Matthias Kaminski

Introduction to Gauge/Gravity & Heavy-Ion-Applications

Exercises IV

Ex. 4.1 Superfluids, Superconductors & Vector mesons

Consider the Einstein-Maxwell action

$$S = \frac{1}{2\kappa}\int d^4x[\mathcal{R} - \frac{1}{4}(F_{\mu\nu})^2 + \frac{6}{L^2}] \tag{1}$$

and use the Ansatz $A = \phi(r)\tau^3 dt + w(r)\tau^1 dx$. Let us see if we can have non-trivial static configurations of ϕ and w in the bulk. Note: This is a *background computation.*

The Euler-Lagrange equations are given by

$$0 = \phi'' + \frac{2}{r}\phi' - \frac{1}{r(r^3-1)}w^2\phi\,, \qquad 0 = w'' + \frac{1+2r^3}{r(r^3-1)}w' + \frac{r^2}{(r^3-1)^2}\phi^2 w\,. \tag{2}$$

with the near-horizon ($r_H = 1$)behavior

$$w = w_0 + w_2(r-1)^2 + \dots\,, \tag{3}$$
$$\phi = \phi_1(r-1) + \dots\,. \tag{4}$$

and the near-boundary ($r_B = \infty$) behavior

$$w = \frac{W_1}{r} + \dots\,, \tag{5}$$
$$\phi = p_0 + \frac{p_1}{r} + \dots\,. \tag{6}$$

a) Determine the solution to these two equations of motion numerically.

b) What is the meaning of the fields w and ϕ on the gauge side?

c) Find a numerical solution such that the non-normalizable mode of the field w vanishes, but its normalizable mode remains finite.

d) What do these boundary conditions imply for the gauge theory object corresponding to w?

Remark: Compare with [arXiv:0805.2960].

Ex. 4.2 Confinement -an entry in the dictionary

In the lectures we have seen a "toy model" for deconfinement, which was in fact just a deconfinement transition for the fundamental matter of the gauge theory.

Is it possible to construct a setup from a stack of N_c D3 branes, one probe brane and two strings that models confinement? Follow your intuition.

Ex. 4.3 Fluid/Gravity Correspondence

a) Show that the AdS_5 metric is a solution of the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} + \Lambda g_{\mu\nu} = T_{\mu\nu} \,. \tag{7}$$

Use Mathematica with the package diffgeo.m (or equivalent software).

Another solution are the boosted black branes

$$ds^2 = -2u_\mu dx^\mu dr - r^2 f(br) u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu \,, \tag{8}$$

with $f(r) = 1 - 1/r^4$, the boost parameters $u^\nu = 1/\sqrt{1-\beta^2}$, $u^i = \beta_i/\sqrt{1-\beta^2}$

Now we give each of the parameters a spacetime dependence on t, x, y, z and expand each parameter $T(x^\mu)$, $u^\nu((x^\mu))$ in derivatives.

b) Verify that the leading (constant) order still satisfies Einstein's equations.

c) Keeping only first order terms, show that this perturbation in general is not a solution of the Einstein equations. Do you think it is possible to find new solutions this way? If 'Yes', explain how? If 'No', explain why!