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Introduction to String Theory

Exercises I

Ex. 1.1 General relativity and curved spacetime

Read pages 1-7 in Poisson's "A relativist's toolkit" and answer the following questions.

a) How are scalars, vectors, tensors and invariants defined (under coordinate transformations)?

b) Show that for a vector V^μ its partial derivative $\frac{\partial V^\mu}{\partial x^\alpha}$ is not a tensor.

c) Show that the definition of the Christoffel-symbols (the connection)

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} (g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu}) \, , \tag{1}$$

implies $\Gamma^\alpha_{\gamma\beta} = \Gamma^\alpha_{\beta\gamma}$ and $g_{\alpha\beta;\gamma} = 0$.

d) What is the length ℓ of a curve along a geodesic?

e)Derive the Einstein equations with $F \equiv 0$ (and Einstein-Maxwell equations with $F \not\equiv 0$) from the Einstein-Hilbert action (Einstein-Maxwell action)

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left[R - 2\Lambda + \frac{\ell^2}{g^2} F^{\mu\nu} F_{\mu\nu} \right] \, , \tag{2}$$

where Λ is the cosmological constant, and the Ricci scalar is given by $R = R^\mu{}_\mu$, $R_{\mu\nu} = R^\alpha_{\mu\alpha\nu}$ and $R^\alpha_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\beta\gamma,\delta} + \Gamma^\alpha_{\mu\gamma}\Gamma^\mu_{\beta\delta} - \Gamma^\alpha_{\mu\delta}\Gamma^\mu_{\beta\gamma}$. Note that all variations with respect to derivatives of the metric cancel, i.e. we can effectively set $\frac{\delta S}{\delta g_{\mu\nu,\rho}} \to 0$.

Ex. 1.2 Polchinski exercise 1.7

Ex. 1.3 Polchinski exercise 2.7