

## Introduction to String Theory

### Group Project: Holographic Superconductors

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**Outline** Recently, it has been found that black holes in Anti deSitter spacetime can coexist with a charged condensate outside their horizon [1]. This setup is dual to a superfluid (or possibly a superconductor) in the dual field theory [2, 3].

**Task 1** Write up a one-page-summary explaining the main idea of and recent progress on higher spin holography.

**Task 2** Construct a p-wave superconductor holographically. Consider the Einstein-Maxwell action

$$S = \frac{1}{2\kappa} \int d^4x [\mathcal{R} - \frac{1}{4}(F_{\mu\nu})^2 + \frac{6}{L^2}] \quad (1)$$

and use the Ansatz  $A = \phi(r)\tau^3 dt + w(r)\tau^1 dx$ . Let us see if we can have non-trivial static configurations of  $\phi$  and  $w$  in the bulk. Note: This is a *background computation*.

The Euler-Lagrange equations are given by

$$0 = \phi'' + \frac{2}{r}\phi' - \frac{1}{r(r^3 - 1)}w^2\phi, \quad 0 = w'' + \frac{1 + 2r^3}{r(r^3 - 1)}w' + \frac{r^2}{(r^3 - 1)^2}\phi^2w. \quad (2)$$

with the near-horizon ( $r_H = 1$ ) behavior

$$w = w_0 + w_2(r - 1)^2 + \dots, \quad (3)$$

$$\phi = \phi_1(r - 1) + \dots \quad (4)$$

and the near-boundary ( $r_B = \infty$ ) behavior

$$w = \frac{W_1}{r} + \dots, \quad (5)$$

$$\phi = p_0 + \frac{p_1}{r} + \dots \quad (6)$$

a) Determine the solution to these two equations of motion numerically.

b) What is the meaning of the fields  $w$  and  $\phi$  on the gauge side?

c) Find a numerical solution such that the non-normalizable mode of the field  $w$  vanishes, but its normalizable mode remains finite.

d) What do these boundary conditions imply for the gauge theory object corresponding to  $w$ ?

Remark: Compare with [arXiv:0805.2960].

## References

- [1] S. S. Gubser, *Breaking an Abelian gauge symmetry near a black hole horizon*, *Phys.Rev.* **D78** (2008) 065034, [arXiv:0801.2977].
- [2] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, *Building a Holographic Superconductor*, *Phys.Rev.Lett.* **101** (2008) 031601, [arXiv:0803.3295].
- [3] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, *Holographic Superconductors*, *JHEP* **0812** (2008) 015, [arXiv:0810.1563].