Matthias Kaminski

Introduction to Gauge/Gravity & Heavy-Ion-Applications

Exercises III

Ex. 3.1 Viscosity Bound

a) Find a numerical solution to the equation

$$0 = \phi'' - \frac{1+u^2}{uf}\phi' + \frac{\omega^2 - q^2 f}{uf^2}\phi\,, \tag{1}$$

with $f = (1-u^2)$. Use the ingoing boundary condition at the horizon $u = 1$, and choose an arbitrary normalization ($\phi(u=1) = 1$ may be convenient). The boundary is located at $u = 0$ in these coordinates.

This equation arises as the equation of motion for the off-diagonal (shear) metric perturbation h_{xy} in $\mathcal{N} = 4$ SYM. This perturbation is holographically dual to (in other words *its boundary value does source*) the energy momentum tensor component T_{xy} in the dual gauge theory. Let us set $\vec{q} = 0$.

b) Compute the two-point correlation function $G^R_{xy,xy}(\omega,\vec{0}) = \langle T_{xy}T_{xy}\rangle(\omega,\vec{0})$, and plot the thermal spectral function against frequency ω.

c) Use the following Kubo formula in order to numerically compute the shear viscosity

$$\eta = -\lim_{\omega\to 0}\frac{1}{2\omega}\mathrm{Im}G^R_{xy,xy}(\omega,\vec{q}=0)\,. \tag{2}$$

d) In which sense is this approach more powerful than the hydrodynamic one?

Ex. 3.2 Charged Hot Plasma from Flavor Branes

Recall the D7-probe-branes from exercise I. These introduced quarks, or more generally matter fields in the fundamental representation of the "color" group $SU(N_c)$. We are able to turn on a gauge field strength $F_{\rho t}$ on the worldvolume of these D7-branes. We want to work in the canonical ensemble, i.e. at a fixed finite charge density. This setup is described by the DBI-action

$$S_{D7} = -N_f T_{D7}\int d^8\sigma \frac{\varrho^3}{4} f\tilde{f}(1-\chi^2)\sqrt{1-\chi^2+\varrho^2(\partial_\varrho\chi)^2 - 2(2\pi l_s^2)^2\frac{\tilde{f}}{f^2}(1-\chi^2)F^2_{\varrho t}}\,, \tag{3}$$

where A_t depends solely on ρ. which was obtained after computing the induced metric and its determinant similarly as we have done in Exercise 1.2.

a) Find the equation of motion for the background field $A_t(\rho)$. There is a conserved quantity, name it d.

b) In order to work in the canonical ensemble in the field theory, Legendre-transform the DBI-action with respect to the quantity d, eliminating A_t.

Varying this Legendre transformed action with respect to the field χ gives the equation of motion for the embeddings $\chi(\rho)$,

$$\begin{aligned} &\partial_\rho \left[\frac{\rho^5 f \tilde{f}(1-\chi^2)\chi'}{\sqrt{1-\chi^2+\rho^2\chi'^2}} \sqrt{1+\frac{8\tilde{d}^2}{\rho^6 \tilde{f}^3 (1-\chi^2)^3}} \right] \\ &= -\frac{\rho^3 f \tilde{f} \chi}{\sqrt{1-\chi^2+\rho^2\chi'^2}} \sqrt{1+\frac{8\tilde{d}^2}{\rho^6 \tilde{f}^3 (1-\chi^2)^3}} \\ &\quad \times \left[3(1-\chi^2) + 2\rho^2\chi'^2 - 24\tilde{d}^2 \frac{1-\chi^2+\rho^2\chi'^2}{\rho^6 \tilde{f}^3 (1-\chi^2)^3 + 8\tilde{d}^2} \right] . \end{aligned} \tag{4}$$

where we have introduced the dimensionless $\tilde{d}$ which now replaces d, and dimensionless radial coordinate $\rho = \varrho/\varrho_H$.

c) Find some embeddings at finite charge density and compare to the one without any charge.

d) Plot the mass parameter m as a function of the horizon parameter χ_0 for distinct values of $\tilde{d}$.

e) Which field theory object do fluctuations of the gauge field on the brane correspond to? What are their charges under $SU(N_c)$ and $SU(N_f)$? What would change if we wanted to embed $N_f = 2$ coincident D7-branes?

*f) Show that the charge on the brane is generated by a bundle of D3-D7 strings located at the horizon.