Matthias Kaminski July 19, 2012

Introduction to String Theory

Exercises IV

Ex. 4.1 Fermionic OPEs

The fermion contribution T_F to the energy-momentum tensor has the following OPEs with the bosonic and fermionic fields

$$T_F(z)X^\mu(0) \sim -i\sqrt{\frac{\alpha'}{2}}\frac{\psi^\mu(0)}{z}\,, \quad T_F(z)\psi^\mu(0) \sim i\sqrt{\frac{\alpha'}{2}}\frac{\partial X^\mu(0)}{z}\,. \tag{1}$$

Show that the residues of the OPEs of X^μ and ψ^μ with the currents (10.1.9) in Polchinski are proportional to the superconformal variations (10.1.10).

Ex. 4.2 Ramond and Neveu-Schwarz algebras

Show that the coefficients of the central charge terms in T_BT_B and T_FT_F, i.e. the central charges, are related to each other. Use the Jacobi identity for the R-NS algebra in (10.2.11) of Polchinski's book.

Ex. 4.3 Fermionic & bosonic correlation functions from gauge/gravity

The gauge/gravity correspondence is a duality between a gauge theory at strong coupling on one side and a (super)gravity theory on the other side. According to the gauge/gravity correspondence, the retarded correlation function of an operator with itself in a field theory is given by a particular ratio A/B. The values of A and B can be extracted from the AdS-boundary asymptotics of a particular bosonic gravity field ϕ, which is dual to the operator. The boundary expansion for bosonic fields is typically of the form $\phi(u) = A + Bu^{\text{some power}} + \dots$, where u is the radial AdS-direction with the AdS-boundary at $u = 0$. Then the retarded correlation function is given by

$$G^R \sim B/A\,, \tag{2}$$

up to factors (see upcoming lectures).

Consider a bosonic operator, namely the energy-momentum tensor $T_{\mu\nu}$ in a particular gauge theory. The gravity field which is dual to this operator is the metric $g_{\mu\nu}$.

a) Find a numerical solution to the equation

$$0 = \phi'' - \frac{1+u^2}{uf}\phi' + \frac{\omega^2 - q^2 f}{uf^2}\phi\,, \tag{3}$$

with $f = (1-u^2)$. Use the ingoing boundary condition at the horizon $u = 1$, and choose an arbitrary normalization. The boundary is located at $u = 0$ in these coordinates.

This equation arises as the equation of motion for the off-diagonal (shear) metric perturbation h_{xy} in $\mathcal{N} = 4$ SYM. This perturbation is holographically dual to (in other words *its boundary value does source*) the energy momentum tensor component T_{xy} in the dual gauge theory. Let us set $\vec{q} = 0$.

b) Compute the two-point correlation function $G^R_{xy,xy}(\omega, \vec{0}) = \langle T_{xy} T_{xy} \rangle(\omega, \vec{0})$, and plot the thermal spectral function against frequency ω.

c) Use the following Kubo formula in order to numerically compute the shear viscosity

$$\eta = - \lim_{\omega \to 0} \frac{1}{2\omega} \mathrm{Im} G^R_{xy,xy}(\omega, \vec{q} = 0) \,. \tag{4}$$

d) In which sense is this approach more powerful than a hydrodynamic one?

Ex. 4.4 Fermionic correlation functions from gauge/gravity

What is the gauge/gravity formula for the correlator of a fermionic operator Ψ with itself (the fermionic analog of equation (2)). Refer to the review paper arXiv:0903.2596, the introductory section 2.1 in arXiv:1003.1134, and especially the introductory level section 5.1 in arXiv:1010.4806. Provide an explicit formula (including all factors).