Fermions in a holographic p-wave superfluid

London Imperial College, HET Seminar, June 23rd 2010

by Matthias Kaminski (Princeton University)

in collaboration with M. Ammon, J. Erdmenger, A. O’Bannon (MPI Munich)

[arXiv: 0810.2316]
[arXiv: 0903.1864]
[arXiv: 1003.1134]
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Outline

I. Invitation: Fermions in Experiments & String Theory

II. Holographic Superfluids

III. Flavored Fermions from String Theory

IV. Results (spec. func., scaling, reduced Fermi surface)

V. Discussion
Why can string theory be useful for condensed matter physics?
I. Invitation: Superfluids & Fermions

*Superfluid*: global symmetry spontaneously broken

*Superconductor*: charged superfluid, gauge symmetry

Weakly gauge AdS-boundary theory

**Weak coupling concepts**
- (charged) condensate of Cooper-pairs (BCS)
- (gauge) symmetry spontaneously broken
- strongly correlated electron systems

**Examples**
- Color superconducting phase at high densities
  *Alford, Rajagopal, Wilczek ‘97*
- superfluid He3, (high Tc) superconductors

What happens to Fermions?
I. Invitation: Fermions in Experiments

Quantum Critical Point

- high Tc cuprate (d-wave)
- unconventional low Tc superconductor (p-wave)

Fermi Surface from ARPES

$\text{Sr}_2\text{RuO}_4$ cleaved at 180 K
$T = 10 \text{ K}$
$E = 28 \text{ eV}$
I. Invitation: Holographic Fermionic Operators

Gravity model [Liu et al. 0903.2477]

charged AdS4 black hole:

\[ ds^2 = g_{MN} dx^M dx^N = \frac{r^2}{R^2} (-f dt^2 + dx^2) + \frac{R^2}{r^2} \frac{dr^2}{f} \]

\[ f = 1 + \frac{Q^2}{r^{2d-2}} - \frac{M}{r^d}, \quad A_t = \mu \left( 1 - r_0^{d-2} \right) \]

Dirac action:

\[ S = \int d^{d+1}x \sqrt{-g} i (\bar{\psi} \Gamma^M D_M \psi - m \bar{\psi} \psi) \]

\[ D_M = \partial_M + \frac{1}{4} \omega_{abM} \Gamma^{ab} - i q A_M \]

Critical Scaling: non-Fermi liquid

\[ \omega_* \sim (k - k_F)^z \]

\[ \text{Im} \ G_{22}(\omega_*, k) \sim (k - k_F)^{-\alpha} \]

fermionic quasiparticle peak:

\[ z \approx 2, \quad \alpha \approx 1 \]

Fermi liquid:

\[ \alpha = 1 = z \]

\[ \text{Im}(G_{22}) \]
I. Invitation: Holographic Fermions

**Gravity model** [Liu et al. 0903.2477]

charged AdS4 black hole:

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Dirac action:

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fermionic quasiparticle peak:

\[ z \approx 2, \quad \alpha \approx 1 \]

chosen \( q = 1 \)

zoo of non-Fermi liquids for arbitrary \( q \)

\[ \alpha \overset{?}{=} 1 = z \]
I. Invitation: Holographic Fermions

**Gravity model** [Liu et al. 0903.2477]

charged AdS4 black hole:

\[
\begin{align*}
    ds^2 &= g_{MN} dx^M dx^N = \frac{r^2}{R^2} (-f dt^2 + dx^2) + \frac{R^2}{r^2} d\tau^2 \frac{f}{f} \\
    f &= 1 + \frac{Q^2}{r^{2d-2}} - M \frac{M}{r^d}, \quad A_t = \mu \left(1 - \frac{r_0^{d-2}}{r^{d-2}} \right)
\end{align*}
\]

Dirac action:

\[
S = \int d^{d+1}x \sqrt{-g} i (\bar{\psi} \Gamma^M D_M \psi - m \bar{\psi} \psi)
\]

\[
D_M = \partial_M + \frac{1}{4} \omega_{abM} \Gamma^{ab} - i q A_M
\]

---

**Critical Scaling: non-Fermi liquid**

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fermionic quasiparticle peak:

\[
z \approx 2, \quad \alpha \approx 1
\]

**Problems**

- arbitrary parameters (fermion charge/mass,...)
- low T but condensation NOT considered

**Solutions**

- fix parameters by generic string construction (D3/D5)
- background with condensate (SC black hole)
Navigator

✓ Invitation: Fermions in Experiments & String Theory

II. Holographic Superfluids

III. Flavored Fermions from String Theory

IV. Results (spec. func., scaling, reduced Fermi surface)

V. Discussion
II. Holo SF: Gauge/Gravity Correspondence

Near AdS-boundary ($\rho \to \infty$)

$$A = A^{(0)} + \frac{A^{(2)}}{\rho^2} + \ldots$$

- non-normalizable (source)
- normalizable (vev)

gravity theory (weak) \leftrightarrow \text{gauge theory (strong)}

AdS

QFT

$\rho$

$\rho_{bdy}$
II. Holo SF: Gauge/Gravity Correspondence

Near AdS-boundary \((\rho \to \infty)\)

\[
A = A^{(0)} + \frac{A^{(2)}}{\rho^2} + \ldots
\]

\(A = A^{(0)} + A^{(2)}\)

\(\rho \to \infty\)

QFT FEATURE \(\leftrightarrow\) GEOMETRY

- operator \(J_\mu\) \(\leftrightarrow\) field \(A_\mu\) (gauge)
- vev \(\leftrightarrow\) \(A^{(2)}\) (charge)
- source \(\leftrightarrow\) \(A^{(0)}\) (chem. pot.)

Dictionary

gravity theory (weak) \(\leftrightarrow\) gauge theory (strong)
Building a Holographic Superfluid

Field Theory

What do we need?

- charged condensate (vev)
- no source
- condensate of charge carriers
- finite temperature, chem. pot.

Gravity

- AdS-boundary
- curvature
- electromag.
- horizon
- gravity
- introduce normalizable mode
- no non-normalizable mode
- condensate hovers over horizon
- black hole, charge

Is this stable?

[Gubser 0801.2977]
[Gubser, Pufu 0805.2960]
Get some intuition

Field Theory

\[ \mathcal{L} \sim D_\nu \phi D^\nu \phi \sim (M_q^2 - \mu_{\text{isospin}}^2) \phi^2 \]

charged particles condense at large enough chemical potential

\[ \mu_{\text{isospin}} \sim M_q \]

Gravity

- strings (D3-D5) give FT charges
- cannot put infinitely many
- second brane is important (probe limit)
Get some intuition

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Gravity

strings (D3-D5) give FT charges

cannot put infinitely many

second brane is important (probe limit)

We need a non-Abelian structure!
II.b) Holographic Superfluid: D3/D5-Branes

- $N_c$ D3-branes
dual to $\mathcal{N} = 4$ SYM with $SU(N_c)$
II.b) Holographic Superfluid: D3/D5-Branes

$N_f$ D5-branes
dual to $\mathcal{N} = 4 \; SU(N_f)$ flavor

$[\text{Karch,Katz hep-th/0205236}]$

$N_c$ D3-branes
dual to $\mathcal{N} = 4 \; \text{SYM with } SU(N_c)$
II.b) Holographic Superfluid: D3/D5-Branes

\[ N_f \text{D}_5\text{-branes} \]

dual to \( \mathcal{N} = 4 \) \( SU(N_f) \) flavor

\[ [Karch,Katz \text{ hep-th/0205236}] \]

\[ N_c \text{D}_3\text{-branes (black)} \]

dual to \( \mathcal{N} = 4 \) SYM with \( SU(N_c) \)
II.b) Holographic Superfluid: D3/D5-Branes

\( N_f \) D5-branes
dual to \( \mathcal{N} = 4 \) SU\((N_f)\) flavor

[Karch, Katz hep-th/0205236]

\( N_c \) D3-branes (black)
dual to \( \mathcal{N} = 4 \) SYM with SU\((N_c)\)
II.b) Holographic Superfluid: D3/D5-Branes

$\hat{A}_\nu \subset U(N_f)$

$N_f$ D5-branes
dual to $\mathcal{N} = 4$ $SU(N_f)$ flavor

$N_c$ D3-branes (black)
dual to $\mathcal{N} = 4$ SYM with $SU(N_c)$

[Karch,Katz hep-th/0205236]
II.b) Holographic Superfluid: D3/D5-Branes

\[ \hat{A}_\nu \subset U(N_f) \]

\[ N_f \text{ D5-branes} \]

dual to \( \mathcal{N} = 4 \ SU(N_f) \) flavor

\[ \rho \]

\[ \sim \text{quark mass} \]

\[ N_c \text{ D3-branes (black)} \]

dual to \( \mathcal{N} = 4 \) SYM with \( SU(N_c) \)

\[ \begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
D_3 & x & x & x & x & & & & & \\
D_5 & x & x & x & & x & x & x & & \\
\end{array} \]

\[ [\text{Karch},\text{Katz hep-th/0205236}] \]
II.b) Holographic Superfluid: D3/D5-Branes

\[ \hat{A}_\nu \subset U(N_f) \]

~quark mass

\[ \hat{A}_\nu = \delta_{\mu 0} A_0 + \tilde{\hat{A}}_\mu \]

Chemical potential: (cf. therm. FT)

\[ \tilde{\hat{A}}_\mu = \delta_{\mu 0} A_0 + \tilde{\hat{A}}_\mu \]

\[ \hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{\hat{A}}_\mu \]

[Myers et al., hep-th/0611099]

[Mateos et al. 0709.1225]
II.b) Holographic Superfluid: D3/D5-Branes

\[ \hat{A}_\nu \subset U(N_f) \]

\[ N_f D_5\text{-branes} \]

dual to \( \mathcal{N} = 4\) \( SU(N_f)\) flavor

\[ \rho \]

\[ \sim \text{quark mass} \]

\[ N_c D_3\text{-branes (black)} \]

dual to \( \mathcal{N} = 4\) \( SYM\) with \( SU(N_c)\)

\[ \hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu \]

(Chemical potential: \[ \text{[Myers et al., hep-th/0611099]} \]
[\text{[Mateos et al. 0709.1225]}]

(cf. therm. FT)
II.b) Holographic Superfluid: D3/D5-Branes

\[ \hat{A}_\mu \subset U(N_f) \]

\[ N_c \text{ D3-branes (black)} \]

dual to \( N = 4 \) SYM with \( SU(N_c) \)

dual to \( N = 4 \) SU\((N_f)\) flavor

Chemical potential:

\[ \hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu \]

(cf. therm. FT)

[Myers et al., hep-th/0611099]

[Mateos et al. 0709.1225]

Mesons

\[ \rho \]

\[ N_f \text{ D5-branes} \]

\[ \subset U(N_f) \]

\[ \sim \text{quark mass} \]
Non-Abelian DBI with Chemical Potential

AdS black hole metric

\[ d s_{Dp}^2 = \frac{1}{u^2} \left( \frac{d u^2}{f(u)} - f(u) d t^2 + d \bar{x}^2 \right) + d s_{SQ}^2 \]

Induced metric on Dp-branes

with \( f(u) = 1 - \frac{u^4}{u_h^4} \), \( u_h = \frac{1}{\pi T} \)

Bosonic DBI action (second order expansion)

\[ S_{Dp} = -T_{Dp} N_f \int d^{p+1} \xi \sqrt{-g_{Dp}} \left[ 1 + (2\pi \alpha')^2 \frac{1}{2} T r \left( F_{\mu\nu} F^{\mu\nu} \right) \right] \]

chemical potential

(cf. thermal QFT)
Flavor Superfluid Phase

General idea

\[ A^3_0 = \mu + \frac{d}{\rho^2} + \ldots \]

[Ernmenger, M.K., Kerner, Rust 0807.2663]
Flavor Superfluid Phase

General idea

\[ A^3_0 = \mu + \frac{d}{\rho^2} + \ldots \]

[Ammon, Erdmenger, M.K., Kerner 0810.2663]

\[ A^3_0 = \mu + \frac{d_0^3}{\rho^2} + \ldots \]

[Ammon, Erdmenger, M.K., Kerner 0810.2316]

\[ A^1_3 = \frac{d_3^1}{\rho^2} + \ldots \]

[Gubser, Pufu 0805.2960]
Flavor Superfluid Phase

General idea

\[ A^3_0 = \mu + \frac{d}{\rho^2} + \ldots \]

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\[ A^1_3 = \frac{d^1_3}{\rho^2} + \ldots \]

[Spontaneous breaks U(1)]

\[ [Gubser, Pufu 0805.2960] \]
Field Theory Picture

Gravity field

\[ A_0^3 = \mu + \frac{d_0^3}{\rho^2} + \ldots \]

dual to current

\[ J_0^3 \propto \bar{\psi} \gamma^3 \gamma_0 \psi + \phi \gamma^3 \partial_0 \phi = n_u - n_d \]

explicitly breaks

\[ U(2) \sim U(1)_B \times SU(2)_I \rightarrow U(1)_B \times U(1)_3 \]

New field

\[ A_3^1 = \frac{d_3^1}{\rho^2} + \ldots \]

dual to current

\[ J_3^1 \propto \bar{\psi} \gamma^1 \gamma_3 \psi + \phi \gamma^1 \partial_3 \phi \]

\[ = \bar{\psi}_u \gamma_3 \psi_d + \bar{\psi}_d \gamma_3 \psi_u + \text{bosons} \]

spontaneously breaks

\[ U(1)_3 \leftrightarrow U(1)_{\text{em}} \quad \text{and} \quad SO(2) \]

Condensation of SU(2) adjoint vector mesons
Thermodynamics results

Grand potential:

Specific heat:

Order parameter:

SC density:

Vanishes linearly.

Critical exponent is $1/2$. 

$S_{DBI} \propto$ grand potential
Background field configuration

Gravity fields:

$$A_0^3 \rightarrow \mu_0^3$$

$$A_3^1 \rightarrow p_3^1$$

Conjugate momenta:

$$p_0^3 \rightarrow d_0^3$$

$$p_3^1 \rightarrow d_3^1$$
String Theory Picture

boundary

ρ

condensate
curvature

electromag.

horizon

gravity
5-5 strings generate $A_3^1$ i.e. they break the U(1) and are thus dual to Cooper pairs.

charged horizon (3-5 strings generate $A_0^3$)
Navigator

✓ Invitation: Fermions in Experiments & String Theory
✓ Holographic Superfluids

III. Flavored Fermions from String Theory

IV. Results (spec. func., scaling, reduced Fermi surface)

V. Discussion
III. Fermions from ST: D3/D5-Brane Setup

\[ \hat{A}_\nu \subset U(N_f) \]

\[ L(\rho) \]

\[ N_f \text{D}5\text{-branes} \]

\[ N_c \text{D}3\text{-branes (black)} \]

SUSY: meson superpartners

Chemical potential:

\[ \hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu \]

(cf. therm. FT)

[Myers et al., hep-th/0611099]
[Mateos et al. 0709.1225]
Worldvolume Fermions

Quadratic action for fermionic fluctuations:

\[ S_{Dp} = N_f T_{Dp} \int d^{p+1}\xi \sqrt{-g_{Dp}} \frac{1}{2} Tr \left[ \hat{\Psi} P_\Gamma \hat{A} \left( D_{\hat{A}} + \frac{i}{8 \cdot 2 \cdot 5!} F_{N\hat{P}\hat{Q}\hat{R}\hat{S}} \Gamma_{N\hat{P}\hat{Q}\hat{R}\hat{S}} \Gamma_{\hat{A}} \right) \hat{\Psi} \right] \]

[Martucci et al. hep-th/0504041]

Dirac equation:

\[ \left[ \Gamma_{\hat{A}} D_{\hat{A}} + \frac{i}{8 \cdot 2 \cdot 5!} \Gamma_{\hat{A}} F_{N\hat{P}\hat{Q}\hat{R}\hat{S}} \Gamma_{N\hat{P}\hat{Q}\hat{R}\hat{S}} \Gamma_{\hat{A}} \right] \hat{\Psi} = 0 \]

reduces to

\[ (\slashed{D}_{AdS_P} - m_l^{\pm}) \Psi_l^{\pm} = 0 \]

Spelling out the gauge and spacetime covariant derivative

\[ \left[ (\slashed{D}_{AdS_P} - m) \Psi \right]_a = \left( u\sqrt{f} \gamma^u \partial_u + \frac{u}{\sqrt{f}} \gamma^t \partial_t + u \gamma^i \partial_i + \left[ -\frac{P - 1}{2} \sqrt{f} + \frac{1}{4} u \frac{f'}{\sqrt{f}} \right] \gamma^u \right) \Psi_a \]

\[ + e^M_A i \gamma^A \left[ A_M, \Psi \right]_a - m \Psi_a \]

adjoint fermions couple through commutator to gauge field
Fixing the parameters

Dirac equation masses in $AdS_P \times S^Q$

$$m^+_\ell = \ell + \frac{P}{2}, \quad m^-_\ell = -\left(\ell + Q - \frac{1}{2}P\right)$$

Corresponding operator dimensions

$$\Delta^\pm_\ell = \frac{P-1}{2} + |m^\pm_\ell|$$

Choose massless fermions

$$P = 4, \quad Q = 2 \quad m^+_\ell = \ell + 2 \quad m^-_\ell = -\ell$$

Fermion charges under $U(1)_3$

$$q = \pm 1, \ 0$$
Dual Field Theory: (2+1)dim SYM with $\mathcal{N} = 4$

\[
SO(4, 2) \rightarrow SO(3, 2) \quad SO(6)_R \rightarrow SU(2)_H \times SU(2)_V
\]

contains quarks and squarks $\psi, \tilde{\psi}$

[DeWolfe et al., hep-th/0111135]
[Ernmeniger et al., hep-th/0203020]

Operator dual to fermionic D5-brane fluctuations $\Psi^{-\ell}$

\[
\mathcal{F}_\ell = \bar{\psi}(X_H)(\ldots)\tilde{\psi} + \tilde{\psi}^\dagger(X_H)(\ldots)\psi
\]

$\Delta^{-\ell} = \frac{3}{2} + \ell$

is neutral under baryon, adjoint under isospin symmetry.

Consider massless $\mathcal{F}_{\ell=0} = \bar{\psi}\tilde{\psi} + \tilde{\psi}^\dagger\psi$

with gauge indices $a=1,2,3$ corresponding to bulk charges $q = \pm 1, 0$
Equations of motion for gravity fermions

\[ \Psi_a(u, x^\mu) = u^{3/2} f(u)^{-1/4} e^{ik \cdot x} \psi_a(u) \]

\[
0 = \left( \sqrt{f} \gamma^u \partial_u - \frac{i \omega}{\sqrt{f}} \gamma^t + ik_i \gamma^i - \frac{1}{u} m \right) \psi_+ + i \frac{A_3^3(u)}{\sqrt{f}} \gamma^t \psi_+ + A_x^1(u) \gamma^x \psi_0 \\
0 = \left( \sqrt{f} \gamma^u \partial_u - \frac{i \omega}{\sqrt{f}} \gamma^t + ik_i \gamma^i - \frac{1}{u} m \right) \psi_- - i \frac{A_3^3(u)}{\sqrt{f}} \gamma^t \psi_- + A_x^1(u) \gamma^x \psi_0 \\
0 = \left( \sqrt{f} \gamma^u \partial_u - \frac{i \omega}{\sqrt{f}} \gamma^t + ik_i \gamma^i - \frac{1}{u} m \right) \psi_0 - \frac{1}{2} A_x^1(u) \gamma^x (\psi_+ + \psi_-).
\]

\[ A_x^1 = 0 \quad \text{diagonal correl.} \quad [\text{Liu et al. 0903.2477}] \]

\[ A_x^1 \neq 0 \quad \text{superfluid phase, coupled bulk eoms, operator mixing} \quad [\text{Cubrovic et al. 0904.1993}] \quad [\text{Kaminski et al. 0911.3610}] \]

\[ \psi_\alpha \equiv \Pi_\alpha \psi = a_\alpha \begin{pmatrix} 0 \\ 1 \end{pmatrix} + b_\alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \ldots \]

\[ G_\alpha^R(\omega, k_x, k_y) = b_\alpha / a_\alpha \]
IV. Results (spec. func., scaling, reduced Fermi surface)

V. Discussion
Main results in wrong phase

- reproduces (multiple) Fermi surfaces
- non-Fermi liquid with critical exponents $\nu = 1$, $\alpha = 2$
- qualitatively very similar to Liu et al
Superfluid phase: 3D Spectral measure

\[ T_c \]

\[ 0.91T_c \]

\[ \omega = 0 \]

\[ 0.69T_c \]

\[ 0.43T_c \]
Superfluid phase: Anisotropic Fermi surface

Recall:
Superfluid phase: Reduced Fermi surface

\[(3+1)\text{dim gravity} \& \text{SU}(2) \text{ field} \]
\[(\text{fundamental fermions})\]

\[\text{T}=0: \text{solve bulk Dirac equation exactly.}\]

\[\text{Green’s function has poles at roots of the fermion representation.}\]

\[k_x = qA_x^1(u_H)\]
Superfluid phase: Reduced Fermi surface

(3+1)dim gravity & SU(2) field (fundamental fermions)

[Gubser et al. 1002.4416]

T=0: solve bulk Dirac equation exactly.

at T=0 the Fermi surface apparently reduces to 3 points (adjoint fermions)

Green's function has poles at roots of the fermion representation.

\[ k_x = q A_x^1(u_H) \]
Superfluid phase: Reduced Fermi surface

(3+1)dim gravity & SU(2) field (fundamental fermions)

\[ [Gubser \ et \ al. \ 1002.4416] \]

T=0: solve bulk Dirac equation exactly.

at T=0 the Fermi surface apparently reduces to 3 points (adjoint fermions)

\[ k_x = q A^1_x (u_H) \]
Superfluid phase: Spectrum

\[ \mathcal{R} \]

\[ \omega / \pi T \]

Graph showing the spectrum of the superfluid phase with plots labeled (0) and (4).
Low frequency poles in $G$ (QNMs)

$q = -1$

wrong phase: $0.9T_c$

superfluid phase: $0.4T_c$

\[
q = -1
\]

\[
0.9T_c
\]

\[
0.4T_c
\]
QNM at vanishing momentum

preliminary analysis suggests: $z = 1$, $\alpha =$?
IV. Discussion

☑️ generic dual to (non)Fermi liquids fixing parameters & considering condensation
☑️ Fermi surface reduces to patches at low T
☑️ left out: holographic renormalization with fermions, correlation functions for mixing operators

□️ critical exponents, (non) Fermi liquid?
□️ backreaction of fermions on metric
□️ zero T: backreaction of flavor branes on metric (speed of sound)
□️ massive fermions
APPENDIX: Spectral measure

\[
T_c \\
0.91T_c \\
0.54T_c \\
0.43T_c
\]

\[
\frac{\omega}{\pi T} = 0.25
\]
APPENDIX: Limits of Probe Limit

\[ \propto \frac{\langle J^x_1 \rangle}{T^2} \]

\[ \frac{T}{T_C} \]
**APPENDIX: Determinant Method**

Quasinormal modes of coupled systems

\[ a_t = b_1 a^I_t + b_2 a^{II}_t \]

\[ a_x = b_1 a^I_x + b_2 a^{II}_x \]

where I and II are distinct sets of boundary conditions.

QNM condition at AdS boundary

\[ \lim_{r \to \infty} a_t = 0 = \lim_{r \to \infty} a_x \]

Normalize and plug in for coeff

\[ 0 = -\frac{a_t^{II}}{a_t^I} a^I_x + a^{II}_x \bigg|_{r \to \infty} \]

\[ 0 = \det \begin{pmatrix} a^I_t & a^{II}_t \\ a^I_x & a^{II}_x \end{pmatrix} \bigg|_{r \to \infty} \]

**Example:**

\[ a_t^{II} = -\omega \alpha, \quad a_x^{II} = k\alpha \]

\[ 0 = \omega \alpha a^I_x + k\alpha a^I_t \equiv \alpha E_x \]
APPENDIX: Determinant Method (2)

General formalism switching on specific operators

\[
\begin{pmatrix}
c_1^1 & \cdots & c_1^n \\
\vdots & \ddots & \vdots \\
c_n^1 & \cdots & c_n^n
\end{pmatrix}
\cdot
\begin{pmatrix}
\phi_1^I & \cdots & \phi_1^{I_n} \\
\vdots & \ddots & \vdots \\
\phi_n^I & \cdots & \phi_n^{I_n}
\end{pmatrix}
\xrightarrow{r \to \infty}
= \text{diagonal}(1, \ldots, 1)
\]

Set right hand side to zero:

General QNM condition

\[
det \mathcal{F}_{r \to \infty} = 0
\]

QNM corresponding to poles in holographic Green functions are zeroes of the determinant of field values at the AdS boundary for a maximal set of linearly independent solutions (infalling at horizon).

Advantages

- no explicit trafo to gauge-invariant fields needed
- elegant formulation including all fields simultaneously
- works for any coupled system of gravitational fluctuations
- need not compute full Green function (but possible)
- coupled EOMs along r dual to operator mixing along RG-flow, cutoff!
APPENDIX: Holographic Operator Mixing

\[
\Psi = \begin{pmatrix} 0 \\ \Psi_{+u} \\ 0 \\ \Psi_{+d} \end{pmatrix} + \begin{pmatrix} \Psi_{-u} \\ 0 \\ \Psi_{-d} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \Psi_{1u} \\ \Psi_{1d} \end{pmatrix} + \begin{pmatrix} \Psi_{2u} \\ \Psi_{2d} \\ 0 \\ 0 \end{pmatrix}
\]

\[
\gamma^u \Psi_\pm = \pm \Psi_\pm
\]

\[
\Psi_\pm = c_\pm(k) u^{\frac{d}{2}\pm m} + O\left(u^{\frac{d}{2}+1\pm m}\right)
\]

\[
S_{ren} = \int d^d x \bar{c}_+ c_-
\]

\[
\langle \mathcal{O} \bar{\mathcal{O}} \rangle_{ren} = -\frac{\delta^2 S_{ren}}{\delta c_- \delta \bar{c}_-} = -\frac{\delta c_+}{\delta c_-}.
\]

\[
\left[ \hat{P}_{\alpha j}^{-}(u, \omega, k) \right] = \begin{pmatrix} \Psi_{1}^{(1)} & \Psi_{2}^{(1)} & \cdots & \Psi_{N}^{(1)} \\ \Psi_{1}^{(2)} & \Psi_{2}^{(2)} & \cdots & \Psi_{N}^{(2)} \\ \cdots & \cdots & \cdots & \cdots \\ \Psi_{1}^{(N)} & \Psi_{2}^{(N)} & \cdots & \Psi_{N}^{(N)} \end{pmatrix},
\]

\[
G_{ab}^R(\omega, k) = \lim_{\epsilon \to 0} \left( P_+^+(\epsilon)_{aj} \ P_-^-(\epsilon)_{jb}^{-1} \right)
\]
APPENDIX: Conductivity

Conductivity:

\[ \sigma = \frac{J}{E} = \frac{A^{(2)}}{\partial_t A^{(0)}} \sim \frac{i A^{(2)}}{\omega A^{(0)}} = -\frac{i}{\omega} \frac{\rho^3 A'}{2 A} = \frac{i}{\omega} G^{\text{ret}}(\omega, q = 0) \]

with flavorelectric current \( J_m \leftrightarrow A_m \in U(1) \)

Only the first of these peaks was seen to second order in \( F \).

[compare: spikes in Horowitz’ model]

Our expansion to fourth order shows all peaks (zero quark \( m \)).

Peaks are higher order effect in \( F \).
APPENDIX: Higgs mechanism & Meissner effect

Peaks at finite mass:

b_0t = 1.0 and \( \chi_0 = 0.92 \)

Peaks in conductivity/spectral function approach
SUSY vector meson spectrum at large quark mass.

Finite magnetic field:

Add background field component: \( H_3^3 = F_{12}^3 = \partial_1 A_2^3 \)

Induced currents in SC phase with H