) \]
# Invitation

## Gauge/Gravity Dictionary

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- Temperature: $T \sim \text{horizon radius}$
- Hawking: $g_{\mu\nu}(r)$
- Horizon formation: $g_{\mu\nu}(r; r_{\text{Horizon}})$
- Shock-wave collision: $g_{\mu\nu}(r, t, \vec{x})$
**Invitation**

**Gauge/Gravity Dictionary**

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\[
g_{\mu\nu}(r) \quad g_{\mu\nu}(r; r_{\text{Horizon}}) \quad g_{\mu\nu}(r, t, \vec{x}) \quad \delta g_{\mu\nu}(r, t, \vec{x})
\]
Outline

✓ Invitation

I. Review: Gauge/Gravity & Heavy-Ion-Collisions
   - Gauge/Gravity
   - Completed Hydrodynamics

II. Gauge/Gravity Models for the Ridge
   - Shock-Wave Metric yields Pre-Equilibrium
   - Fluctuations give Correlation Functions

III. Other Possibilities

IV. Conclusions
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III. Other Possibilities
     \(\rightarrow\) Toy models of full collision

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     - Systematic scan for origin of ridge
     - Toy models for hydrodynamic flow vs. toy models of jets

IV. Conclusions
I. Gauge/Gravity & Heavy-Ion-Collisions

What has been done to holographically model HIC?

A lot

Not much

We are going to discuss only examples here. This is not a full review. 

Review: [Gubser, Karch 0901.0935]
I. Gauge/Gravity & Heavy-Ion-Collisions

Chiral vortex effect

Heavy-ion-collision
Chiral vortex effect

Heavy-ion-collision

I. Gauge/Gravity & Heavy-Ion-Collisions

**Fluid/Gravity**

\[ \text{Einstein equations} = \text{hydrodyn. conservation} + \text{EOMs for gravity fields} \]

[Baier et al. 2007]
[Bhattacharyya et al. 0712.2456]
Chiral vortex effect

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Fluid/Gravity

Einstein equations = hydrodyn. dynamical conservation + EOMs for equations gravity fields

Complete constitutive relation for EM-tensor, values for transport coefficients.
(completes Israel-Stewart)

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It gives you all there is!
I. Gauge/Gravity & Heavy-Ion-Collisions

**Chiral vortex effect**

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**Fluid/Gravity derivation of chiral vortex effect.**

[Baier et al. 2007]
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Computed all first/second order transport coefficients in a gravity dual without B.
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Computed all first/second order transport coefficients in a gravity dual without B.

Pure field theory derivation.

[Son, Surowka 0906.5044]

In parallel: chiral magnetic effect.

[Kharzeev et al., 2007]
[Fukushima et al., 2008]
I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamics

Hydrodynamics is an effective field theory, an expansion in gradients (equivalently: low frequencies and large momenta).

Constitutive equations

\[ T_{\mu \nu} = \frac{\epsilon}{3} (4u^\mu u^\nu + g^{\mu \nu}) + \tau_{\mu \nu} \]

\[ j^\mu = n u^\mu - \sigma T (g^{\mu \nu} + u^\mu u^\nu) \partial_\nu \left( \frac{\mu}{T} \right) + \xi \omega^\mu \]

\[ =: \Delta_{\mu \nu} \]

Example: Relativistic fluids with one conserved charge, with an anomaly (chiral)

Vorticity

\[ \omega^\mu = \frac{1}{2} \epsilon^{\mu \nu \lambda \rho} u_\nu \partial_\lambda u_\rho \]

NEW!
I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamics

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Example: Relativistic fluids with one conserved charge, with an anomaly (chiral)

Vorticity \( \omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho \)

NEW!

from writing down all possible terms (respecting symmetries) with one derivative, built from \( \{ u, \epsilon, T, n, \mu, \epsilon^{\mu\nu\rho\ldots} \} \).

Examples

\[ \{ \nabla^\nu u^\mu, \nabla^\nu T, nu^\nu, \]

\[ u^\nu u^K \nabla^K n, u^\nu n \nabla^K u^K, \ldots \} \]
I. Gauge/Gravity & Heavy-Ion-Collisions

*Hydrodynamics: first order traditional procedure*

1. Write down all first order (pseudo)vectors and (pseudo)tensors

2. Restricted by conservation equations

\[ \nabla_{\mu} T^{\mu\nu} = F^{\nu\lambda} j_{\lambda} \quad \nabla_{\mu} j^{\mu} = CE^\mu B_\mu \]

*Example: no external fields*

\[ 0 = \nabla_{\mu} nu^{\mu} = n\nabla_{\mu} u^{\mu} + u^{\mu} \nabla_{\mu} n \]

Possibly restricted by conformal symmetry

*Example:*

\[ \nabla^{\nu} \left( \mu \over T \right) \]  

invariant under Weyl rescaling

3. Further restricted by positivity of entropy production

\[ \nabla_{\mu} J_{\mu}^{\mu} \geq 0 \]  

*Landau, Lifshitz*
I. Gauge/Gravity & Heavy-Ion-Collisions

(non-conformal) hydrodynamics in 3+1

[Son,Surowka 0906.5044]

Complete constitutive equations in 3+1 (with external gauge field)

\[ T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu} - \eta \Delta^{\mu\alpha} \Delta^{\nu\beta}(\partial_\alpha u_\beta + \partial_\beta u_\alpha) - (\zeta - \frac{2}{3}\eta)\Delta^{\mu\nu}\nabla_\gamma u^\gamma \]

\[ j^\mu = nu^\mu + \sigma V^\mu + \xi \omega^\mu + \xi_B B^\mu \]

\[ V^\mu = \frac{E^\mu - T\Delta^{\mu\nu}\nabla_\nu \left( \frac{\mu}{T} \right)}{E^\mu} \]

\[ E^\mu = F^{\mu\nu}u_\nu \]

\[ B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta} \]

\[ \omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu \nabla_\rho u_\sigma \]
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\]

\[
j^\mu = nu^\mu + \sigma V^\mu + \xi \omega^\mu + \xi_B B^\mu
\]

New transport coefficients restricted

\[
\xi = C \left( \mu^2 - \frac{2}{3} \frac{n\mu^3}{\epsilon + P} \right), \quad \xi_B = C \left( \mu - \frac{1}{2} \frac{n\mu^2}{\epsilon + P} \right)
\]

Chiral vortex effect
Chiral magnetic effect

Observable in heavy-ion collisions

\[
V^\mu = E^\mu - T \Delta^{\mu\nu} \nabla_\nu \left( \frac{\mu}{T} \right)
\]

\[
E^\mu = F^{\mu\nu} u^\nu
\]

\[
B^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}
\]

\[
\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} u_\nu \nabla_\rho u_\sigma
\]

Predicted values:

[Kharzeev, Son 1010.0038]
I. Gauge/Gravity & Heavy-Ion-Collisions

Un-biased predictive power

What we did not know:

Chiral magnetic effect predicted: [Kharzeev 2004]
Chiral vortical effect proposed: [Kharzeev, Zhitnitsky 2007]

Needs corrections: [Landau, Lifshitz]

Ignorance is bliss:
Complete first order constitutive equations in 3+1dim discovered in gravity without prejudice.

Gauge/Gravity method gives you everything there is inside a model.

Word of caution:
Gauge/Gravity is not entirely universal. Values of e.g. transport coefficients and features are generally model-dependent. But within the model you get “everything”.

Matthias Kaminski
LECTURE X: Gauge/Gravity Correspondence Applications
Complete first order constitutive equations in 3+1dim discovered in gravity without prejudice.

Gauge/Gravity method gives you everything there is inside a model.

Take a model, check for ridge, change model

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I. Gauge/Gravity & Heavy-Ion-Collisions

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I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamic two-point-functions

Simplified example in 2+1 dim:

\[ J^\mu = \rho_0 u^\mu + \sigma E^\mu \]

External sources \( A_t, A_x \propto e^{-i\omega t + ikx} \)

\[ u^\mu = (1, 0, 0) \]

Allow response \( \rho_0 = \delta \rho \quad (\text{fix } T \text{ and } u) \)
I. Gauge/Gravity & Heavy-Ion-Collisions

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Allow response \( \rho_0 = \delta \rho \) (fix \( T \) and \( u \))

One-point-functions from solving \( \nabla_\mu J^\mu = 0 \)

\[
\langle J^t \rangle = \delta \rho = -\frac{i\sigma k}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + kA_t)
\]

\[
\langle J^x \rangle = \delta \rho = -\frac{i\sigma \omega}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + kA_t)
\]

\[
\langle J^y \rangle = 0
\]

Einstein relation for diffusion: \( D = \frac{\sigma}{\chi} \)
I. Gauge/Gravity & Heavy-Ion-Collisions

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\[ \langle J^y \rangle = 0 \]

\[ \langle J^t J^x \rangle = \langle J^t \rangle \frac{\delta \langle J^t \rangle}{\delta A_x} = -\frac{i\sigma \omega k}{\omega + i D k^2} \]

\[ \Rightarrow \text{Kubo formulae for transport coefficients} \]

\( D = \frac{\sigma}{\chi} \) Einstein relation for diffusion:
I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamic two-point-functions

Simplified example in 2+1 dim:

\[ J^\mu = \rho_0 u^\mu + \sigma E^\mu \]

External sources

\[ A_t, A_x \propto e^{-i\omega t + ikx} \]

possible: more sources

\[ u^\mu = (1, 0, 0) \]

Allow response

\[ \rho_0 = \delta \rho \quad \text{(fix } T \text{ and } u) \]

generally: \( T \) and \( u \) respond as well

One-point-functions from solving \( \nabla_\mu J^\mu = 0 \)

\[ \langle J^t \rangle = \delta \rho = -\frac{i\sigma k}{\omega + ik^2\frac{\sigma}{\chi}} (\omega A_x + k A_t) \]

Einstein relation for diffusion:

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⇒ Two-point-functions

\[ \langle J^t J^x \rangle = \frac{\delta \langle J^t \rangle}{\delta A_x} = -\frac{i\sigma \omega k}{\omega + iDk^2} \]

⇒ Kubo formulae for transport coefficients
I. Gauge/Gravity & Heavy-Ion-Collisions

Hydrodynamic Frames

Decomposition (Lorentz-invariance implied)

\[ T_{\mu\nu} = \mathcal{E} u_\mu u_\nu + \mathcal{P} \Delta_{\mu\nu} + (q_\mu u_\nu + q_\nu u_\mu) + t_{\mu\nu} \]

\[ J_\mu = N u_\mu + j_\mu \]

\[ u_\mu q^\mu = 0, \quad u_\mu t^{\mu\nu} = 0, \quad u_\mu j^\mu = 0 \]

Example: Temperature gradient

\[ j_\mu = \cdots + \chi_T \Delta_{\mu}^{\nu} \nabla_\nu T + \cdots \]

Field redefinition ambiguity out-of-equilibrium

\[ u_\nu(x) \rightarrow \hat{u}_\nu(x) \]

\[ T(x) \rightarrow \hat{T}(x) \]

\[ \mu(x) \rightarrow \hat{\mu}(x) \]

Fix by choice of a particular hydrodynamic frame

Example: Landau frame

\[ q_\mu = 0 \quad \mathcal{E} = \epsilon_0 \quad N = \rho_0 \]
I. Gauge/Gravity & Heavy-Ion-Collisions

Hydro without entropy current

Two-point functions together with “equilibrium correlators“ replace the entropy argument.

Proven for 2+1 dimensions:
[Jensen, MK, Kovtun, Meyer, Ritz, Yarom 1112.4498]

Proven for “equality type” conditions in d dimensions:
[Jensen, MK, Kovtun, Meyer, Ritz, Yarom 1203.3556]
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Proven for “equality type” conditions in d dimensions:
[Jensen, MK, Kovtun, Meyer, Ritz, Yarom 1203.3556]

Inequality type: \[ \sigma \geq 0 \quad \eta \geq 0 \] (from two-point functions)

Generating functional
\[ W_m = \int d^d x \mathcal{L}[\text{sources}(x)]. \]

Example: Ideal superfluid
\[ W_0 = \int d^d x \sqrt{-g} P(T, \mu, \xi^2) \]

Example: Equality type \[ \chi_T = 0 \]

Generally: m-point functions, simplifies higher order hydro (zero frequency)
I. Gauge/Gravity & Heavy-Ion-Collisions

Summary of part I

- Relativistic hydrodynamics was completed at first and second order (Careful with “Causal Viscous Hydro”).
  
  [Baier et al, Minwalla et al 2007]
  [Erdmenger, Haack, MK, Yarom 0809.2488]
  [Banerjee et al. 0809.2596]

- Chiral transport effects measured in heavy-ion-collisions?
  
  [Kharzeev, Son]

- New methods for hydrodynamic correlation functions

- New method restricting transport coefficients

- Gauge/Gravity provides playground without prejudice

- Various models of particle collisions exist
Outline

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✓ Review: Gauge/Gravity & Heavy-Ion-Collisions
  - Gauge/Gravity
  - Completed Hydrodynamics

II. Gauge/Gravity Models for the Ridge
  - Shock-Wave Metric yields Pre-Equilibrium
  - Fluctuations give Correlation Functions

A “first guess”

Toy models of full collision

III. Other Possibilities

IV. Conclusions
II. Gauge/Gravity Models for the Ridge

Pre-Equilibrium Model I

Single gravitational shock-wave metric

\[ ds^2 = \frac{L^2}{z^2} \left\{ -2\, dx^+\, dx^- + t_1(x^-)\, z^4\, dx^{-2} + dx_\perp^2 + dz^2 \right\} \]

\[ t_1(x^-) \equiv \frac{2\, \pi^2}{N_c^2} \langle T_{1--}(x^-) \rangle \]

\( z \) is the radial AdS-direction

\( L \) is the AdS-radius

Energy-momentum tensor component

Solves Einstein’s equations in AdS5

\[ R_{\mu\nu} + \frac{4}{L^2} \, g_{\mu\nu} = 0 \]
**II. Gauge/Gravity Models for the Ridge**

*Pre-Equilibrium Model I*

Single gravitational shock-wave metric

\[ ds^2 = \frac{L^2}{z^2} \left\{ -2 dx^+ dx^- + t_1(x^-) z^4 dx^-^2 + dx_\perp^2 + dz^2 \right\} \]

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\( z \) is the radial AdS-direction
\( L \) is the AdS-radius

Solves Einstein’s equations in AdS5

\[ R_{\mu\nu} + \frac{4}{L^2} g_{\mu\nu} = 0 \]

Collide two shock waves with

\[ t_1(x^-) = \mu_1 \delta(x^-), \quad t_2(x^+) = \mu_2 \delta(x^+) \]

This gives

\[ ds^2 = \frac{L^2}{z^2} \left\{ - \left[ 2 + G(x^+, x^-, z) \right] dx^+ dx^- + \left[ t_1(x^-) z^4 + F(x^+, x^-, z) \right] dx^-^2 \right. \]

\[ + \left. \left[ t_2(x^+) z^4 + \tilde{F}(x^+, x^-, z) \right] dx^+^2 + \left[ 1 + H(x^+, x^-, z) \right] dx_\perp^2 + dz^2 \right\}. \]

which is analytically known (perturbatively)
II. Gauge/Gravity Models for the Ridge

Pre-Equilibrium Model II

“Holography and colliding gravitational shock waves in asymptotically AdS_5 spacetime”

[Chesler, Yaffe 1011.3562]

Evolution of two colliding initial states with finite energy density, finite thickness, Gaussian profile, in N=4 Super-Yang-Mills theory at strong coupling.

Full planar shock-wave, non-singular, time-dependent, numerical solution to Einstein’s equations.

Contains strong coupling and “medium” effects.

Ansatz:

\[ ds^2 = -A dv^2 + \Sigma^2 \left[ e^B dx_\perp^2 + e^{-2B} dz^2 \right] + 2 dv (dr + F dz) \]
II. Gauge/Gravity Models for the Ridge

Pre-Equilibrium Model II

“Holography and colliding gravitational shock waves in asymptotically AdS_5 spacetime”  

[Chesler, Yaffe 1011.3562]

FIG. 1: Energy density $\mathcal{E}/\mu^4$ as a function of time $\nu$ and longitudinal coordinate $z$.

FIG. 2: Energy flux $S/\mu^4$ as a function of time $\nu$ and longitudinal coordinate $z$.

Initial data:

$$ds^2 = r^2[-dx_+dx_- + dx_{\perp}^2] + \frac{1}{r^2}[dr^2 + h(x_\pm)dx_{\perp}^2]$$

Pick Gaussian (arbitrary)

$$h(x_\pm) \equiv \mu^2 (2\pi w^2)^{-1/2} e^{-\frac{1}{2}x_\pm^2/w^2}$$
II. Gauge/Gravity Models for the Ridge

“Long-Range Rapidity Correlations in Heavy Ion Collisions at Strong Coupling from AdS/CFT”  
[Grigoryan, Kovchegov 1012.5431]

Basic idea:

Gauge

Collision of nuclei

Correlations at early times

Correlations at late times

Gravity

Metric of Model I

Fluctuations around this

Fluctuations around dual to ideal Bjorken
II. Gauge/Gravity Models for the Ridge

Recipe: Two-point correlator from fluctuations [Son, Starinets 2002]

Action for gravity scalar field fluctuation (dual to glueball)

\[ S^\phi = -\frac{N_c^2}{16\pi^2 L^3} \int d^4x \, dz \sqrt{-g} \, g^{MN} \partial_M\phi(x, z) \partial_N\phi(x, z) \]

Solve equation of motion for that scalar

\[ \frac{1}{\sqrt{-g}} \partial_M \left[ \sqrt{-g} \, g^{MN} \partial_N\phi(x, z) \right] = 0 \]

On-shell action

\[ S_{cl}^\phi = \frac{N_c^2}{16\pi^2 L^3} \int d^4x \left[ \sqrt{-g} \, g^{zz} \phi(x, z) \partial_z\phi(x, z) \right] \bigg|_{z=0} = \frac{N_c^2}{16\pi^2} \int d^4x \phi_B(x) \left[ \frac{1}{z^3} \partial_z\phi(x, z) \right] \bigg|_{z=0} \]

Real-time retarded Green’s function

\[ G_R(x_1, x_2) = \frac{\delta^2[S_{cl}^\phi - S_0]}{\delta\phi_B(x_1) \delta\phi_B(x_2)} \]
II. Gauge/Gravity Models for the Ridge

Implications

Large-rapidity glueball correlations in simplest background look very different from ridge data. But there are large-rapidity correlations at early times.

\[ C'(k_1, k_2) \big|_{|\Delta y| \gg 1} \sim \cosh(4 \Delta y) \]

Computation in background dual to ideal Bjorken hydrodynamics gives no large-rapidity correlations at late times.
Outline

✓ Invitation

✓ Review: Gauge/Gravity & Heavy-Ion-Collisions
  • Gauge/Gravity
  • Completed Hydrodynamics

✓ Gauge/Gravity Models for the Ridge
  • Shock-Wave Metric yields Pre-Equilibrium
  • Fluctuations give Correlation Functions

III. Other Possibilities
   Systematic scan for origin of ridge
   Toy models for hydrodynamic flow vs. toy models of jets

IV. Conclusions
III. Other Possibilities

*Correlations after collision of two nulei in a medium*

**PROPOSAL**

Compute fluctuations around the full numerical background metric of model II at different times to scan the full time evolution of correlations.

**Step in this direction:**

>[Chesler, Teaney 2011]

Compute fluctuations around simplified version of model II (dual to two-point correlation functions). Check fluctuation dissipation theorem and equilibration.
III. Other Possibilities

Model of a jet

Take a string falling/being torn apart (backreacted)

Initial conditions?

see also [Hofman, Maldacena 2008]
III. Other Possibilities

Model of a jet

Take a string falling/being torn apart (backreacted)

Compute fluctuations around this background (dual to two-point correlation functions)

Initial conditions?  

Toy model for jets?

see also [Hofman, Maldacena 2008]
IV. Conclusions

✓ complete first and second order hydro

✓ new method for restricting transport coeffs

✓ new method for zero-frequency m-point correlators

✓ candidate model for collision (ridge)

⇒ fluctuations at different times, unique features?

⇒ use “more of hydro”: fluctuations, 2nd O(), methods...

⇒ measure chiral transport effects
APPENDIX

-Entropy production

Structure of divergence

\[ \nabla_\alpha J_\alpha = \]  
\[ + \left( \text{products of first order data} \right) , \]  
\[ \implies \tilde{\nu}_2 = \nu_0 = \nu_1 = \nu_2 = \nu_3 = 0 \]
-Entropy production

Structure of divergence

\[ \nabla_\alpha J_s^\alpha = + \left( \nu_2 - \frac{\nu_3}{T} \right) \nabla_\mu E^\mu + \nu_3 \Delta^{\mu\nu} \nabla_\mu \partial_\nu \frac{\mu}{T} + (\nu_0 + \nu_1) u^\alpha \nabla_\alpha \nabla_\mu u^\mu - \nu_1 u^\alpha u^\mu R_{\alpha\mu} - \tilde{\nu}_2 u^\alpha \nabla_\alpha B + \text{ (products of first order data)} \]

\[ \Rightarrow \tilde{\nu}_2 = \nu_0 = \nu_1 = \nu_2 = \nu_3 = 0 \]
-Entropy production

Structure of divergence

\[ \nabla_\alpha J_\alpha^s = + \left( \nu_2 - \frac{\nu_3}{T} \right) \nabla_\mu E^\mu + \nu_3 \Delta^{\mu\nu} \nabla_\mu \partial_\nu \frac{\mu}{T} \]

\[ + (\nu_0 + \nu_1) u^\alpha \nabla_\alpha u^\mu - \nu_1 u^\alpha u^\mu R_{\alpha\mu} \]

\[ - \tilde{\nu}_2 u^\alpha \nabla_\alpha B + \text{(products of first order data)} \]

\[ \implies \tilde{\nu}_2 = \nu_0 = \nu_1 = \nu_2 = \nu_3 = 0 \]

Products of first order data

\[ \partial_\alpha J_\alpha^s = + \partial_\alpha J_\alpha^s_{\text{canon}} \]

- \( \Omega(\partial \cdot u) \)

- \( B(\partial \cdot u) \)

+ \( U_2 \cdot \tilde{U}_3 \)

+ \( U_1 \cdot \tilde{U}_3 \)

+ \( U_1 \cdot \tilde{U}_2 \)
- Entropy production

Structure of divergence

\[
\nabla_\alpha J^\alpha_s = + \left( \nu_2 - \frac{\nu_3}{T} \right) \nabla_\mu E^\mu + \nu_3 \Delta^{\mu\nu} \nabla_\mu \partial_\nu \frac{\mu}{T} \\
+ (\nu_0 + \nu_1) u^\alpha \nabla_\alpha \nabla_\mu u^\mu - \nu_1 u^\alpha u^\mu R_{\alpha\mu} \\
- \tilde{\nu}_2 u^\alpha \nabla_\alpha B + (\text{products of first order data}),
\]

\[\implies \tilde{\nu}_2 = \nu_0 = \nu_1 = \nu_2 = \nu_3 = 0\]

Products of first order data

\[
\partial_\alpha J^\alpha_s = + \partial_\alpha J^\alpha_s_{\text{canon}} \\
- \Omega (\partial \cdot u) \left[ T \left( \frac{\partial P_0}{\partial \epsilon_0} \right)_{\rho_0} (\partial T \tilde{\nu}_5 + \tilde{\nu}_1) + \frac{1}{T} \left( \frac{\partial P_0}{\partial \rho_0} \right)_{\epsilon_0} (\partial \tilde{\mu} \tilde{\nu}_5 + \tilde{\nu}_3) \right] \\
- B (\partial \cdot u) \left[ T \left( \frac{\partial P_0}{\partial \epsilon_0} \right)_{\rho_0} \partial_T \tilde{\nu}_4 + \frac{1}{T} \left( \frac{\partial P_0}{\partial \rho_0} \right)_{\epsilon_0} \partial \tilde{\mu} \tilde{\nu}_4 \right] \\
+ U_2 \cdot \tilde{U}_3 \left[ R_0 T (\partial_T \tilde{\nu}_3 - \partial \tilde{\mu} \tilde{\nu}_1) - \partial \tilde{\mu} \tilde{\nu}_4 + R_0 T^2 \partial_T \tilde{\nu}_4 \right] \\
+ U_1 \cdot \tilde{U}_3 \left[ - R_0 T^2 (\partial_T \tilde{\nu}_5 + \tilde{\nu}_1) + (\partial \tilde{\mu} \tilde{\nu}_5 + \tilde{\nu}_3) + T (\partial \tilde{\mu} \tilde{\nu}_1 - \partial_T \tilde{\nu}_3) \right] \\
+ U_1 \cdot \tilde{U}_2 \left[ \frac{\partial \tilde{\mu} \tilde{\nu}_5 + \tilde{\nu}_3}{T} + \partial \tilde{\mu} \tilde{\nu}_1 - \partial_T \tilde{\nu}_3 - T \partial_T \tilde{\nu}_4 \right],
\]
APPENDIX

-Entropy production

Canonical part

\[
\partial_\alpha J^\alpha_{\text{canon}} = - \left( \frac{1}{2} \Delta_{\mu\nu} \tau^{\mu\nu} - \left( \frac{\partial P_0}{\partial \epsilon_0} \right)_{\rho_0} u_\mu u_\nu \tau^{\mu\nu} + \left( \frac{\partial P_0}{\partial \rho_0} \right)_{\epsilon_0} u_\mu \Upsilon_\mu \right) \frac{\partial \cdot u}{T} \\
- (R_0 u_\mu \tau^{\mu\nu} + \Upsilon_\nu) \Delta_{\nu\alpha} U^\alpha_3 \\
- \tau^{\mu\nu} \sigma_{\mu\nu} \right) \frac{1}{2T}.
\]

Transform back to Landau frame

Thermodynamic response parameters

\[
\begin{align*}
\tilde{\chi}_B &= \frac{\partial P_0}{\partial \epsilon_0} \left( T \frac{\partial M_B}{\partial T} + \mu \frac{\partial M_B}{\partial \mu} - M_B \right) + \frac{\partial P_0}{\partial \rho_0} \frac{\partial M_B}{\partial \mu}, \\
\tilde{\chi}_\Omega &= \frac{\partial P_0}{\partial \epsilon_0} \left( T \frac{\partial M_\Omega}{\partial T} + \mu \frac{\partial M_\Omega}{\partial \mu} + f_\Omega(T) - 2M_\Omega \right) + \frac{\partial P_0}{\partial \rho_0} \left( \frac{\partial M_\Omega}{\partial \mu} - M_B \right), \\
\tilde{\chi}_E &= \frac{\partial M_B}{\partial \mu} - R_0 \left( \frac{\partial M_\Omega}{\partial \mu} - M_B \right), \\
T\tilde{\chi}_T &= \left( T \frac{\partial M_B}{\partial T} + \mu \frac{\partial M_B}{\partial \mu} - M_B \right) - R_0 \left( T \frac{\partial M_\Omega}{\partial T} + \mu \frac{\partial M_\Omega}{\partial \mu} + f_\Omega(T) - 2M_\Omega \right),
\end{align*}
\]

Matching to two-point functions later gives: \( M_B = \frac{\partial P}{\partial B}, \quad M_\Omega = \frac{\partial P}{\partial \Omega} \)
Most general parity-violating case is more complicated

\[
\begin{pmatrix}
  k^2 \sigma - i \omega \frac{\partial \rho_0}{\partial \mu} - k^2 \left( \frac{\mu}{T} \sigma + \chi T \right) - i \omega \frac{\partial \rho_0}{\partial T} \\
  -i \omega \frac{\partial \sigma_0}{\partial \mu} & -i \omega \frac{\partial \sigma_0}{\partial T} \\
  ik \rho_0 & ik \sigma_0 \\
  0 & 0
\end{pmatrix}
\begin{pmatrix}
  ik \rho_0 \\
  ik (\epsilon_0 + P_0) \\
  k^2 (\epsilon + \zeta - i \omega (\epsilon_0 + P_0)) \\
  -k^2 \eta
\end{pmatrix}
\begin{pmatrix}
  \delta \mu \\
  \delta T \\
  \delta u^x \\
  \delta u^y
\end{pmatrix}
= \text{vector containing external sources } h_{\mu \nu}, A_\mu
\]

For example, we get a Kubo formula for

\[
\lim_{k \to 0} \frac{1}{ik} \langle C^0 T^{02} \rangle_R(0, k) = \tilde{\chi} \Omega
\]
-Two-point-functions

Most general parity-violating case is more complicated

\[
\begin{pmatrix}
k^2 \sigma - i \omega \frac{\delta \rho_0}{\delta \mu} & -k^2 \left( \mu_T \sigma + \chi T \right) - i \omega \frac{\delta \rho_0}{\delta T} \\
- i \omega \frac{\delta \rho_0}{\delta \mu} & - i \omega \frac{\delta \rho_0}{\delta T} \\
 i k \rho_0 & i k \rho_0 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
 i k \rho_0 \\
i k (\epsilon_0 + P_0) \\
0 \\
k^3 (\eta + \zeta) - i \omega (\epsilon_0 + P_0) \\
k^2 (\lambda \Omega + \eta) \\
- k^2 \eta \\
k^2 \eta - i \omega (\epsilon_0 + P_0)
\end{pmatrix}
= \text{vector containing external sources } h_{\mu \nu}, A_\mu

For example, we get a Kubo formula for

\[
\lim_{k \to 0} \frac{1}{i k} \langle C^0 T^{02} \rangle_R(0, k) = \tilde{\chi} \Omega
\]
APPENDIX

-Two-point-functions

Restrictions from Onsager relations

\[ G^{ij}_R(\omega, k; b_a) = n_i n_j G^{ji}_R(\omega, -k; -b_a) \]

where under time-reversal \( \Theta \mathcal{O}_i \Theta^{-1} = n_i \mathcal{O}_i \)
APPENDIX

-Two-point-functions

Restrictions from Onsager relations

\[
G_{R}^{ij}(\omega, k; b_{a}) = n_{i}n_{j}G_{R}^{ji}(\omega, -k; -b_{a})
\]

where under time-reversal \( \Theta \Theta^{-1} = n_{i}O_{i} \)

From time-reversal covariance plus translation invariance

\[
G_{R}^{ij}(x) \equiv i\theta(t) \text{Tr} (\varrho [O_{i}(t, x), O_{j}(0)]) = i\theta(t) n_{i}n_{j} \text{Tr} (\varrho^{'} [O_{j}(t, -x), O_{i}(0)])
\]

Parameters \( b_{a} \) break time-reversal invariance,

i.e. time-reversal and \( b_{a} \rightarrow -b_{a} \)

together are a symmetry
**APPENDIX**

*-Two-point-functions*

Restrictions from susceptibility constraints

\[
\lim_{k \to 0} \langle J^0 J^0 \rangle (\omega = 0, k) = \left( \frac{\partial \rho_0}{\partial \mu} \right)_T
\]

Examples

Partition function in grand canonical ensemble

\[
Z[T, \mu] = \text{Tr} \left[ \exp \left( -\frac{H}{T} + \frac{\mu Q}{T} \right) \right]
\]

Constant external sources \( A_0, h_{00}, h_{0i} \)
can be eliminated by shifting thermodynamic variables

\[
Z[T, \mu; A_0, h_{00}, h_{0i}] = Z \left[ T \left( 1 + \frac{h_{00}}{2} \right), \mu \left( 1 + \frac{h_{00}}{2} \right) + A_0; 0, 0, 0 \right]
\]

Thus we get relations for zero-momentum limits of zero-frequency correlators.
**APPENDIX**

-Magnetovortical frame

Thermodynamics depending on vorticity and magnetic field

\[ P = P(T, \mu, B, \Omega) \]

\[ dP = s\,dT + \rho\,d\mu + \frac{\partial P}{\partial B} B + \frac{\partial P}{\partial \Omega} \Omega , \]

\[ \epsilon + P = sT + \mu \rho . \]

Constitutive relations

\[ T^{\mu\nu} = (\epsilon - \mathcal{M}_\Omega \Omega + f_\Omega \Omega) \, u^\mu \, u^\nu \]

\[ + (P - \zeta \nabla_\alpha u^\alpha - \tilde{x}_B B - \tilde{x}_\Omega \Omega) \, \Delta^{\mu\nu} - \eta \sigma^{\mu\nu} - \tilde{\eta} \tilde{\sigma}^{\mu\nu} , \]

\[ J^\mu = (\rho - \mathcal{M}_B \Omega) \, u^\mu + \sigma V^\mu + \tilde{\sigma} \tilde{V}^\mu + \tilde{\chi}_E \tilde{E}^\mu + \tilde{\chi}_T \epsilon^{\mu\nu\rho} u\nu \nabla_\rho T , \]

where

\[ \mathcal{M}_B = \frac{\partial P}{\partial B} , \quad \mathcal{M}_\Omega = \frac{\partial P}{\partial \Omega} \]

Matching

\[ \tilde{x}_B = \frac{\partial P}{\partial B} , \]

\[ T\tilde{x}_T = \frac{\partial \epsilon}{\partial B} + R_0 \left( \frac{\partial P}{\partial \Omega} - \frac{\partial \epsilon}{\partial \Omega} - f_\Omega \right) , \]

\[ \tilde{x}_\Omega = \frac{\partial P}{\partial \Omega} , \]

\[ \tilde{\chi}_E = \frac{\partial \rho}{\partial B} + R_0 \left( \frac{\partial P}{\partial B} - \frac{\partial \rho}{\partial \Omega} \right) . \]
APPENDIX

-2+1 dimensional results

Conservation equations

\[ \nabla_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \]

\[ \nabla_\mu J^\mu = 0 \]

[Jensen, MK, Kovtun, Meyer, Ritz, Yarom 1112.4498]
**APPENDIX**

-2+1 dimensional results

Conservation equations

\[ \nabla_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \]

\[ \nabla_\mu J^{\mu} = 0 \]

Constitutive equations

\[ T^{\mu\nu} = \epsilon_0 u^\mu u^\nu + (P_0 - \zeta \nabla_\alpha u^\alpha - \tilde{\chi}_B B - \tilde{\chi}_\Omega \Omega) \Delta^{\mu\nu} - \eta \sigma^{\mu\nu} - \tilde{\eta} \tilde{\sigma}^{\mu\nu} \]

\[ J^{\mu} = \rho_0 u^\mu + \sigma V^\mu + \tilde{\sigma} \tilde{V}^\mu + \tilde{\chi}_E \tilde{E}^\mu + \tilde{\chi}_T \epsilon^{\mu\nu\rho} u_\nu \nabla_\rho T \]

“New” transport terms arise!
APPENDIX

-2+1 dimensional results

Conservation equations

\[ \nabla_\mu T^{\mu \nu} = F^{\nu \lambda} J_\lambda \]
\[ \nabla_\mu J^\mu = 0 \]

Constitutive equations

\[
T^{\mu \nu} = \epsilon_0 u^\mu u^\nu + (P_0 - \zeta \nabla_\alpha u^\alpha - \tilde{\chi}_B B - \tilde{\chi}_\Omega \Omega) \Delta^{\mu \nu} - \eta \sigma^{\mu \nu} - \tilde{\eta} \tilde{\sigma}^{\mu \nu}
\]
\[
J^\mu = \rho_0 u^\mu + \sigma V^\mu + \tilde{\sigma} \tilde{V}^\mu + \tilde{\chi}_E \tilde{E}^\mu + \tilde{\chi}_T \epsilon^{\mu \nu \rho} u_\nu \nabla_\rho T
\]

“New” transport terms arise!

\[ \tilde{\eta} \quad \text{Hall viscosity} \]

thermodynamic interpretation of

\[ \tilde{\chi}_E, \tilde{\chi}_\Omega, \tilde{\chi}_B \]

off-diagonal conductivity (anomalous Hall conductivity)

\[ \tilde{\chi}_T \quad \text{“thermal Hall conductivity”} \]